GRAVITATIONAL POLARIZATION AND MOND

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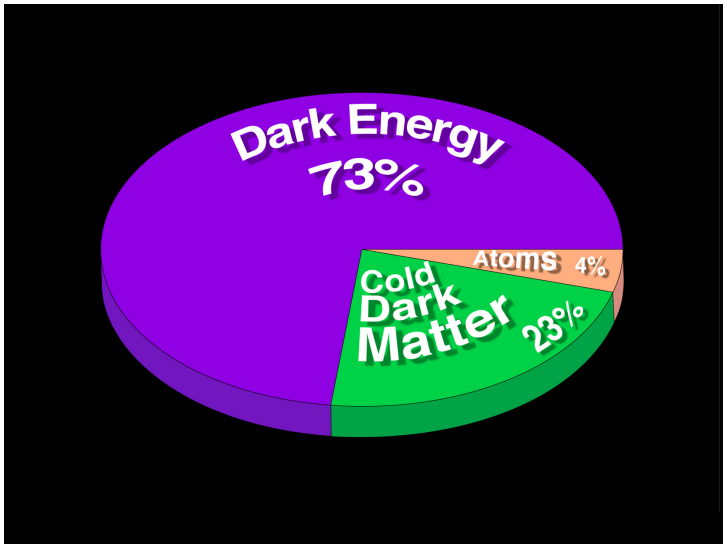
Mass-energy content of the Universe



The total mass-energy of the Universe is made of

1. $\Omega_{\rm de} = 73\%$ of dark energy, maybe in the form of a cosmological constant Λ, as measured from the Hubble diagram of supernovas
2. $\Omega_{\rm dm} = 23\%$ of non-baryonic dark matter, a perfect fluid without pressure whose nature is unknown
3. $\Omega_{\rm b} = 4\%$ of baryonic matter, measured by the Big Bang nucleosynthesis and from CMB fluctuations

The concordance model Λ-CDM

This model brilliantly accounts for

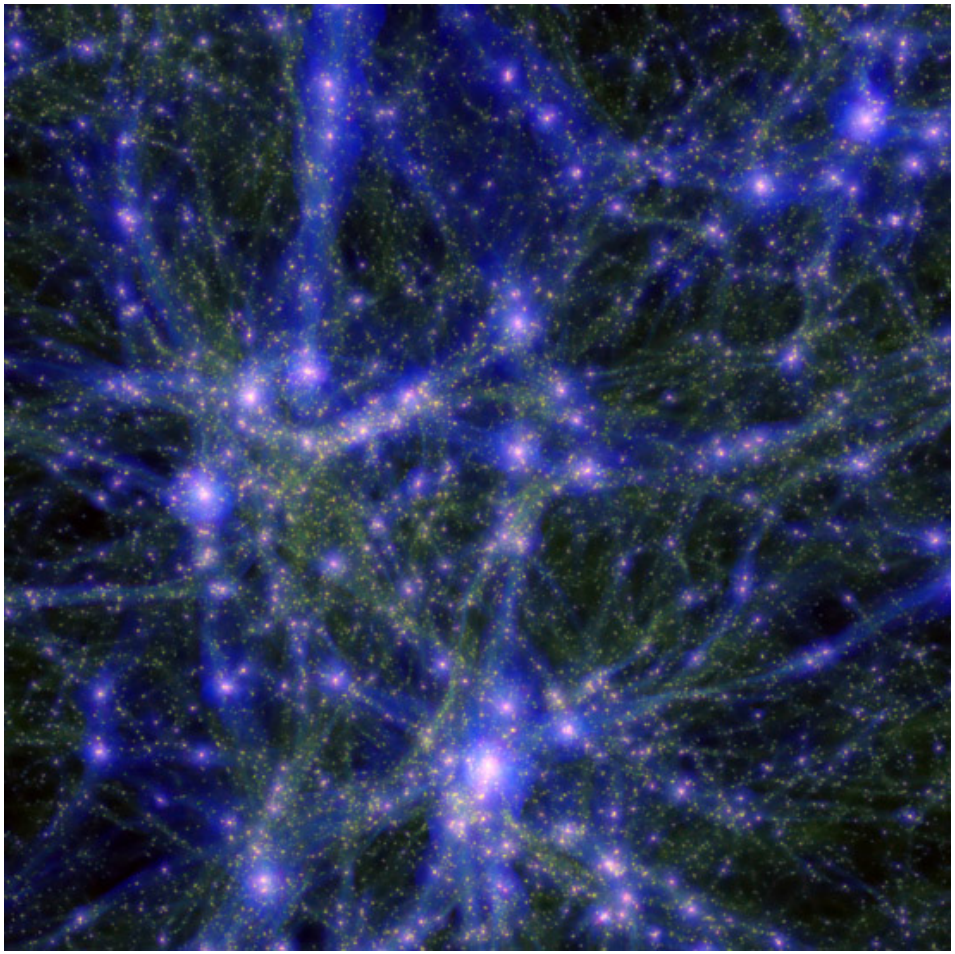
1. the mass discrepancy between the dynamical and luminous masses of clusters of galaxies
2. the precise measurements of the anisotropies of the cosmic microwave background (CMB)
3. the formation and growth of large scale structures as seen in deep redshift and weak lensing surveys
4. the fainting of the light curves of distant supernovae

DM appears to be made by non-relativistic (cold) particles at large scales

Candidates include [e.g. Bertone, Hooper & Silk, 2004]

- the neutralino predicted by super-symmetric extensions of the standard model
- the axion introduced in an attempt to solve the problem of CP violation
- Kaluza-Klein states predicted by models with extra dimensions
- ...

Cosmic N-body simulations



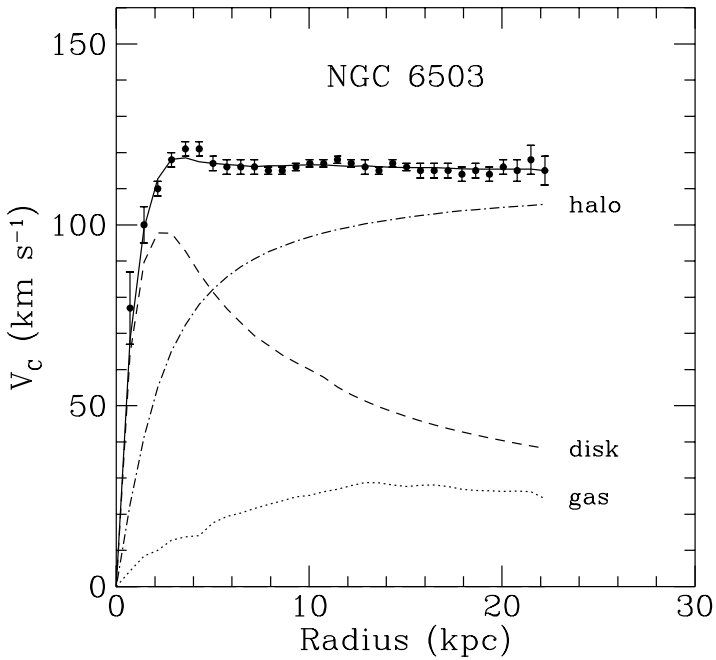
Thanks to high precision N-body simulations in cosmology the model Λ-CDM can be extrapolated and tested at the smaller scale of galaxies

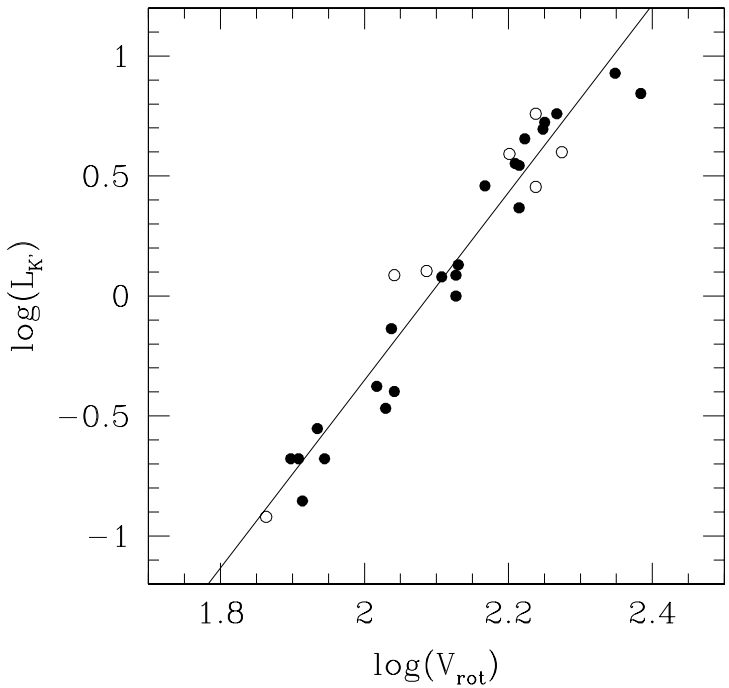
Problems of CDM with galactic halos [talks by M. Milgrom and F. Combes]

The CDM paradigm faces severe challenges when compared to observations at galactic scales [McGaugh & Sanders 2004; Famaey 2007]

1. Prediction of numerous but unseen satellites of large galaxies
2. Generic formation of cusps of DM in central regions of galaxies while the rotation curves seem to favor a constant density profile in the core
3. Evidence that tidal dwarf galaxies are dominated by DM contrary to CDM predictions [Bournaud *et al.* 2007; Gentile *et al.* 2007]
4. Failure to explain in a natural way Milgrom's law, that DM arises only in regions where gravity falls below some universal acceleration scale a_0
5. Difficulty at explaining in a natural way the flat rotation curves of galaxies and the Tully-Fisher relation

Basic phenomenology of galactic halos





The rotation velocity is constant and scales with the baryonic mass like

$$v_{\text{flat}} \simeq (G\,M\,a_0)^{1/4}$$

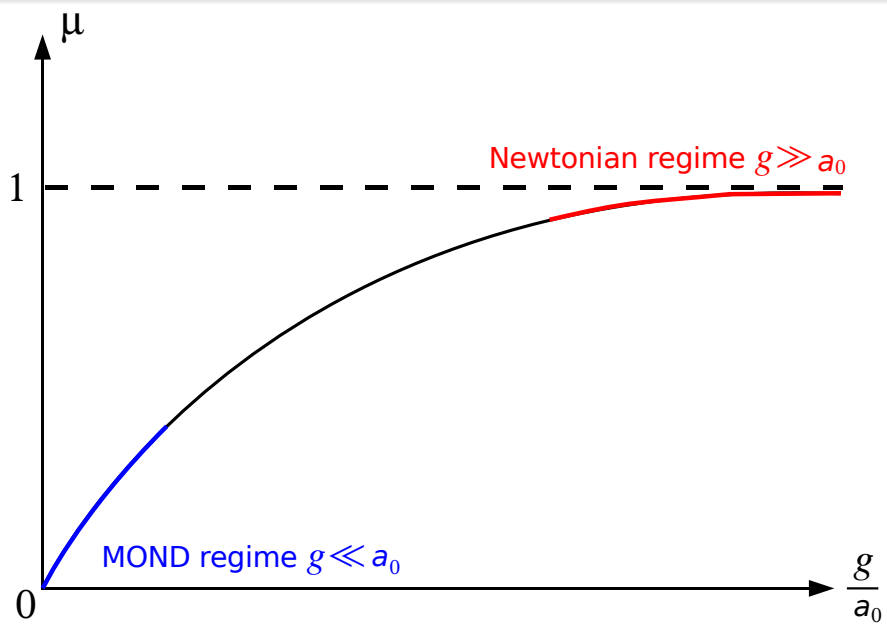
The measured value of the acceleration scale is $a_0 \simeq 1.2\,10^{-10}\,\text{m/s}^2$, very close to the natural scale $a_\Lambda = \frac{1}{2\pi}\left(\frac{\Lambda}{3}\right)^{1/2}$ associated with the cosmological constant

Modified Newtonian dynamics [Milgrom 1983]

MOND states that there is no dark matter and we witness a violation of the fundamental law of gravity in the regime of weak gravity

The Newtonian gravitational field is modified in an *ad hoc* way

$$\mu\left(\frac{g}{a_0}\right)\boldsymbol{g} = \boldsymbol{g}_{\rm Newton}$$



In the MOND regime we have $\mu = g/a_0 + \mathcal{O}(g^2)$

Different approaches to the DM problem

Faced with the unreasonable effectiveness of MOND, three solutions are possible

1. Standard: MOND could be explained within the CDM paradigm
2. Modified Gravity: There is a fundamental modification of the law of gravity in a regime of weak gravity (this is the traditional approach of MOND and its relativistic extensions like TeVeS [talk by M. Milgrom])
3. Modified Matter: The law of gravity is not modified but DM is endowed with special properties which make it able to explain the phenomenology of MOND

Modified gravity or modified matter?

We consider that the Standard scenario (CDM) is excluded by observations

1. To solve the problems of CDM in galactic halos one must invoke complicated astrophysical processes which have to be fine tuned for each galaxies
2. No convincing mechanism has been found to incorporate in a natural way the universal acceleration scale a_0 in the simulated CDM halos

Only the Modified Gravity and Modified Matter approaches remain

In these two approaches we shall have to explain why DM seems to be made of particles at cosmological scales

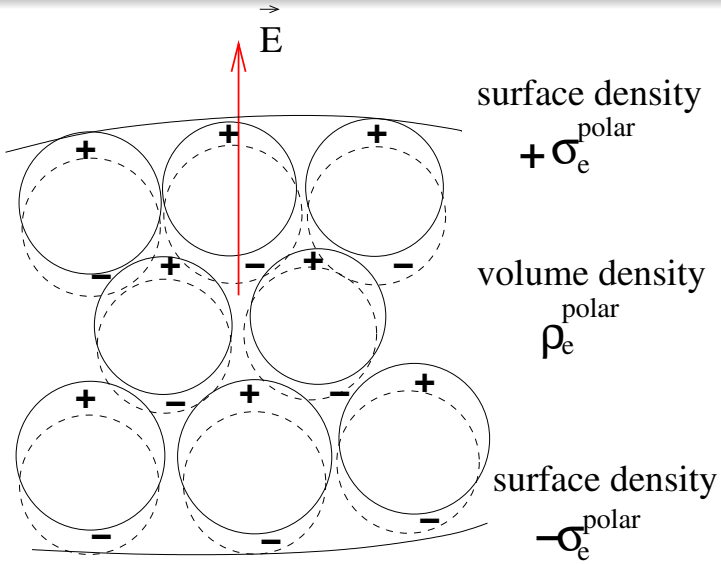
Modified matter approach

1. Keep the standard law of gravity namely general relativity
2. Use the phenomenology of MOND to guess what could be the nature of DM

This approach is based on a striking analogy between

- MOND which takes the form of a modified Poisson equation
- The electrostatics of dielectric media described by a modified Gauss equation

Electrostatics of dielectric media



In the presence of a polarization field the Gauss equation can be written as

$$\boldsymbol{\nabla}\cdot\boldsymbol{E}=\frac{\rho_e+\rho_e^{\text{polar}}}{\varepsilon_0}\qquad\Longleftrightarrow\qquad\boldsymbol{\nabla}\cdot\Big[(1+\chi_e)\boldsymbol{E}\Big]=\frac{\rho_e}{\varepsilon_0}$$

The density of polarization charges is

$$\rho_e^{\text{polar}}=-\boldsymbol{\nabla}\cdot\boldsymbol{\Pi}_e$$

and the polarization vector $\boldsymbol{\Pi}_e$ is aligned with the electric field

$$\boldsymbol{\Pi}_e=\varepsilon_0\,\chi_e\,\boldsymbol{E}$$

where χ_e denotes the coefficient of electric susceptibility

Interpretation of MOND [Blanchet 2006]

The MOND equation in the form of a modified Poisson equation

$$\boldsymbol{\nabla}\cdot\left[\underbrace{\mu\left(\frac{g}{a_0}\right)}_{\text{MOND function}}\boldsymbol{g}\right]=-4\pi\,G\,\rho_{\mathsf{b}}$$

is formally analogous to the equation of electrostatics inside a dielectric. We pose

$$\mu=1+\underbrace{\chi(g)}_{\substack{\text{gravitational}\\ \text{susceptibility}}}\qquad\text{and}\qquad\underbrace{\boldsymbol{\Pi}}_{\substack{\text{gravitational}\\ \text{polarization}}}=-\frac{\chi}{4\pi\,G}\,\boldsymbol{g}$$

The MOND equation is equivalent to

$$\Delta U=-4\pi\,G\,\left(\rho_{\mathsf{b}}+\rho_{\mathrm{polar}}\right)$$

In this interpretation the Newtonian law of gravity is not violated but we are postulating a new form of DM consisting of polarization masses with density

$$\rho_{\mathrm{polar}}=-\boldsymbol{\nabla}\cdot\boldsymbol{\Pi}$$

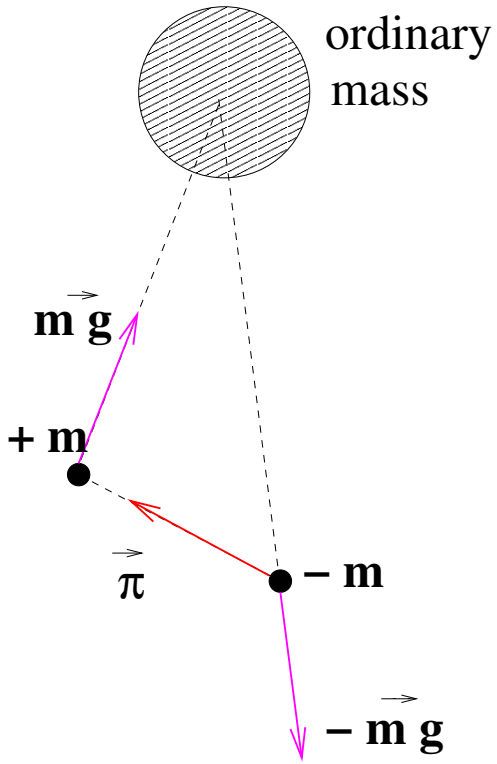
Gravitational dipolar medium

Following the electrostatic analogy we suppose that

1. Dark matter consists of a digravitational medium made of individual gravitational dipole moments
$$\boldsymbol{\pi} = m\,\boldsymbol{\xi}$$
2. Each dipole moment is interpreted as a doublet of particles
$$(m_{\mathrm{i}}, m_{\mathrm{g}}) = (m, \pm m)$$
3. The polarization field is the density of dipole moments
$$\boldsymbol{\Pi} = m\,n\,\boldsymbol{\xi}$$
where n is the number density of the dipolar particles

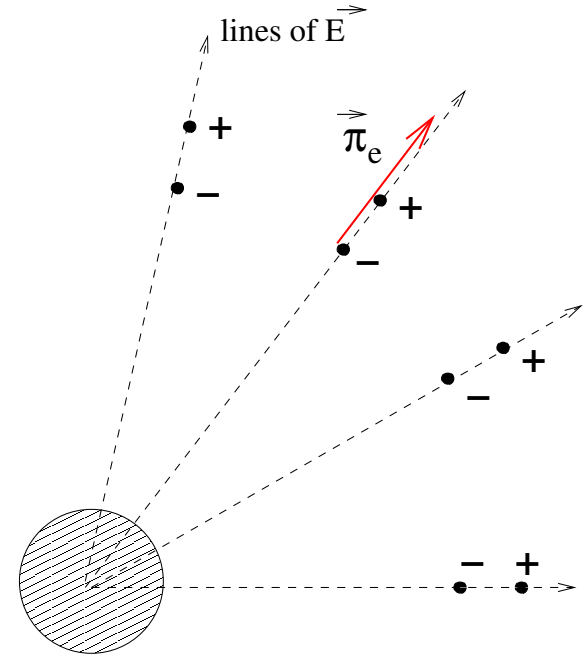
The model involves negative gravitational masses so violates the equivalence principle at a fundamental level

Properties of the dipolar medium



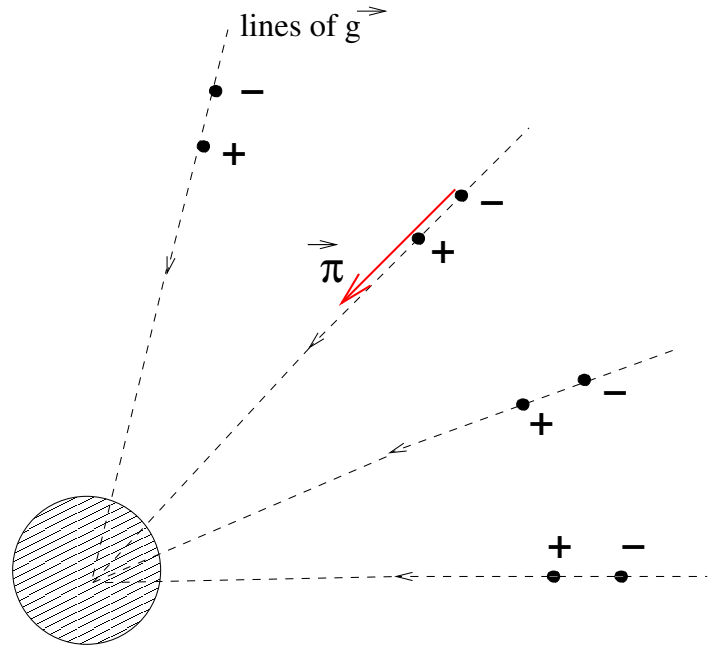
1. The dipole moments tend to align in the same direction as the gravitational field thus $\chi < 0$ which is exactly what MOND predicts
2. Since the constituents of the dipole will repel each other we need to invoke a non-gravitational force (i.e. a fifth force) to stabilize the dipolar medium

Gravitational anti-screening



electric charge

$O > 0$



ordinary mass

$M > 0$

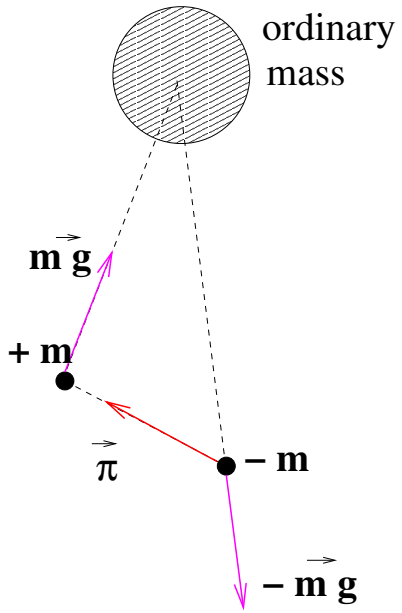
Screening by polarization charges

$\chi_e > 0$

Anti-screening by polarization masses

$\chi < 0$

Equations of motion of dipolar particles



$$m\,\frac{\mathrm{d}^2\mathbf{x}_+}{\mathrm{d}t^2} = m\,\mathbf{g}(\mathbf{x}_+) - \mathbf{F}(|\mathbf{x}_+ - \mathbf{x}_-|)$$
$$m\,\frac{\mathrm{d}^2\mathbf{x}_-}{\mathrm{d}t^2} = -m\,\mathbf{g}(\mathbf{x}_-) + \mathbf{F}(|\mathbf{x}_+ - \mathbf{x}_-|)$$

where $\mathbf{F}$ is an attractive non-gravitational force.

$$\mathbf{x} \equiv \frac{\mathbf{x}_+ + \mathbf{x}_-}{2} \qquad \boldsymbol{\xi} \equiv \mathbf{x}_+ - \mathbf{x}_-$$

The equations of motion are

$$\left.\begin{aligned} 2\,\frac{\mathrm{d}^2\mathbf{x}}{\mathrm{d}t^2} &= (\boldsymbol{\xi}\cdot\boldsymbol{\nabla})\,\mathbf{g} \\ m\,\frac{\mathrm{d}^2\boldsymbol{\xi}}{\mathrm{d}t^2} &= 2m\,\mathbf{g} - 2\mathbf{F} \end{aligned}\right\} + \mathcal{O}(\xi^2)$$

The dipolar particles are weakly influenced by the distribution of ordinary matter since they are accelerated by the tidal gravitational field

Interpretation of the dark matter medium

To recover MOND we find that the internal force should be

$$\mathbf{F} = \frac{1}{2} m \, \omega^2 \, \boldsymbol{\xi}$$

The dipole moments obey the equation of an harmonic oscillator

$$\frac{\mathrm{d}^2 \boldsymbol{\xi}}{\mathrm{d}t^2} + \omega^2 \, \boldsymbol{\xi} = 2 \, \mathbf{g}$$

in which ω is the usual plasma frequency given here by

$$\omega = \sqrt{-\frac{8\pi \, G \, m \, n}{\chi}}$$

1. The dark matter medium is interpreted as the gravitational analogue of a plasma made of particles $(m_{\mathrm{i}}, m_{\mathrm{g}}) = (m, \pm m)$
2. The plasma oscillates at the natural plasma frequency and is polarized by the external gravitational field

Need for a relativistic model

The quasi-Newtonian model

1. Suggests that the gravitational analogue of the electric polarization is possible
2. Yields a simple and natural explanation of the MOND phenomenology
3. Requires the existence of a new non-gravitational force

But this model is not viable

1. Involves negative gravitational masses so violates the equivalence principle
2. Is not relativistic so does not allow to answer questions related to cosmology

In particular it is a challenge to find a relativistic model which

1. Reproduces MOND by a mechanism of gravitational polarization
2. Recovers the standard cosmological model Λ-CDM at large scales

See the next talk by Alexandre Le Tiec