f(R) Theories of Gravity with Non-minimal Curvature-Matter Coupling

f(R) theories of gravity with non-minimal curvature-matter coupling

Implications

Energy conditions and stability

INSTITUTO SUPERIOR TÉCNICO

Orfeu Bertolami

Instituto Superior Técnico
Departamento de Física
(http://web.ist.utl.pt/orfeu.bertolami/homeorfeu.html)

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General Relativity

($\gamma = \beta = 1$)

- GR has survived all tests so far…

  [C. Will, gr-qc/0510072]
  [O.B., J. Páramos, S. Turyshev, gr-qc/0602016]

- Parametrized Post-Newtonian Formalism (U-gravitational potential, v_i velocity)

$$g_{00} = -1 + 2U - 2\beta U^2 + \ldots, \quad g_{ij} = (1 + 2\gamma U)\delta_{ij} + \ldots, \quad g_{0i} = -\frac{1}{2}(4\gamma + 3)v_i + \ldots$$

- Local (solar system) tests

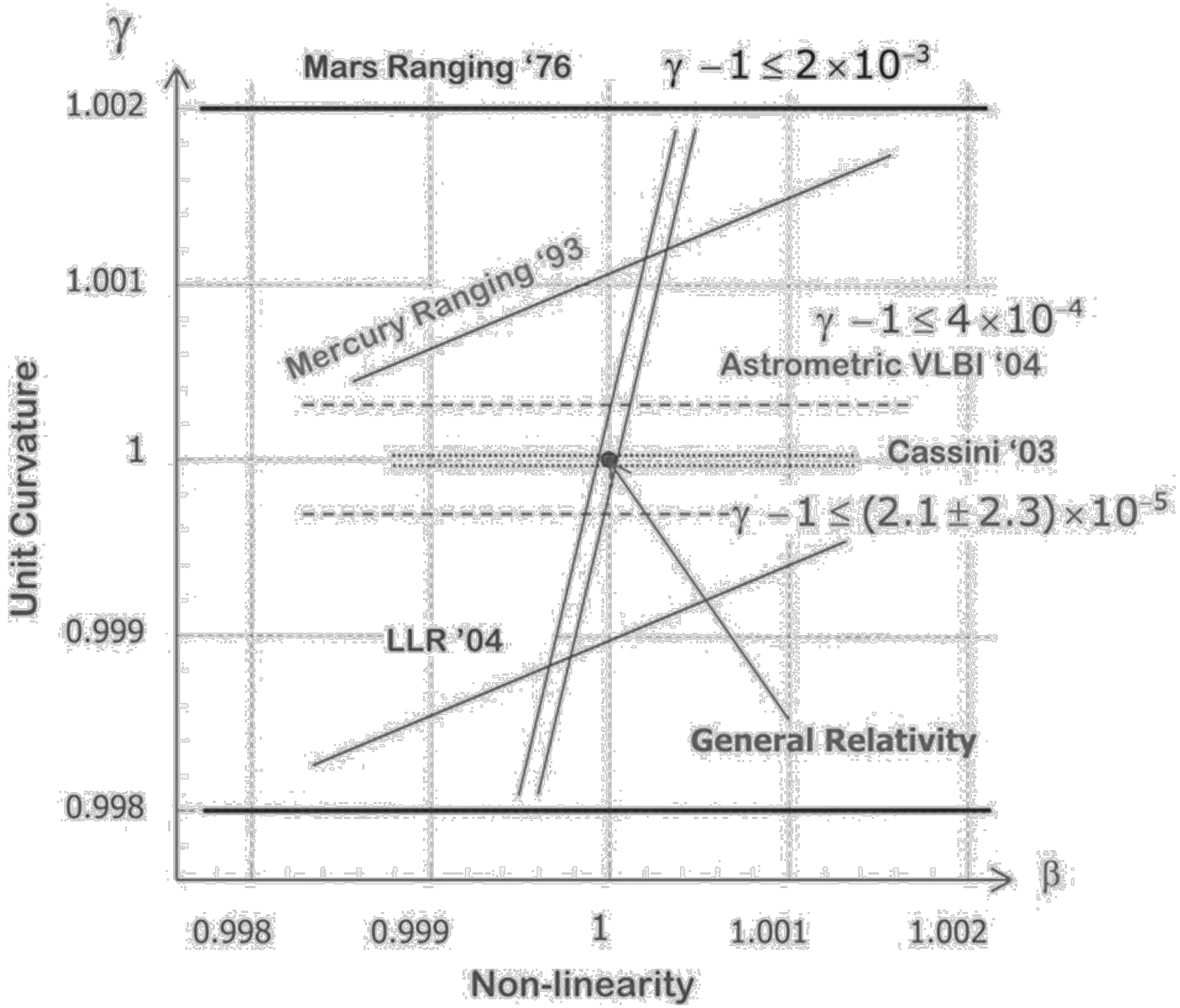
Mercury´s perihelion shift: $|2\gamma - \beta - 1| < 3 \times 10^{-3}$ [Shapiro 1990]

Lunar Laser Ranging: $4\beta - \gamma - 3 = (4.4 \pm 4.5) \times 10^{-4}$ [Williams, Turyshev, Boggs 2004]

LBLI light deflection: $|\gamma - 1| < 4 \times 10^{-4}$ [Eubanks et al. 1997]

Cassini Experiment: $\gamma - 1 = (2.1 \pm 2.3) \times 10^{-5}$ [Bertotti, Iess, Tortora 2003]

Summary of the General Relativity Tests



Cosmological Tests of General Relativity

- Outstanding challenges (GR + Quantum Field Theory)
  - Singularity Problem
  - Cosmological Constant Problem
  - Underlying particle physics theory for Inflation

- Theory provides in the context of the Big Bang model an impressive picture of the history of the Universe
  - Nucleosynthesis ($N_\nu < 4$, $\Omega_B h^2 = 0.023 \pm 0.001$)
  - Cosmic Microwave Background Radiation
  - Large Scale Structure
  - Gravitational lensing
  - …

- Required entities (missing links):
  - Dark Matter
  - Dark Energy

Dark Matter

- **Evidence:**

  Flatness of the rotation curve of galaxies
  Large scale structure
  Gravitational lensing
  N-body simulations and comparison with observations
  Merging galaxy cluster 1E 0657-56
  Massive Clusters Collision Cl 0024+17
  Dark core of the cluster A520

- **Cold Dark Matter (CDM) Model**

  Weakly interacting non-relativistic massive particle at decoupling

- **Candidates:**

  Neutralinos (SUSY WIMPS), axions, scalar fields, self-interacting scalar particles (adamastores), ***etc.***

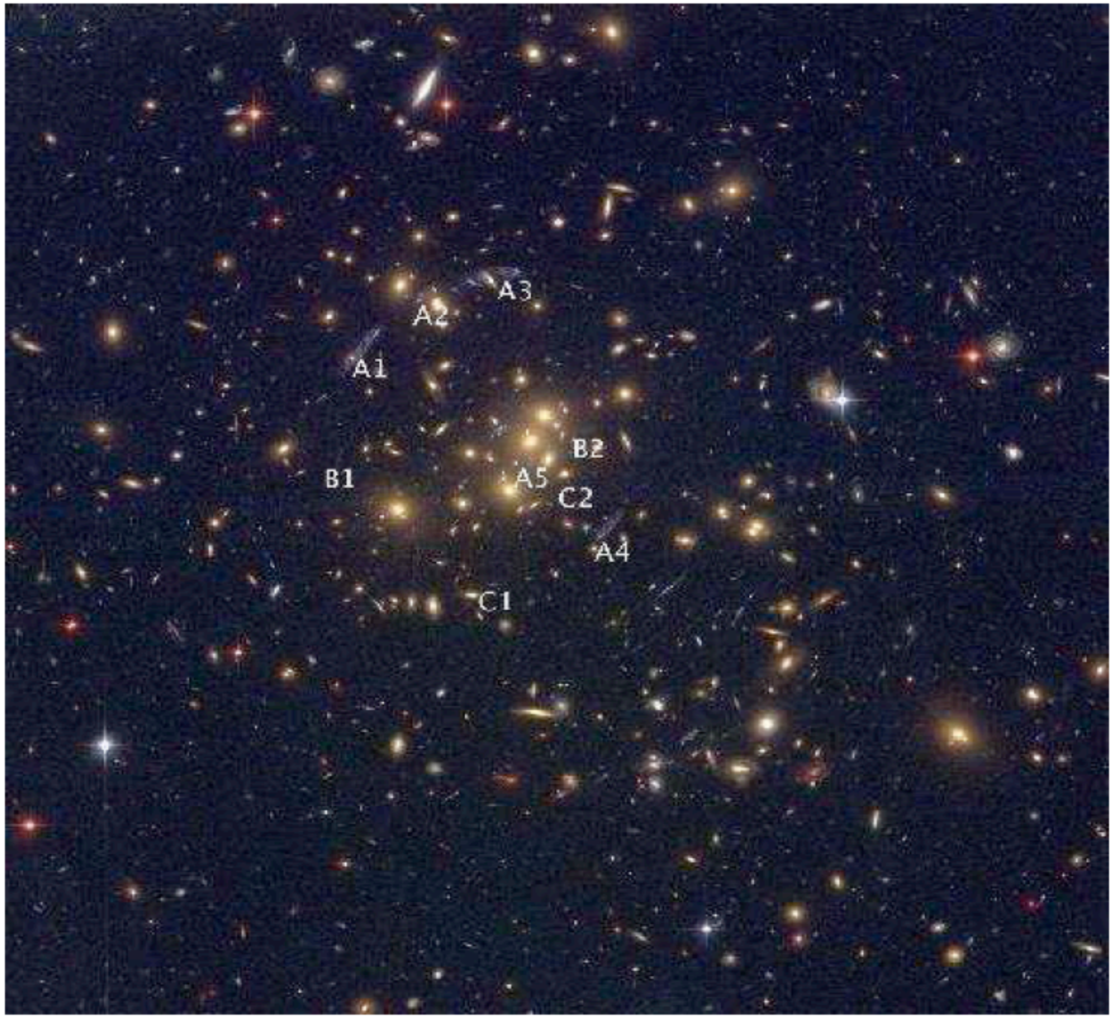
Merging Galaxy Cluster 1E 0657-56

[Clowe et al., astro-ph/0608407]

“Bullet” Cluster

Massive Clusters Collision Cl 0024+17

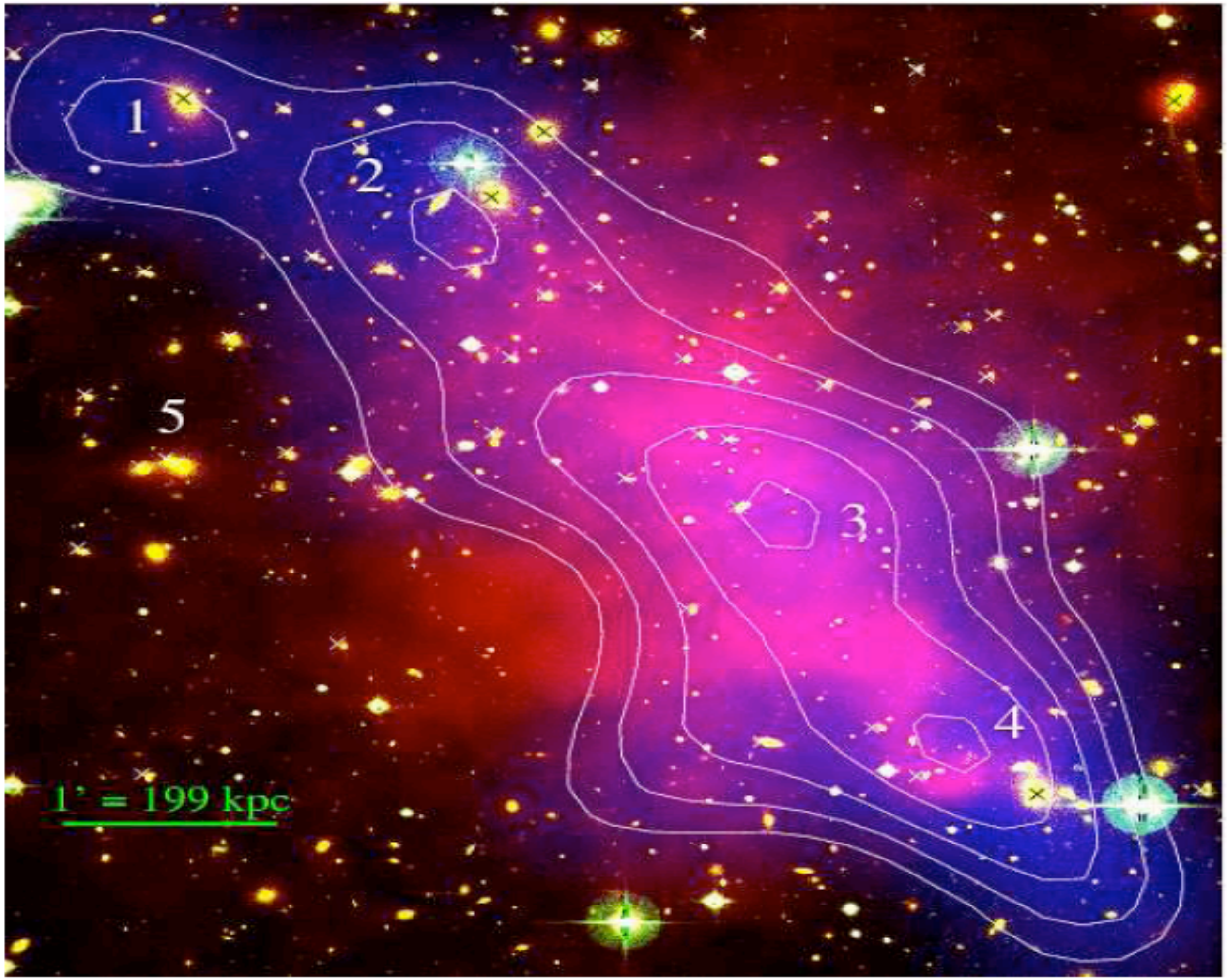
[Jee et al., astro-ph/0705.2171]



Ring-like dark matter structure

Dark core of the Abell 520

[Mahdavi et al., 0706.3048(astro-ph)]



Collisional dark matter ?

Self-Interacting Dark Matter

[Spergel, Steinhardt 2000]

Motivation: **“cuspy core” problem**

Model:

$$\mathcal{L} = \frac{1}{2}(\partial_\mu \phi)^2 - \frac{1}{2} m_\phi^2 \phi^2 - \frac{g}{4!}\phi^4 + g' v \phi^2 h$$

Higgs decay width

$$\Gamma(h \to \phi\phi) = 5.23 \left(\frac{m_h}{115\ \mathrm{GeV}}\right)^{-1} g'^2\ \mathrm{GeV}$$

[Bento, O.B., Rosenfeld, Teodoro 2000]
[Siveira, Zee 1988]
[Bento, O.B., Rosenfeld 2001]

Unified model for dark energy – dark matter: $g' \Phi^2 H^2$

[O.B., Rosenfeld 2008]

Dark Energy

- **Evidence:**

  Dimming of type Ia Supernovae with $z > 0.35$

  Accelerated expansion (negative deceleration parameter): $q_0 \equiv -\frac{\ddot{a}a}{\dot{a}^2} \leq -0.47$

  [Perlmutter et al. 1998; Riess et al. 1998, …]

- **Homogeneous and isotropic expanding geometry**

  Driven by the vacuum energy density Ω_Λ and matter density Ω_M

  Equation of state: $p = \omega\rho$ $\omega \leq 1$

- **Friedmann and Raychaudhuri equations imply:** $q_0 = \frac{1}{2}(3\omega + 1)\Omega_m - \Omega_\Lambda$

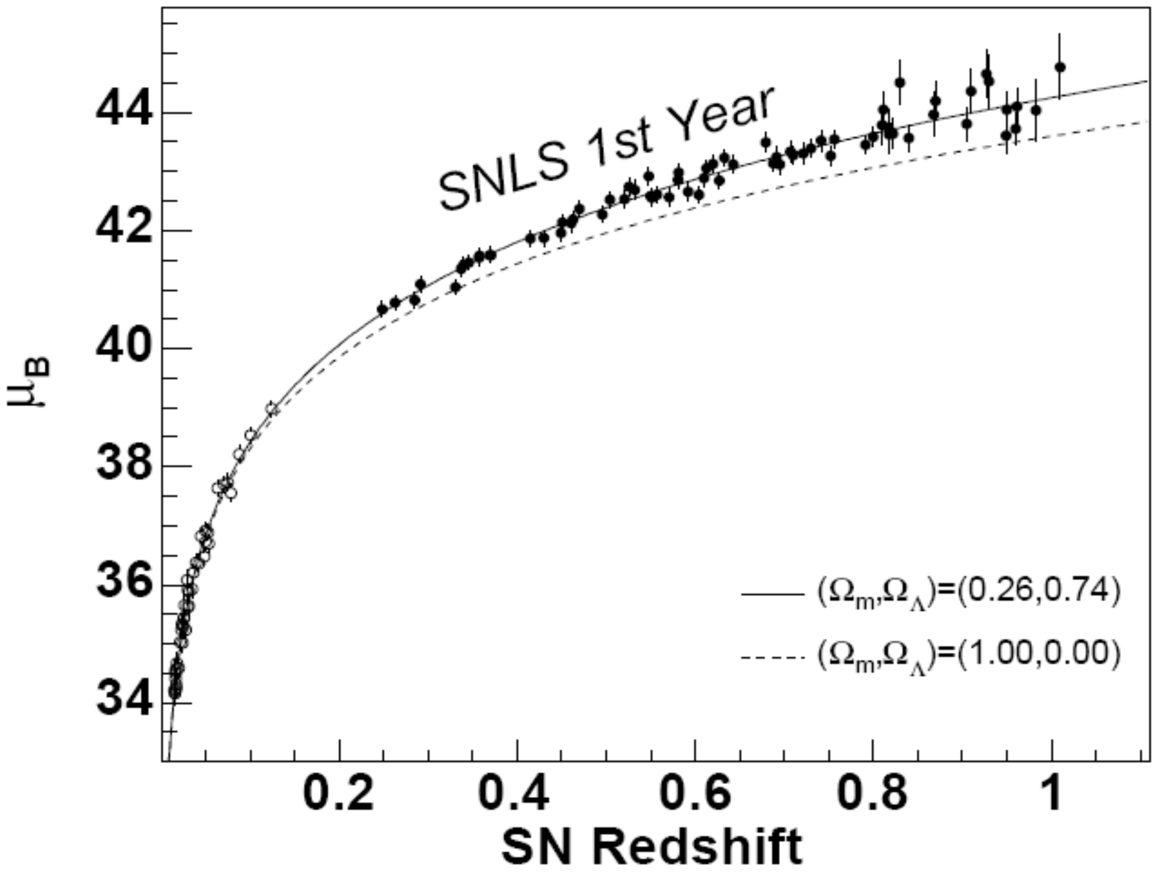
  $q_0 < 0$ suggests an invisible smooth energy distribution

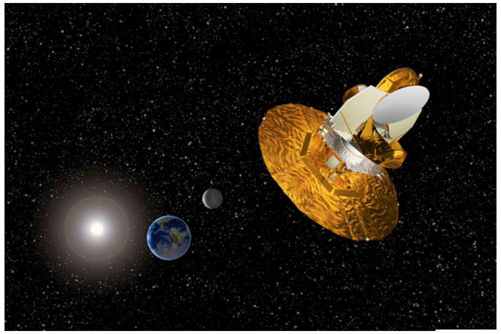
- **Candidates:**

  Cosmological constant, quintessence, more complex equations of state, etc.

Supernova Legacy Survey (SNLS)

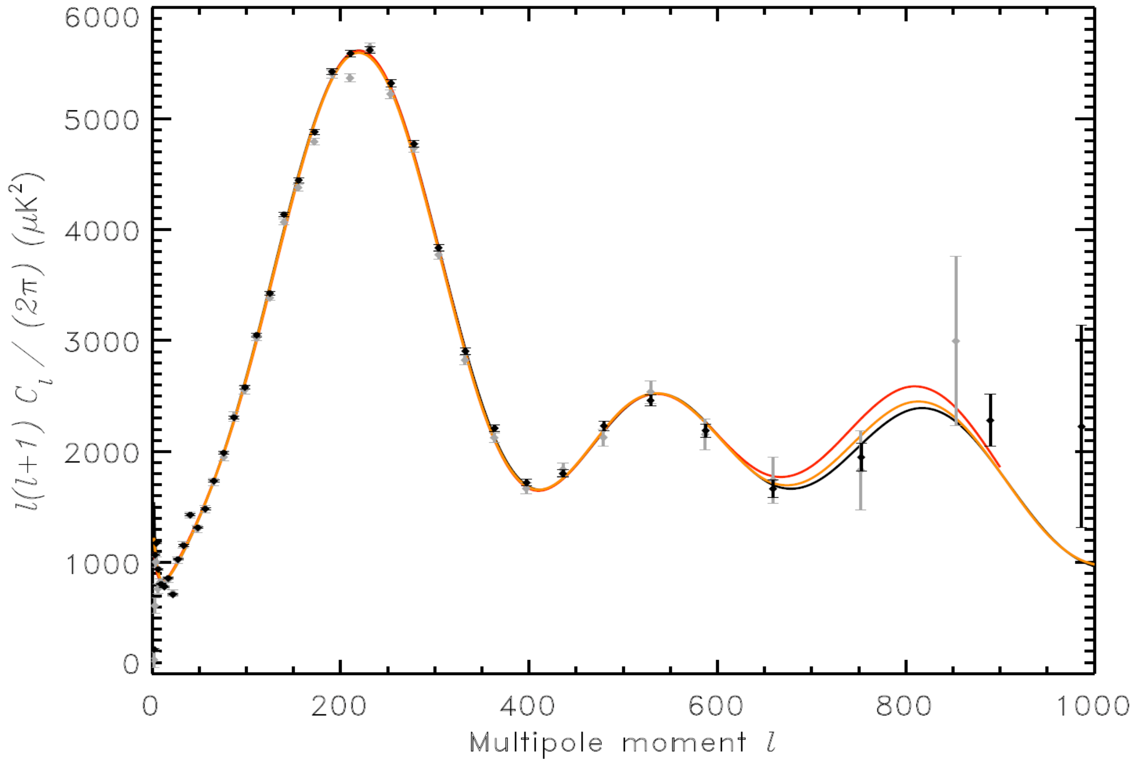
[Astier et al., astro-ph/0510447]





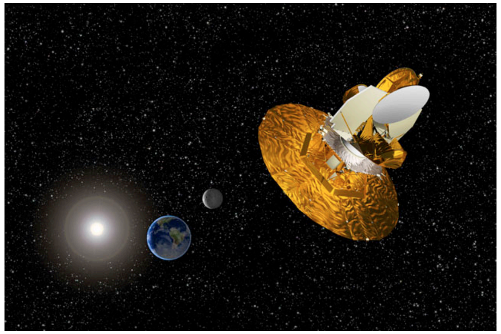
WMAP 3 Year Results

D.N. Spergel et al., astro-ph/0603449



$$(\Omega_m h^2, \Omega_b h^2, h, n_s, \tau, \sigma_8) \quad =$$

$$(0.127^{+0.007}_{-0.013}, 0.0223^{+0.0007}_{-0.0009}, 0.73^{+0.03}_{-0.03}, 0.951^{+0.015}_{-0.019}, 0.09^{+0.03}_{-0.03}, 0.74^{+0.05}_{-0.06})$$



WMAP 3 Year Results

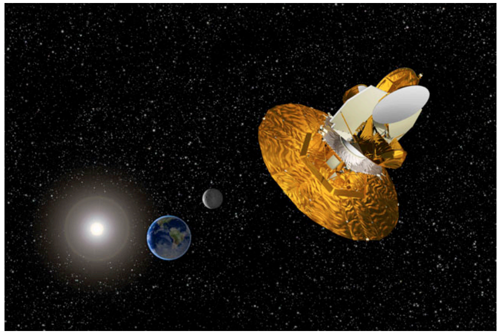
D.N. Spergel et al., astro-ph/0603449

ΛCDM Model

Parameter	WMAP+ SDSS	WMAP+ LRG	WMAP+ SNLS	WMAP + SN Gold	WMAP+ CFHTLS
$100\Omega_b h^2$	$2.233^{+0.062}_{-0.086}$	$2.242^{+0.062}_{-0.084}$	$2.233^{+0.069}_{-0.088}$	$2.227^{+0.065}_{-0.082}$	$2.255^{+0.062}_{-0.083}$
$\Omega_m h^2$	$0.1329^{+0.0056}_{-0.0075}$	$0.1337^{+0.0044}_{-0.0061}$	$0.1295^{+0.0056}_{-0.0072}$	$0.1349^{+0.0056}_{-0.0071}$	$0.1408^{+0.0034}_{-0.0050}$
h	$0.709^{+0.024}_{-0.032}$	$0.709^{+0.016}_{-0.023}$	$0.723^{+0.021}_{-0.030}$	$0.701^{+0.020}_{-0.026}$	$0.687^{+0.016}_{-0.024}$
A	$0.813^{+0.042}_{-0.052}$	$0.816^{+0.042}_{-0.049}$	$0.808^{+0.044}_{-0.051}$	$0.827^{+0.045}_{-0.053}$	$0.846^{+0.037}_{-0.047}$
τ	$0.079^{+0.029}_{-0.032}$	$0.082^{+0.028}_{-0.033}$	$0.085^{+0.028}_{-0.032}$	$0.079^{+0.028}_{-0.034}$	$0.088^{+0.026}_{-0.032}$
n_s	$0.948^{+0.015}_{-0.018}$	$0.951^{+0.014}_{-0.018}$	$0.950^{+0.015}_{-0.019}$	$0.946^{+0.015}_{-0.019}$	$0.953^{+0.015}_{-0.019}$
σ_8	$0.772^{+0.036}_{-0.048}$	$0.781^{+0.032}_{-0.045}$	$0.758^{+0.038}_{-0.052}$	$0.784^{+0.035}_{-0.049}$	$0.826^{+0.022}_{-0.035}$
Ω_m	$0.266^{+0.026}_{-0.036}$	$0.267^{+0.018}_{-0.025}$	$0.249^{+0.024}_{-0.031}$	$0.276^{+0.023}_{-0.031}$	$0.299^{+0.019}_{-0.025}$

WMAP 3 + SNLS: $w = -0.97^{+0.07}_{-0.09}$

$$\omega = \frac{p}{\rho}$$



WMAP 5 Year Results

E. Komatsu et al., 0803.0547 [astro-ph]

SUMMARY OF THE COSMOLOGICAL PARAMETERS OF ΛCDM MODEL AND THE CORRESPONDING 68% INTERVALS

Class	Parameter	*WMAP* 5-year ML[a]	*WMAP*+BAO+SN ML	*WMAP* 5-year Mean[b]	*WMAP*+BAO+SN Mean
Primary	$100\Omega_b h^2$	2.268	2.263	2.273 ± 0.062	2.265 ± 0.059
	$\Omega_c h^2$	0.1081	0.1136	0.1099 ± 0.0062	0.1143 ± 0.0034
	Ω_Λ	0.751	0.724	0.742 ± 0.030	0.721 ± 0.015
	n_s	0.961	0.961	$0.963^{+0.014}_{-0.015}$	$0.960^{+0.014}_{-0.013}$
	τ	0.089	0.080	0.087 ± 0.017	0.084 ± 0.016
	$\Delta^2_{\mathcal{R}}(k_0{}^e)$	2.41×10^{-9}	2.42×10^{-9}	$(2.41 \pm 0.11) \times 10^{-9}$	$(2.457^{+0.092}_{-0.093}) \times 10^{-9}$
Derived	σ_8	0.787	0.811	0.796 ± 0.036	0.817 ± 0.026
	H_0	72.4 km/s/Mpc	70.3 km/s/Mpc	$71.9^{+2.6}_{-2.7}$ km/s/Mpc	70.1 ± 1.3 km/s/Mpc
	Ω_b	0.0432	0.0458	0.0441 ± 0.0030	0.0462 ± 0.0015
	Ω_c	0.206	0.230	0.214 ± 0.027	0.233 ± 0.013
	$\Omega_m h^2$	0.1308	0.1363	0.1326 ± 0.0063	0.1369 ± 0.0037
	$z_{\rm reion}{}^f$	11.2	10.5	11.0 ± 1.4	10.8 ± 1.4
	$t_0{}^g$	13.69 Gyr	13.72 Gyr	13.69 ± 0.13 Gyr	13.73 ± 0.12 Gyr

Summary of the 95% confidence limits on deviations from the simple (flat, Gaussian, adiabatic, power-law) ΛCDM model

Section	Name	Type	*WMAP* 5-year	*WMAP*+BAO+SN
§ 3.2	Gravitational Wave[a]	No Running Ind.	$r < 0.43^b$	$r < 0.20$
§ 3.1.3	Running Index	No Grav. Wave	$-0.090 < dn_s/d\ln k < 0.019^c$	$-0.0728 < dn_s/d\ln k < 0.0087$
§ 3.4	Curvature[d]		$-0.063 < \Omega_k < 0.017^e$	$-0.0175 < \Omega_k < 0.0085^f$
	Curvature Radius[g]	Positive Curv.	$R_{\rm curv} > 12\ h^{-1}{\rm Gpc}$	$R_{\rm curv} > 23\ h^{-1}{\rm Gpc}$
		Negative Curv.	$R_{\rm curv} > 23\ h^{-1}{\rm Gpc}$	$R_{\rm curv} > 33\ h^{-1}{\rm Gpc}$
§ 3.5	Gaussianity	Local	$-9 < f_{NL}^{\rm local} < 111^h$	N/A
		Equilateral	$-151 < f_{NL}^{\rm equil} < 253^i$	N/A
§ 3.6	Adiabaticity	Axion	$\alpha_0 < 0.16^j$	$\alpha_0 < 0.067^k$
		Curvaton	$\alpha_{-1} < 0.011^l$	$\alpha_{-1} < 0.0037^m$
§ 4	Parity Violation	Chern-Simons[n]	$-5.9^\circ < \Delta\alpha < 2.4^\circ$	N/A
§ 5	Dark Energy	Constant w^o	$-1.37 < 1+w < 0.32^p$	$-0.11 < 1+w < 0.14$
		Evolving $w(z)^q$	N/A	$-0.38 < 1+w_0 < 0.14^r$
§ 6.1	Neutrino Mass[s]		$\sum m_\nu < 1.3\ {\rm eV}^t$	$\sum m_\nu < 0.61\ {\rm eV}^u$
§ 6.2	Neutrino Species		$N_{\rm eff} > 2.3^v$	$N_{\rm eff} = 4.4 \pm 1.5^w$ (68%)

[a] In the form of the tensor-to-scalar ratio, r, at $k = 0.002\ {\rm Mpc}^{-1}$

[b] Dunkley et al. (2008)

[c] Dunkley et al. (2008)

[d] (Constant) dark energy equation of state allowed to vary ($w \neq -1$)

[e] With the HST prior, $H_0 = 72 \pm 8$ km/s/Mpc. For $w = -1$, $-0.052 < \Omega_k < 0.013$ (95% CL)

[f] For $w = -1$, $-0.0181 < \Omega_k < 0.0071$ (95% CL)

[g] $R_{\rm curv} = (c/H_0)/\sqrt{|\Omega_k|} = 3/\sqrt{|\Omega_k|}\ h^{-1}{\rm Gpc}$

[h] Cleaned V+W map with $l_{\rm max} = 500$ and the *KQ75* mask, after the point source correction

[i] Cleaned V+W map with $l_{\rm max} = 700$ and the *KQ75* mask, after the point source correction

[j] Dunkley et al. (2008)

[k] In terms of the adiabaticity deviation parameter, $\delta_{adi}^{(c,\gamma)} = \sqrt{\alpha}/3$ (Eq. [39]), the axion-like dark matter and photons are found to obey the adiabatic relation (Eq. [36]) to 8.6%.

[l] Dunkley et al. (2008)

[m] In terms of the adiabaticity deviation parameter, $\delta_{adi}^{(c,\gamma)} = \sqrt{\alpha}/3$ (Eq. [39]), the curvaton-like dark matter and photons are found to obey the adiabatic relation (Eq. [36]) to 2.0%.

[n] For an interaction of the form given by $(\phi/M)F_{\alpha\beta}\tilde{F}^{\alpha\beta}$, the polarization rotation angle is $\Delta\alpha = M^{-1}\int \frac{dt}{a}\dot{\phi}$

[o] For spatially curved universes ($\Omega_k \neq 0$)

[p] With the HST prior, $H_0 = 72 \pm 8$ km/s/Mpc

[q] For a flat universe ($\Omega_k = 0$)

[r] $w_0 \equiv w(z=0)$

[s] $\sum m_\nu = 94(\Omega_\nu h^2)$ eV

[t] Dunkley et al. (2008)

[u] For $w = -1$. For $w \neq -1$, $\sum m_\nu < 0.66$ eV (95% CL)

[v] Dunkley et al. (2008)

[w] With the HST prior, $H_0 = 72 \pm 8$ km/s/Mpc. The 95% limit is $1.9 < N_{\rm eff} < 7.8$

Generalized Chaplygin gas model

- **Unified model for Dark Energy and Dark Matter**

Generalized d-brane

$$\mathcal{L} = -A^{\frac{1}{1+\alpha}} \left(1 - (g^{\mu\nu}\Theta_{,\mu}\Theta_{,\nu})^{\frac{\alpha+1}{2\alpha}}\right)^{\frac{\alpha}{\alpha+1}}$$

$\alpha = 1$: d-brane

Generalized Chaplygin gas

$$p = -\frac{A}{\rho^\alpha}$$

$\alpha = 1$: Chaplygin gas

$p \ll \rho$

Dust

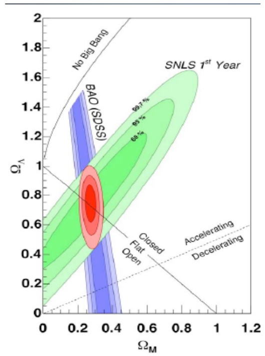
$p = \alpha\rho$

$\alpha = 1$: stiff matter

$p = -\rho$

De Sitter

[Bento, O.B., Sen 2002]



Dark Energy - Dark Matter Unification: Generalized Chaplygin Gas Model

- CMBR Constraints [Bento, O. B., Sen 2003, 2004; Amendola et al. 2004, Barreiro, O.B., Torres 2008]
- SNe Ia [O. B., Sen, Sen, Silva 2004; Bento, O.B., Santos, Sen 2005]
- Gravitational Lensing [Silva, O. B. 2003]
- Structure Formation *

  [Sandvik, Tegmark, Zaldarriaga, Waga 2004; Bento, O. B., Sen 2004; Bilic, Tupper, Viollier 2005; …]
- Gamma-ray bursts [O. B., Silva 2006]
- Cosmic topology [Bento, O. B., Rebouças, Silva 2006]
- Inflation [O.B., Duvvuri 2006]
- Coupling with electromagnetic coupling [Bento, O.B., Torres 2007]
- Coupling with neutrinos [Bernardini, O.B. 2007]

Background tests: $\alpha \leq 0.35, \quad 0.8 \leq A_s \leq 0.9 \qquad A_s \equiv \dfrac{A}{\rho_{Ch0}^{1+\alpha}}$

Structure formation and BAO: $\alpha \leq 0.2$

Modified Newtonian Dynamics (MOND)

f(R)

[O. Bertolami, Böhmer, Harko, Lobo, Phys. Rev. D 75 (2007) 104016]

[Milgrom 1983, Bekenstein, Milgrom 1984, ..., Bekenstein 2004]

- **Model:** $$S = \int \left\{ \frac{1}{2} f_1(R) + [1 + \lambda f_2(R)]\, \mathcal{L}_m \right\} \sqrt{-g}\; d^4x$$

$f_i(R)$ – arbitrary functions of R ($F_i = \dfrac{df_i(R)}{dR}$)

- **Energy-momentum tensor of matter is not necessarily conserved:**

$$\nabla^\mu T^{(m)}_{\mu\nu} = \frac{\lambda F_2}{1 + \lambda f_2} \left[g_{\mu\nu} \mathcal{L}_m - T^{(m)}_{\mu\nu} \right] \nabla^\mu R$$

- **Motion is non-geodesic:** $$\frac{Du^\alpha}{ds} \equiv \frac{du^\alpha}{ds} + \Gamma^\alpha_{\mu\nu} u^\mu u^\nu = f^\alpha$$

- **For a perfect fluid:** $$T^{(m)}_{\mu\nu} = (\epsilon + p)\, u_\mu u_\nu - p g_{\mu\nu}$$

$$f^\alpha = \frac{1}{\epsilon + p} \left[\frac{\lambda F_2}{1 + \lambda f_2} (\mathcal{L}_m + p) \nabla_\nu R + \nabla_\nu p \right] h^{\alpha\nu} \quad h_{\mu\lambda} = g_{\mu\lambda} - u_\mu u_\lambda$$

Dark Matter

Modified Newtonian Dynamics (MOND)

[Milgrom 1983, Bekenstein, Milgrom 1984, ..., Bekenstein 2004]

Motivation: Flatness Rotation Curve of Galaxies

$$\vec{a} = \mu\left(\frac{|\vec{g}|}{a_0}\right)\vec{g} = -\mu\left(\frac{|\vec{g}|}{a_0}\right)\nabla\phi$$

$$\mu(x) = \begin{cases} 1 \;\; if \;\; x \gg 1 \\ x \;\; if \;\; x \ll 1 \end{cases}$$

$a_0 \approx 1.2 \times 10^{-10}$ m/s² **- universal acceleration**

Tully-Fisher Law: $L_H \propto v_c^4$ **as** $L_H \propto M = (Ga_0)^{-1} v_c^4$

TeVeS² version: F-function problem

MOND

Tensor-Vector-Scalar field theory, $S = S_g + S_s + S_v + S_m$:

$$S_g = (16\pi G)^{-1} \int g^{\alpha\beta} R_{\alpha\beta} (-g)^{1/2} d^4x$$

$$S_s = -\frac{1}{2} \int \left[\sigma^2 h^{\alpha\beta} \phi_{,\alpha} \phi_{,\beta} + \frac{1}{2} G \ell^{-2} \sigma^4 F(kG\sigma^2) \right] (-g)^{1/2} d^4x$$

$$S_v = -\frac{K}{32\pi G} \int \left[g^{\alpha\beta} g^{\mu\nu} \mathfrak{U}_{[\alpha,\mu]} \mathfrak{U}_{[\beta,\nu]} - 2(\lambda/K)(g^{\mu\nu} \mathfrak{U}_\mu \mathfrak{U}_\nu + 1) \right] (-g)^{1/2} d^4x$$

$$S_m = \int \mathcal{L}(\tilde{g}_{\mu\nu}, f^\alpha, f^\alpha{}_{|\mu}, \cdots)(-\tilde{g})^{1/2} d^4x$$

Conformal transformation to the physical metric: $(-\tilde{g})^{1/2} = e^{-2\phi}(-g)^{1/2}$

Consistency

- **PPN:** $\beta = 1, \gamma = 1$ **(see however O.B., Páramos 2006)**

  i) (Potentially) compatible

  [Skordis, Mota, Ferreira, Boehm 2005]

- **CMBR**

  ii) Problem with the third peak $\frac{P_{\Lambda CDM}}{P_{MOND}} \cong 2 \times 10^2$

  [Slosar, Melchiorri, Silk 2005]

- **Gravitational lensing – great potential for testing**

  [Zhao, Bacon, Taylor, Horne 2005]

MOND in Post-Newtonian regime

Scalar field: $\phi(r) = \phi_c - \frac{kGm}{4\pi r}$

Vector field:

$$\left(\mathfrak{U}^{[\alpha;\beta]}{}_{;\beta} + \mathfrak{U}^\alpha \mathfrak{U}_\gamma \mathfrak{U}^{[\gamma;\beta]}{}_{;\beta}\right) + 8\pi G\sigma^2 \left[\mathfrak{U}^\beta \phi_{,\beta}\, g^{\alpha\gamma} \phi_{,\gamma} + \mathfrak{U}^\alpha (\mathfrak{U}^\beta \phi_{,\beta})^2\right]$$
$$= 8\pi G(1 - e^{-4\phi})\left[g^{\alpha\mu}\mathfrak{U}^\beta \tilde{T}_{\mu\beta} + \mathfrak{U}^\alpha \mathfrak{U}^\beta \mathfrak{U}^\gamma \tilde{T}_{\gamma\beta}\right]$$

- **Timelike vector tracks the metric** [Bekenstein 2004]

$$\mathfrak{U}^\alpha = (\sqrt{-g^{00}}, 0, 0, 0)$$ **consistent with eq. of motion**

- **Einstein eq.**

$$G_{\alpha\beta} = 8\pi G\left[\tilde{T}_{\alpha\beta} + (1 - e^{-4\phi})\mathfrak{U}^\mu \tilde{T}_{\mu(\alpha}\mathfrak{U}_{\beta)} + \tau_{\alpha\beta}\right] + \Theta_{\alpha\beta}$$

$$\tau_{\alpha\beta} \equiv \sigma^2\left[\phi_{,\alpha}\phi_{,\beta} - \frac{1}{2}g^{\mu\nu}\phi_{,\mu}\phi_{,\nu}\, g_{\alpha\beta} - \mathfrak{U}^\mu \phi_{,\mu}\left(\mathfrak{U}_{(\alpha}\phi_{,\beta)} - \frac{1}{2}\mathfrak{U}^\nu \phi_{,\nu}\, g_{\alpha\beta}\right)\right]$$

$$\Theta_{\alpha\beta} \equiv K\left(g^{\mu\nu}\mathfrak{U}_{[\mu,\alpha]}\mathfrak{U}_{[\nu,\beta]} - \frac{1}{4}g^{\sigma\tau}g^{\mu\nu}\mathfrak{U}_{[\sigma,\mu]}\mathfrak{U}_{[\tau,\nu]}\, g_{\alpha\beta}\right) - \lambda \mathfrak{U}_\alpha \mathfrak{U}_\beta$$

- **Parametrization of the metric**

$$g_{\alpha\beta}\, dx^{\alpha} dx^{\beta} = -e^{\nu} dt^2 + e^{\varsigma}[d\varrho^2 + \varrho^2 (d\theta^2 + \sin^2\theta d\varphi^2)]$$

$$-g_{00} = e^{\nu} = 1 - R/r + \alpha_2 (R/r)^2 + ...$$

$$g_{rr} = e^{\sigma} = 1 + \beta_1 R/r + \beta_2 (R/r)^2 + ...$$

- **Expansion of Einstein eq. up to order *r* $^{-4}$**

$$\lambda = \frac{K(2+\beta_1-4\alpha_2)}{4}\frac{R^2}{r^4} \qquad 8\pi G\tau_{00} = 8\pi G\tau_{rr} = \frac{kR^2}{16\pi r^4}$$

$$\theta_{00} = \frac{K(-2\beta_1-3+8\alpha_2)}{8}\frac{R^2}{r^4} \quad , \quad \theta_{rr} = -\frac{K}{8}\frac{R^2}{r^4}$$

- **Solution:** $\beta_1 = 1 \;\; , \;\; \alpha_2 = \frac{1}{2} \;\; , \;\; \beta_2 = \frac{3}{8} + \frac{1}{16}K - \frac{k}{32\pi}\left(\frac{R}{r}\right)^2$

- **Transformation into physical, isotropic PPN metric yields**

$$\beta = 1 \quad , \quad \gamma = 1$$

(like GR !)

Dynamical solution for the vector field

- **Assume** $\mathfrak{U}^\alpha = (\mathfrak{U}^0(r), \mathfrak{U}^r(r), 0, 0) \quad , \quad \mathfrak{U}^\alpha \mathfrak{U}_\alpha = -1$
- **Solve eq. of motion of the vector field**

$$\left[r^2\sqrt{-\frac{g_{rr}}{g_{00}}}\,\mathfrak{U}_0'\right]' = Ar^2\sqrt{-\frac{g_{rr}}{g_{00}}}\phi'^2\mathfrak{U}_0 = \frac{C}{r^2\sqrt{-g_{00}^3 g_{rr}}}\mathfrak{U}_0$$

with the expansion $\mathfrak{U}_0(r) = -1 + \gamma_1 \frac{R}{r} + \gamma_2 \left(\frac{R}{r}\right)^2$

- **Solve for γ_1 and γ_2 and expand quantities in Einstein eq.**

$$8\pi G\tau_{00} \approx 8\pi G\tau_{rr} = \frac{kR^2}{16\pi r^4} \quad , \quad \lambda \approx [\gamma_1(1+\beta_1) + 4\gamma_2]\frac{KR^2}{4r^4}$$

$$\Theta_{00} \approx -[(1+\beta_1-\gamma_1)\gamma_1 + 4\gamma_2]\frac{KR^2}{2r^4} \quad , \quad \Theta_{rr} \approx -\gamma_1^2\frac{KR^2}{2r^4}$$

- **Solution:**

$$\alpha_2 = \frac{1}{2} + \frac{k}{16\pi} + \frac{K}{2\left(1+9\frac{K\pi}{k}\right)^2} \quad , \quad \beta_1 = 1 \quad , \quad \beta_2 = \frac{3}{8} - \frac{3k}{32\pi} + \frac{K}{4\left(1+9\frac{K\pi}{k}\right)^2}$$

• **Transformation into physical, isotropic PPN metric yields:**

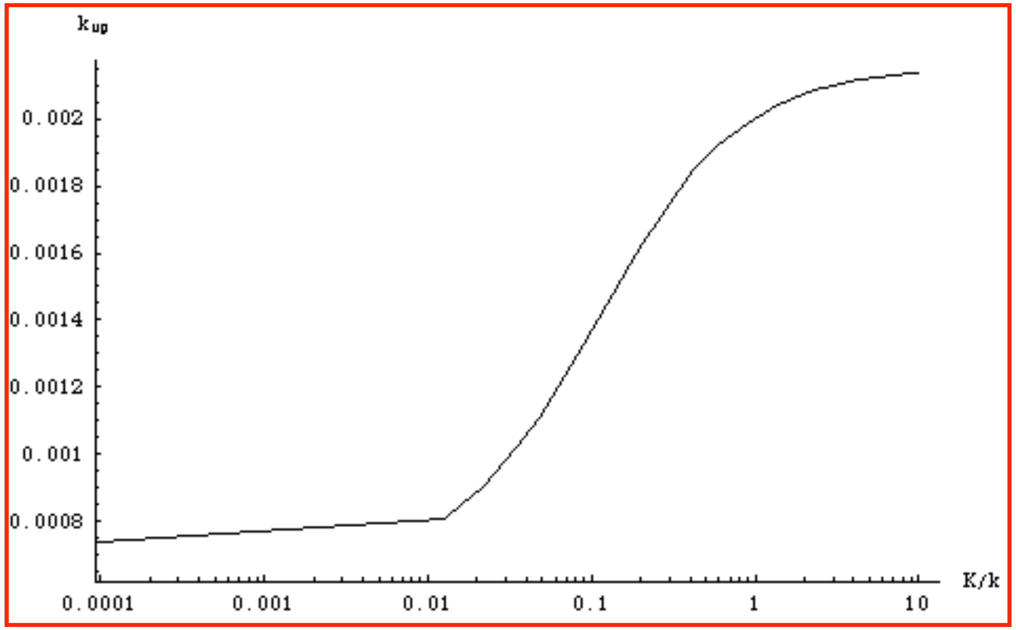
$$g_{00} = -1 + 2\frac{G_N m}{r} - 2\left[2\alpha_2 + \left(2\gamma_1 - \alpha_2 - \frac{1}{2}\right)\frac{k}{\pi}\right]\left(\frac{G_N m}{r}\right)^2$$

$$g_{rr} = 1 + 2\frac{G_N R}{r} \quad , \quad g_{0r} = -\frac{\sqrt{2(1-2\gamma_1)}k}{\pi}\left(\frac{G_N m}{r}\right)^{3/2}$$

• **Gives rise to a non-diagonal component and**

$$\beta = 1 + \frac{K}{\left(1 + 9\frac{K\pi}{k}\right)^2} - \frac{k}{\pi}\left(\frac{7}{8} + \frac{2}{1 + 9\frac{K\pi}{k}}\right) \quad , \quad \gamma = 1$$

(different from GR!)



Constraint $|\beta - 1| < 6 \times 10^{-4}$

allows for $k < k_{up}$

[Giannios 2005]
[O.B., Páramos 2006]

Can MOND take a bullet ?

[Angus, Shan, Zhao, Famaey 2006]

- **Dark halo made of neutrinos:** $m_\nu = (2-3)eV$
- **Not quite !** [Takahashi, Chiba, astro-ph/0701365]

Neutrino oscillations: $\Delta m_\nu{}^2 \leq 10^{-3}\, eV^2$

Tremaine-Gunn bound: $\rho_{\nu Max.} = 4.8\times 10^{-27}\left(\frac{m_\nu}{2eV}\right)^4\left(\frac{T_X}{keV}\right)^{3/2} g/cm^3$

Core density (Hernquist profile): $M(<r) = \frac{M_0 r^2}{(r+r_0)^2}$ $\rho_{core} = \frac{3M(<r_0)}{4\pi r^3}$

$$\rho_{core} < \rho_{\nu Max.}: \quad m_\nu > 6.1\left(\frac{M_0}{10^{14} M_{sun}}\right)^{1/4}\left(\frac{r_0}{100kpc}\right)^{-3/4}\left(\frac{T_X}{keV}\right)^{-3/8} eV$$

A1689: $M_0 = (6.2\pm 1.2)\times 10^{14} M_{sun}$ $r_0 = (125\pm 52)kpc$ $T_X = (9.00\pm 0.13)keV$

$$m_\nu > (3.6\pm 1.1)eV$$

New f(R) modified theory of gravity

[O.B., Böhmer, Harko, Lobo, Phys. Rev. D 75 (2007) 104016]

$$S = \int \left[\frac{1}{2} f_1(R) + f_2(R) \mathcal{L}_m \right] \sqrt{-g} d^4 x \,,$$

- **Implications:**

- **MOND-like behaviour: extra force and Tully-Fisher law** ($L \sim v_\infty^4$)

If $a_N \ll a$: $\vec{a}_N \approx \frac{a}{a_E} \vec{a}$ $\qquad$ $\frac{1}{a_E} \equiv \frac{1}{2f}\left(1 - \frac{f^2}{a^2}\right)$

Hence $a \approx \sqrt{a_E a_N}$ **and as** $a_N = GM/r^2$ **it follows that**

$$a \approx \sqrt{a_E GM}/r = v_{tg}^2/r \qquad v_{tg}^2 \rightarrow v_\infty^2 = \sqrt{a_E GM}$$

and the Tully-Fisher law as L~M

New f(R) modified theory of gravity

[O.B., Böhmer, Harko, Lobo, Phys. Rev. D 75 (2007) 104016]

- Implications:

- Pioneer-like acceleration:

$$a_E = \frac{f^2 r^2}{GM} + 2f \qquad f \sim GM\alpha/r$$

$$f \to 0, \; a_E \approx \alpha^2 \qquad \alpha - const.$$

[Anderson, Laing, Lau, Liu, Nieto, Turyshev 2002]

[O.B., Páramos 2004]

- However, most likely the Pioneer anomalous acceleration is due to on-board thermal effects

[O.B., Francisco, Gil, Páramos 2008]

New f(R) modified theory of gravity

[O.B., Böhmer, Harko, Lobo, Phys. Rev. D 75 (2007) 104016]

- Stellar stability
  [O.B., Páramos, Phys. Rev. D77 (2008)]

- On the non-trivial gravitational coupling to matter
  [O.B., Páramos, Class. Quant. Grav. 25 (2008)]

- Non-minimal coupling of perfect fluids to curvature (Poster)
  [O.B., Lobo, Páramos, Phys. Rev. D78 (2008)]

- Non-minimal curvature-matter couplings in modified gravity (Review)
  [O.B., Páramos, Harko, Lobo, arXiv:0811.2876 [gr-qc]]

- Energy Conditions and Stability in f(R) theories of gravity with non-minimal coupling to matter
  [O.B., Sequeira, Phys. Rev. B79 (2009)]

- Mimicking dark matter through a non-minimal gravitational coupling with matter
  [O.B., Páramos, arXiv:0906.4757 [astro-ph.GA]]

Energy Conditions and Stability

[O.B., Sequeira, Phys. Rev. B79 (2009)]

- **Physical Viability**

  - Match GR Parametrized Post-Newtonian behaviour at solar system
    [O.B., Páramos, Class. Quant. Grav. 25 (2008)]

  - Can lead to a phenomenologically consistent cosmology if
    Energy Conditions are satisfied:

    Strong Energy Condition (SEC) (Gravity is attractive)

    Null Energy Condition (NEC) (Gravity is attractive)

    Dominant Energy Condition (DEC) ($v_{sound} \leq c$)

    Weak Energy Condition (WEK) (Positive energy density)

  - Instability Free

    Dolgov-Kawasaki instability

-Ghost free, well posed Cauchy problem, correct cosmological perturbations, ...

Action and Field equations

Action:

$$S=\int\left[\frac{1}{2}f_1(R)+f_2(R)\mathcal{L}_m\right]\sqrt{-g}d^4x\,,$$

Field equations:

$$(f_1'+2\mathcal{L}_m f_2')R_{\mu\nu}-\frac{1}{2}f_1 g_{\mu\nu}-\Delta_{\mu\nu}(f_1'+2\mathcal{L}_m f_2')=f_2 T_{\mu\nu}$$

$$\Delta_{\mu\nu}=\nabla_\mu\nabla_\nu-g_{\mu\nu}g^{\alpha\beta}\nabla_\alpha\nabla_\beta$$

Effective energy-momentum tensor non-conservation:

$$\nabla^\mu T_{\mu\nu}=\frac{f_2'}{f_2}\left[g_{\mu\nu}\mathcal{L}_m-T_{\mu\nu}\right]\nabla^\mu R\,.$$

Eq. motion test particle:
(Perfect fluid)

$$u^\nu\nabla_\nu u^\lambda=\frac{1}{\epsilon+p}\left(\frac{f_2'}{f_2}(\mathcal{L}_m+p)\nabla_\nu R+\nabla_\nu p\right)h^{\nu\lambda}$$
$$\equiv f^\lambda\,.$$

Action and Field equations

Field equations:

$$(f_1' + 2\mathcal{L}_m f_2')R_{\mu\nu} - \frac{1}{2} f_1 g_{\mu\nu} - \Delta_{\mu\nu}(f_1' + 2\mathcal{L}_m f_2') = f_2 T_{\mu\nu}$$

$$G_{\mu\nu} = \hat{k}\left(\hat{T}_{\mu\nu} + T_{\mu\nu}\right)$$

Effective energy-momentum tensor:

$$\hat{T}_{\mu\nu} = \frac{1}{2}\left(\frac{f_1}{f_2} - \frac{f_1' + 2\mathcal{L}_m f_2'}{f_2} R\right) g_{\mu\nu} + \frac{1}{f_2}\Delta_{\mu\nu}\left(f_1' + 2\mathcal{L}_m f_2'\right)$$

Effective gravitational coupling: $\hat{k} = \dfrac{f_2}{f_1' + 2\mathcal{L}_m f_2'}$

Effective quantities

Perfect fluid:

$$T_{\mu\nu} = (\rho + p)u_\mu u_\nu - p g_{\mu\nu}$$

Robertson-Walker metric:

$$ds^2 = dt^2 - a^2(t) ds_3^2$$

$$\Delta_{\mu\nu} h(R, \mathcal{L}_m) = (\nabla_\mu \nabla_\nu - g_{\mu\nu}\Box) h(R, \mathcal{L}_m)$$

$$= (\partial_\mu \partial_\nu - g_{\mu\nu}\partial_0\partial_0)h - (\Gamma^0_{\mu\nu} + g_{\mu\nu} 3H)\partial_0 h$$

Effective energy-density:

$$\hat{\rho} = \frac{1}{2}\left(\frac{f_1}{f_2} - \frac{f_1' + 2\mathcal{L}_m f_2'}{f_2} R\right) - 3H \frac{f_1'' + 2\mathcal{L}_m f_2''}{f_2} \dot{R}$$

Effective pressure:

$$\hat{p} = -\frac{1}{2}\left(\frac{f_1}{f_2} - \frac{f_1' + 2\mathcal{L}_m f_2'}{f_2} R\right) + (\ddot{R} + 2H\dot{R}) \frac{f_1'' + 2\mathcal{L}_m f_2''}{f_2} + \frac{f_1''' + 2\mathcal{L}_m f_2'''}{f_2} \dot{R}^2$$

Effective gravitational coupling:

$$\hat{k} = \frac{f_2}{f_1' + 2\mathcal{L}_m f_2'} > 0$$

Kinematical Quantities

- **Flat Robertson-Walker metric**

$$R = -6\left(H^2 + \frac{\ddot{a}}{a}\right)$$

- **Deceleration (q), jerk (j), snap (s) parameters**

$$q = -\frac{1}{H^2}\frac{\ddot{a}}{a}, \quad j = \frac{1}{H^3}\frac{\dddot{a}}{a}, \quad s = \frac{1}{H^4}\frac{\ddddot{a}}{a}$$

$$\dot{R} = -6H^3(j - q - 2), \quad \ddot{R} = -6H^4(s + q^2 + 8q + 6),$$

Energy Conditions

- Raychaudhuri eq. for the expansion parameter for a congruence of timelike geodesics

$$\frac{d\theta}{d\tau} = -\frac{1}{3}\theta^2 - \sigma_{\mu\nu}\sigma^{\mu\nu} + \omega_{\mu\nu}\omega^{\mu\nu} - R_{\mu\nu}u^\mu u^\nu$$

- Raychaudhuri eq. for a congruence of null geodesics

$$\frac{d\theta}{d\tau} = -\frac{1}{2}\theta^2 - \sigma_{\mu\nu}\sigma^{\mu\nu} + \omega_{\mu\nu}\omega^{\mu\nu} - R_{\mu\nu}k^\mu k^\nu$$

- Condition for attractive gravity

$$\frac{d\theta}{d\tau} < 0$$

Energy Conditions

- **SEC** $R_{\mu\nu}u^\mu u^\nu \geq 0$
- **NEC** $R_{\mu\nu}k^\mu k^\nu \geq 0$

Warrant that gravity is geometrically attractive

$$G_{\mu\nu} = \hat{k}\left(\hat{T}_{\mu\nu} + T_{\mu\nu}\right) \qquad R_{\mu\nu} = \hat{k}\left(\hat{T}_{\mu\nu} + T_{\mu\nu} - \frac{1}{2}g_{\mu\nu}(\hat{T} + T)\right)$$

- **SEC**

$$\rho + 3p - \left(\frac{f_1}{f_2} - \frac{f_1' + 2\mathcal{L}_m f_2'}{f_2}R\right) + 3(\ddot{R} + H\dot{R})\frac{f_1'' + 2\mathcal{L}_m f_2''}{f_2} + 3\frac{f_1''' + 2\mathcal{L}_m f_2'''}{f_2}\dot{R}^2 \geq 0$$

- **NEC**

$$\rho + p + (\ddot{R} - H\dot{R})\frac{f_1'' + 2\mathcal{L}_m f_2''}{f_2} + \frac{f_1''' + 2\mathcal{L}_m f_2'''}{f_2}\dot{R}^2 \geq 0$$

Energy Conditions

- **DEC**

$$\rho - p + \left(\frac{f_1}{f_2} - \frac{f_1' + 2\mathcal{L}_m f_2'}{f_2} R\right) - (\ddot{R} + 5H\dot{R})\frac{f_1'' + 2\mathcal{L}_m f_2''}{f_2} - \frac{f_1''' + 2\mathcal{L}_m f_2'''}{f_2}\dot{R}^2 \geq 0$$

- **WEC**

$$\rho + \frac{1}{2}\left(\frac{f_1}{f_2} - \frac{f_1' + 2\mathcal{L}_m f_2'}{f_2} R\right) - 3H\frac{f_1'' + 2\mathcal{L}_m f_2''}{f_2}\dot{R} \geq 0$$

Transformations

(GR → f(R) theory with non-minimal coupling)

$$\rho \rightarrow \rho + \hat{\rho} \qquad\qquad p \rightarrow p + \hat{p}$$

Energy Conditions

- **Models**

$$f_1(R) = R + \epsilon R^n \qquad\qquad f_2(R) = 1 + \lambda R^m$$

- **Energy conditions:**

$$\frac{\hat{\epsilon}|R|^n}{1+\hat{\lambda}|R|^m}\left(a - \alpha_n - \frac{2\hat{\lambda}\mathcal{L}_m}{\hat{\epsilon}}\alpha_m|R|^{m-n}\right) \geq b$$

$$\hat{\epsilon} = (-1)^n\epsilon \qquad\qquad \hat{\lambda} = (-1)^m\lambda$$

Energy Conditions

- **Results**

$$f_1(R) = R + \epsilon R^n \qquad\qquad f_2(R) = 1 + \lambda R^m$$

$$\frac{\hat{\epsilon}|R|^n}{1+\hat{\lambda}|R|^m}\left(a - \alpha_n - \frac{2\hat{\lambda}\mathcal{L}_m}{\hat{\epsilon}}\alpha_m |R|^{m-n}\right) \geq b$$

b=0		$\hat{\epsilon} > 0$	$\hat{\epsilon} < 0$
$\hat{\lambda} > 0$		$\frac{2\hat{\lambda}\mathcal{L}_m}{\hat{\epsilon}} \leq \frac{a-\alpha_n}{\alpha_m}\|R\|^{n-m}$	$\frac{2\hat{\lambda}\mathcal{L}_m}{\|\hat{\epsilon}\|} \leq \frac{\alpha_n-a}{\alpha_m}\|R\|^{n-m}$
$\hat{\lambda} < 0$	$1-\|\hat{\lambda}\|\|R\|^m > 0$	$\frac{2\|\hat{\lambda}\|\mathcal{L}_m}{\hat{\epsilon}} \geq \frac{\alpha_n-a}{\alpha_m}\|R\|^{n-m}$	$\frac{2\|\hat{\lambda}\|\mathcal{L}_m}{\|\hat{\epsilon}\|} \geq \frac{a-\alpha_n}{\alpha_m}\|R\|^{n-m}$
	$1-\|\hat{\lambda}\|\|R\|^m < 0$	$\frac{2\|\hat{\lambda}\|\mathcal{L}_m}{\hat{\epsilon}} \leq \frac{\alpha_n-a}{\alpha_m}\|R\|^{n-m}$	$\frac{2\|\hat{\lambda}\|\mathcal{L}_m}{\|\hat{\epsilon}\|} \leq \frac{a-\alpha_n}{\alpha_m}\|R\|^{n-m}$

- **SEC**

$$a^{SEC} = -1\,, \quad b^{SEC} = -(\rho + 3p)\,,$$

$$\alpha_n^{SEC} = -n\left[1 + 3(n-1)(\ddot{R} + H\dot{R})R^{-2} + 3(n-1)(n-2)R^{-3}\dot{R}^2\right]$$

$$\alpha_n^{SEC} = n\left[-1 + \frac{q^2 + 7q + j + s + 4}{2(q-1)^2}(n-1) - \frac{(j-q-2)^2}{2(q-1)^3}(n-1)(n-2)\right]$$

- **NEC**

$$a^{NEC} = 0\,, \quad b^{NEC} = -(\rho + p)\,,$$

$$\alpha_n^{NEC} = -n(n-1)\left[(\ddot{R} - H\dot{R})R^{-2} + (n-2)R^{-3}\dot{R}^2\right]$$

$$\alpha_n^{NEC} = n(n-1)\left[\frac{q^2 + 9q - j + s + 8}{6(q-1)^2} - \frac{(j-q-2)^2}{6(q-1)^3}(n-2)\right]$$

- **DEC**

$$a^{DEC} = 1\,, \quad b^{DEC} = -(\rho - p)\,,$$

$$\alpha_n^{DEC} = n\left[1 + (n-1)(\ddot{R} + 5H\dot{R})R^{-2} + (n-1)(n-2)R^{-3}\dot{R}^2\right]$$

$$\alpha_n^{DEC} = n\left[1 - \frac{q^2 + 3q + 5j + s - 4}{6(q-1)^2}(n-1) + \frac{(j-q-2)^2}{6(q-1)^3}(n-1)(n-2)\right]$$

- **WEC**

$$a^{WEC} = \frac{1}{2}\,, \quad b^{WEC} = -\rho\,,$$

$$\alpha_n^{WEC} = n\left[\frac{1}{2} + 3(n-1)HR^{-2}\dot{R}\right]$$

$$\alpha_n^{WEC} = n\left[\frac{1}{2} - \frac{j-q-2}{2(1-q)^2}(n-1)\right]$$

- **Positive gravitational coupling**

$$a^{AG} = 1\,, \quad b^{AG} = 0\,,$$

$$\alpha_n^{AG} = -\frac{n}{6H^2(1-q)}\,.$$

- **Dolgov-Kawasaki criterion** ($a^{DK}=b^{DK}=0$)

$$\frac{\alpha_n^{DK}}{\alpha_m^{DK}} = \frac{n(n-1)}{m(m-1)}$$

Dolgov-Kawasaki Instability

- **Dynamical eq. for the scalar curvature**

$$3(f_1'' + 2\mathcal{L}_m f_2'')\Box R + 3(f_1''' + 2\mathcal{L}_m f_2''')\nabla^\mu R \nabla_\mu R + 12 f_2'' \nabla^\mu \mathcal{L}_m \nabla_\mu R + 6 f_2' \Box \mathcal{L}_m + (f_1' + 2\mathcal{L}_m f_2')R - 2f_1 = f_2 T$$

- **Perturbative equation**

$$R = R_0 + R_1 \qquad T = T_0 + T_1 \qquad g_{\mu\nu} = \eta_{\mu\nu} + h_{\mu\nu}$$

$$\frac{\partial^2 R_1}{\partial t^2} - \nabla^2 R_1 + m_{eff}^2 R_1 = -\frac{f_2}{3(f_1'' + 2\mathcal{L}_m f_2'')} T_1$$

$$m_{eff}^2 = \frac{1}{3(f_1'' + 2\mathcal{L}_m f_2'')} \left[f_1' + f_2'(T_0 - 2\mathcal{L}_m) - (f_1'' + 2\mathcal{L}_m f_2'') R_0 \right]$$

- **Stability Criterion** $\quad f_1(R) = R + \epsilon \varphi_1(R) \qquad f_2(R) = 1 + \lambda \varphi_2(R)$

$$f_1''(R) + 2\mathcal{L}_m f_2''(R) \geq 0$$

Dolgov-Kawasaki Criterion

$$f_1''(R) + 2\mathcal{L}_m f_2''(R) \geq 0$$

- **Models**

$$f_1(R) = R + \epsilon R^n \qquad\qquad f_2(R) = 1 + \lambda R^m$$

- **Stability conditions**

$$\hat{\epsilon} n(n-1)|R|^{n-2} + 2\hat{\lambda}\mathcal{L}_m m(m-1)|R|^{m-2} \geq 0$$

$$\hat{\epsilon} = \begin{cases} (-1)^n \epsilon, & \text{if } R < 0 \\ \epsilon, & \text{if } R > 0 \end{cases}, \quad \hat{\lambda} = \begin{cases} (-1)^m \lambda, & \text{if } R < 0 \\ \lambda, & \text{if } R > 0 \end{cases}$$

Dolgov-Kawasaki Criterion

- **Results**

$$f_1(R) = R + \epsilon R^n \qquad\qquad f_2(R) = 1 + \lambda R^m$$

$$\frac{\hat{\epsilon}|R|^n}{1+\hat{\lambda}|R|^m}\left(a - \alpha_n - \frac{2\hat{\lambda}\mathcal{L}_m}{\hat{\epsilon}}\alpha_m|R|^{m-n}\right) \geq b$$

a=b=0	$\hat{\epsilon} > 0$	$\hat{\epsilon} < 0$
$\hat{\lambda} > 0$	$\frac{2\hat{\lambda}\mathcal{L}_m}{\hat{\epsilon}} \leq \frac{a-\alpha_n}{\alpha_m}\|R\|^{n-m}$	$\frac{2\hat{\lambda}\mathcal{L}_m}{\|\hat{\epsilon}\|} \leq \frac{\alpha_n - a}{\alpha_m}\|R\|^{n-m}$
$\hat{\lambda} < 0$	$\frac{2\|\hat{\lambda}\|\mathcal{L}_m}{\hat{\epsilon}} \geq \frac{\alpha_n - a}{\alpha_m}\|R\|^{n-m}$	$\frac{2\|\hat{\lambda}\|\mathcal{L}_m}{\|\hat{\epsilon}\|} \geq \frac{a-\alpha_n}{\alpha_m}\|R\|^{n-m}$

Dolgov-Kawasaki Criterion

$$f_1''(R) + 2\mathcal{L}_m f_2''(R) \geq 0$$

- **Models**

$$f_1(R) = \sum_{n=1}^{k} a_n R^n \qquad\qquad f_2(R) = 1 + \lambda \sum_{m=1}^{k'} b_m R^m$$

- **Stability condition**

$$\sum_{n=2}^{k} a_n n(n-1) R^{n-2} + 2\lambda\mathcal{L}_m \sum_{m=2}^{k'} b_m m(m-1) R^{m-2} \geq 0$$

<table>
<tr><th>R>0</th><th colspan="2">$k' = 2$</th><th colspan="2">$k' = 3$</th></tr>
<tr><td rowspan="2">$k = 2$</td><td colspan="2" rowspan="2">$a_2 + 2\lambda\mathcal{L}_m b_2 \geq 0$</td><td>$a_3 > 0$</td><td>$R \geq -\frac{a_2+2\lambda\mathcal{L}_m b_2}{3a_3}$</td></tr>
<tr><td>$a_3 < 0$</td><td>$R \leq \frac{a_2+2\lambda\mathcal{L}_m b_2}{3\lvert a_3\rvert}$</td></tr>
<tr><td rowspan="2">$k = 3$</td><td>$\lambda\mathcal{L}_m b_3 > 0$</td><td>$R \geq -\frac{a_2+2\lambda\mathcal{L}_m b_2}{6\lambda\mathcal{L}_m b_3}$</td><td>$a_3 + 2\lambda\mathcal{L}_m b_3 > 0$</td><td>$R \geq -\frac{a_2+2\lambda\mathcal{L}_m b_2}{3(a_3+2\lambda\mathcal{L}_m b_3)}$</td></tr>
<tr><td>$\lambda\mathcal{L}_m b_3 < 0$</td><td>$R \leq \frac{a_2+2\lambda\mathcal{L}_m b_2}{6\lvert\lambda\mathcal{L}_m b_3\rvert}$</td><td>$a_3 + 2\lambda\mathcal{L}_m b_3 < 0$</td><td>$R \leq \frac{a_2+2\lambda\mathcal{L}_m b_2}{3\lvert a_3+2\lambda\mathcal{L}_m b_3\rvert}$</td></tr>
</table>

Conclusions

- New **f(R)** modied theories of gravity with non-minimal curvature-matter coupling have quite interesting phenomenological implications

- They are shown to be physically consistent and all energy conditions depend on the geometry as well as on the matter Langragian and parameters (ε, λ)

$$f_1(R) = R + \epsilon\varphi_1(R) \qquad f_2(R) = 1 + \lambda\varphi_2(R)$$

- For a given model all energy conditions, positive effective gravitational coupling and stabiltiy condition can be expressed in terms of a single type of inequality with suitable parameters

New f(R) modified theory of gravity

[O.B., Böhmer, Harko, Lobo, Phys. Rev. D 75 (2007) 104016]

- Implications:

- MOND-like behaviour: extra force and Tully-Fisher law ($L \sim v_\infty^4$)

$$\vec{a} = \vec{a}_N + \vec{f}$$

If $a_N \ll a$: $\vec{a}_N \approx \frac{a}{a_E}\vec{a}$ $\frac{1}{a_E} \equiv \frac{1}{2f}\left(1 - \frac{f^2}{a^2}\right)$

Hence $a \approx \sqrt{a_E a_N}$ and as $a_N = GM/r^2$ it follows that

$$a \approx \sqrt{a_E GM}/r = v_{tg}^2/r \qquad v_{tg}^2 \to v_\infty^2 = \sqrt{a_E GM}$$

and the Tully-Fisher law as L~M

- Pioneer-like acceleration: $a_E = \frac{f^2 r^2}{GM} + 2f$ $f \sim GM\alpha/r$

$$f \to 0,\ a_E \approx \alpha^2$$

[Anderson, Laing, Lau, Liu, Nieto, Turyshev 2002]

[O.B., Páramos 2004]

$$\alpha - const.$$