Cosmography and Large Scale Structure by f(R)-gravity: New Results

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What about the Universe?

DATA   **RESULTS**

DATA		RESULTS
Supernovae Hubble Diagram Galaxy power spectrum and BAO Cosmic Microwave Background	⟷	Accelerating expansion Low matter content Spatially flat universe

SOLUTIONS WITHIN GENERAL RELATIVITY   **SHORTCOMINGS**

Flatness $\Omega_k = \mp 0.046^{+0.0066}_{-0.0067}$	Inflationary Epoch	——
Acceleration $q_0 < 0$	Cosmological constant (ΛCDM) Quintessence	120 orders of difference What kind?
Low matter content $\Omega_b = 0.0462 \pm 0.0015$	Dark Energy $\quad \Omega_{de} \approx 0.72$ Dark Matter $\quad \Omega_m \approx 0.25$	Undetected components Coincidence problem Fine tuning problems

Extended Theories of Gravity

SOLUTIONS OUTSIDE GENERAL RELATIVITY

MOTIVATIONS:

- General Relativity tested only up to Solar System

- Effective actions from fundamental field theories

- Inflationary models

- No need of dark components

REQUIREMENTS:

- Reproducing Newtonian Dynamics in Solar System

- Flat Rotation Curves of Spiral Galaxies by Baryonic constituents

- Reproducing Large Scale Structure (Galaxy Clusters scale)

- Successful Fit of SNeIa + CMB + BAO data

- Accelerated Hubble fluid and Dark Energy phenomenology

f(R) Theories of Gravity : Résumé

From Extended Theories of Gravity ➡ f(R) gravity

- **Gravity action :**

$$\mathcal{A} = \int \sqrt{-g} \; [f(R) + \mathcal{L}_M] \, d^4 x$$

- **Field equations :**

$$f'(R)R_{\alpha\beta} - \frac{1}{2}f(R)g_{\alpha\beta} - f'(R)^{;\alpha\beta}(g_{\alpha\mu}g_{\beta\nu} - g_{\alpha\beta}g_{\mu\nu}) = \tilde{T}^M_{\alpha\beta}$$

- **Cosmological equations :**

$$H^2 = \frac{1}{3}\left[\frac{\rho_m}{f'(R)} + \rho_{\text{curv}}\right]$$

1st Friedmann eq.

$$2\,\frac{\ddot{a}}{a} + \left(\frac{\dot{a}}{a}\right)^2 = -(p_{curv} + p_m)$$

2nd Friedmann eq.

- **Curvature Fluid :**

$$\rho_{\text{curv}} = \frac{1}{f'(R)}\left\{\frac{1}{2}[f(R) - Rf'(R)] - 3H\dot{R}f''(R)\right\}.$$

$$w_{\text{curv}} = -1 + \frac{\ddot{R}f''(R) + \dot{R}[\dot{R}f'''(R) - Hf''(R)]}{[f(R) - Rf'(R)]/2 - 3H\dot{R}f''(R)}$$

Constraining Extended Theories of Gravity

by Cosmography

- e.g. **Constraining f(R)-gravity by Cosmography**

  Capozziello, S., Cardone, V., Salzano, V., PRD 78 (2008) 063504

- Constraining f(R)– gravity by Clusters of Galaxies

  Capozziello, S., De Filippis, E., **Salzano**, **V.**, MNRAS 394 (2009) 947

Cosmography

GR based models vs f(R) gravity

⬇

Agreement with Data…

How we can discriminate?

- No a priori dynamical model = **Model Independent** Approach;

- Robertson – Walker metric;

- Expansion series of the scale factor with respect to cosmic time:

$$\frac{a(t)}{a(t_0)} = 1 + H_0(t-t_0) - \frac{q_0}{2}H_0^2(t-t_0)^2 + \frac{j_0}{3!}H_0^3(t-t_0)^3 + \frac{s_0}{4!}H_0^4(t-t_0)^4 + \frac{l_0}{5!}H_0^5(t-t_0)^5 + O[(t-t_0)^6]$$

$$q(t) = -\frac{1}{a}\frac{d^2a}{dt^2}\frac{1}{H^2} \qquad j(t) = \frac{1}{a}\frac{d^3a}{dt^3}\frac{1}{H^3} \qquad s(t) = \frac{1}{a}\frac{d^4a}{dt^4}\frac{1}{H^4} \qquad l(t) = \frac{1}{a}\frac{d^5a}{dt^5}\frac{1}{H^5}$$

Deceleration   **Jerk**   **Snap**   **Lerk**

Expansion up to fifth order :

$d_L (z)$

error on   less than 10% up to z = 1

$^1 (z)$

error on   less than   3% up to z = 2

Cosmography by f(R): How many parameters…

- Definition:
$$H(t) = \frac{1}{a}\frac{da}{dt}, \quad q(t) = -\frac{1}{a}\frac{d^2a}{dt^2}\frac{1}{H^2}, \quad j(t) = \frac{1}{a}\frac{d^3a}{dt^3}\frac{1}{H^3}, \quad s(t) = \frac{1}{a}\frac{d^4a}{dt^4}\frac{1}{H^4}, \quad l(t) = \frac{1}{a}\frac{d^5a}{dt^5}\frac{1}{H^5}$$

- Derivatives of H(t):

$$\dot{H} = -H^2(1+q)$$

$$\ddot{H} = H^3(j + 3q + 2)$$

$$d^3H/dt^3 = H^4\left[s - 4j - 3q(q+4) - 6\right]$$

$$d^4H/dt^4 = H^5\left[l - 5s + 10(q+2)j + 30(q+2)q + 24\right]$$

- Derivatives of scalar curvature:

$$R = -6(\dot{H} + 2H^2)$$

$$R_0 = -6H_0^2(1 - q_0)$$

$$\dot{R}_0 = -6H_0^3(j_0 - q_0 - 2)$$

$$\ddot{R}_0 = -6H_0^4\left(s_0 + q_0^2 + 8q_0 + 6\right)$$

$$d^3R_0/dt^3 = -6H_0^5\left[l_0 - s_0 + 2(q_0+4)j_0 - 6(3q_0+8)q_0 - 24\right]$$

Cosmography by f(R): What equations…?

- 1st Friedmann eq. :

$$H_0^2 = \frac{H_0^2 \Omega_M}{f'(R_0)} + \frac{f(R_0) - R_0 f'(R_0) - 6H_0 \dot{R}_0 f''(R_0)}{6 f'(R_0)},$$

- 2nd Friedmann eq. :

$$-\dot{H}_0 = \frac{3H_0^2 \Omega_M}{2 f'(R_0)} + \frac{\dot{R}_0^2 f'''(R_0) + (\ddot{R}_0 - H_0 \dot{R}_0) f''(R_0)}{2 f'(R_0)}$$

- Derivative of 2nd Friedmann eq. :

$$\ddot{H} = \frac{\dot{R}^2 f'''(R) + (\ddot{R} - H\dot{R}) f''(R) + 3H_0^2 \Omega_M a^{-3}}{2\left[\dot{R} f''(R)\right]^{-1} [f'(R)]^2} - \frac{\dot{R}^3 f^{(iv)}(R) + (3\dot{R}\ddot{R} - H\dot{R}^2) f'''(R)}{2 f'(R)}$$

$$- \frac{(d^3 R/dt^3 - H\ddot{R} + \dot{H}\dot{R}) f''(R) - 9H_0^2 \Omega_M H a^{-3}}{2 f'(R)}$$

- Constraint from gravitational constant:

$$H^2 = \frac{8\pi G}{3 f'(R)}[\rho_m + \rho_{curv.} f'(R)] \qquad \Longrightarrow \qquad G_{eff}(z=0) = G \rightarrow f'(R_0) = 1.$$

f(R) equations and Cosmographic Parameters

- Final solutions:

$$\frac{f(R_0)}{6H_0^2} = -\frac{\mathcal{P}_0(q_0, j_0, s_0, l_0)\Omega_M + \mathcal{Q}_0(q_0, j_0, s_0, l_0)}{\mathcal{R}(q_0, j_0, s_0, l_0)}$$

$$f'(R_0) = 1$$

$$\frac{f''(R_0)}{(6H_0^2)^{-1}} = -\frac{\mathcal{P}_2(q_0, j_0, s_0)\Omega_M + \mathcal{Q}_2(q_0, j_0, s_0)}{\mathcal{R}(q_0, j_0, s_0, l_0)}$$

$$\frac{f'''(R_0)}{(6H_0^2)^{-2}} = -\frac{\mathcal{P}_3(q_0, j_0, s_0, l_0)\Omega_M + \mathcal{Q}_3(q_0, j_0, s_0, l_0)}{(j_0 - q_0 - 2)\mathcal{R}(q_0, j_0, s_0, l_0)}$$

- Taylor expand *f(R)* in series of *R* up to third order (higher not necessary)
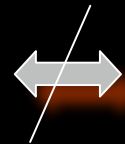
- Linear equations in f(R) and derivatives

- Ω_M is model dependent: $\Omega_M = 0.041$
  $\Omega_M = 0.250$

f(R) derivatives and CPL models

"Precision cosmology" ⟷̸ Values of cosmographic parameters?

Cosmographic parameters ⟷ Dark energy parameters = equivalent f(R)

CPL approach:

(Chevallier, Polarski, Linder)

$$w = w_0 + w_a(1 - a) = w_0 + w_a z(1 + z)^{-1}$$

Cosmographic

parameters:

$$q_0 = \frac{1}{2} + \frac{3}{2}(1 - \Omega_M)w_0$$

$$j_0 = 1 + \frac{3}{2}(1 - \Omega_M)\left[3w_0(1 + w_0) + w_a\right]$$

$$\begin{aligned}
s_0 = & -\frac{7}{2} - \frac{33}{4}(1 - \Omega_M)w_a - \frac{9}{4}(1 - \Omega_M)\left[9 + (7 - \Omega_M)w_a\right]w_0 + \\
& - \frac{9}{4}(1 - \Omega_M)(16 - 3\Omega_M)w_0^2 - \frac{27}{4}(1 - \Omega_M)(3 - \Omega_M)w_0^3
\end{aligned}$$

$$\begin{aligned}
l_0 = & \frac{35}{2} + \frac{1 - \Omega_M}{4}\left[213 + (7 - \Omega_M)w_a\right]w_a + \frac{(1 - \Omega_M)}{4}\left[489 + 9(82 - 21\Omega_M)w_a\right]w_0 + \\
& + \frac{9}{2}(1 - \Omega_M)\left[67 - 21\Omega_M + \frac{3}{2}(23 - 11\Omega_M)w_a\right]w_0^2 + \frac{27}{4}(1 - \Omega_M)(47 - 24\Omega_M)w_0^3 + \\
& + \frac{81}{2}(1 - \Omega_M)(3 - 2\Omega_M)w_0^4
\end{aligned}$$

CPL Cosmography and f(R): the ΛCDM Model

ΛCDM model: $(w_0, w_a) = (-1, 0)$

$$q_0 = \frac{1}{2} - \frac{3}{2}\Omega_M; \quad j_0 = 1; \quad s_0 = 1 - \frac{9}{2}\Omega_M; \quad l_0 = 1 + 3\Omega_M + \frac{27}{2}\Omega_M^2$$

$$f(R_0) = R_0 + 2\Lambda, \qquad f''(R_0) = f'''(R_0) = 0,$$

ΛCDM fits well many data ⟹ cosmographic values strictly depend on Ω_M

$$q_0 = q_0^\Lambda \times (1 + \varepsilon_q), \qquad j_0 = j_0^\Lambda \times (1 + \varepsilon_j),$$

$$s_0 = s_0^\Lambda \times (1 + \varepsilon_s), \qquad l_0 = l_0^\Lambda \times (1 + \varepsilon_l),$$

$$\eta_{20} = \frac{64 - 6\Omega_M(9\Omega_M + 8)}{[3(9\Omega_M + 74)\Omega_M - 556]\Omega_M^2 + 16} \times \frac{\varepsilon}{27}$$

$$\eta_{30} = \frac{6\left[(81\Omega_M - 110)\Omega_M + 40\right]\Omega_M + 16}{[3(9\Omega_M + 74)\Omega_M - 556]\Omega_M^2 + 16} \times \frac{\varepsilon}{243\Omega_M^2}$$

$$\begin{cases} \eta_{20} \simeq 0.15 \times \varepsilon & \text{for} \quad \Omega_M = 0.041 \\ \\ \eta_{20} \simeq -0.12 \times \varepsilon & \text{for} \quad \Omega_M = 0.250 \end{cases}$$

$$\eta_{20} = f''(R_0)/f(R_0) \times H_0^4$$

$$\eta_{30} = f'''(R_0)/f(R_0) \times H_0^6$$

$$\begin{cases} \eta_{30} \simeq 4 \times \varepsilon & \text{for} \quad \Omega_M = 0.041 \\ \\ \eta_{30} \simeq -0.18 \times \varepsilon & \text{for} \quad \Omega_M = 0.250 \end{cases}$$
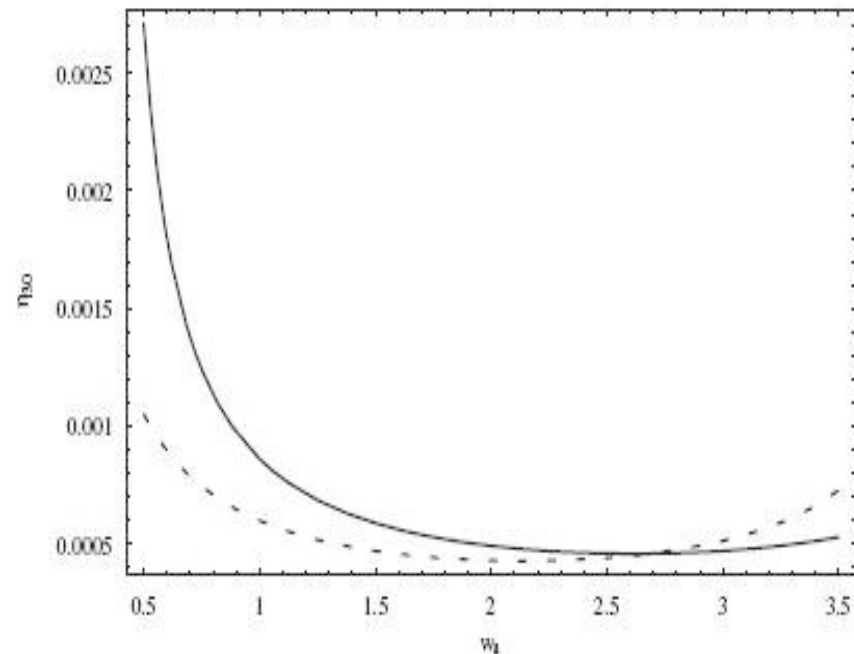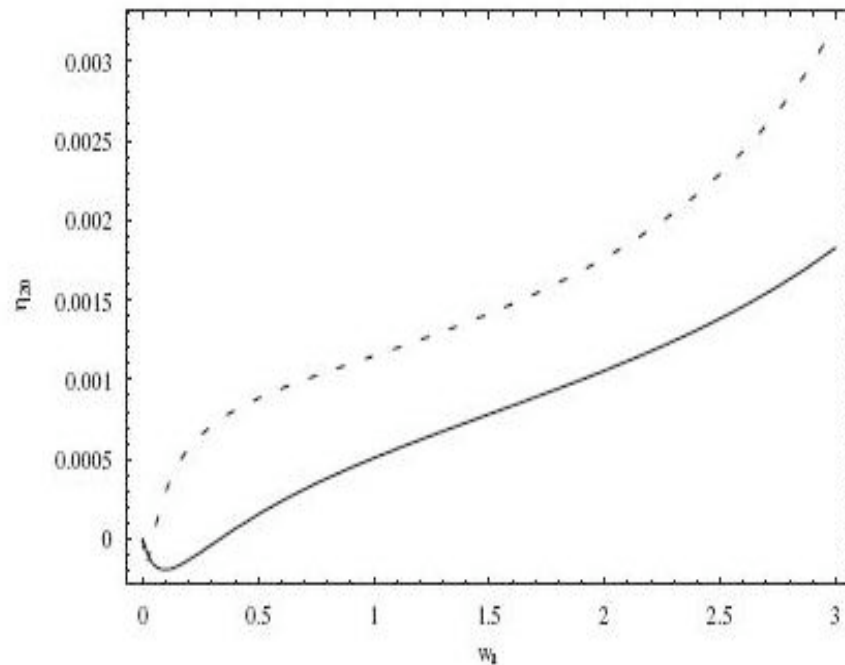
CPL Cosmography and f(R): constant EoS case

- Constant EoS: $w_a = 0$

- Beware of divergences in the *f(R)* derivatives

- Small deviations from GR

- Large deviations for baryonic dominated universe
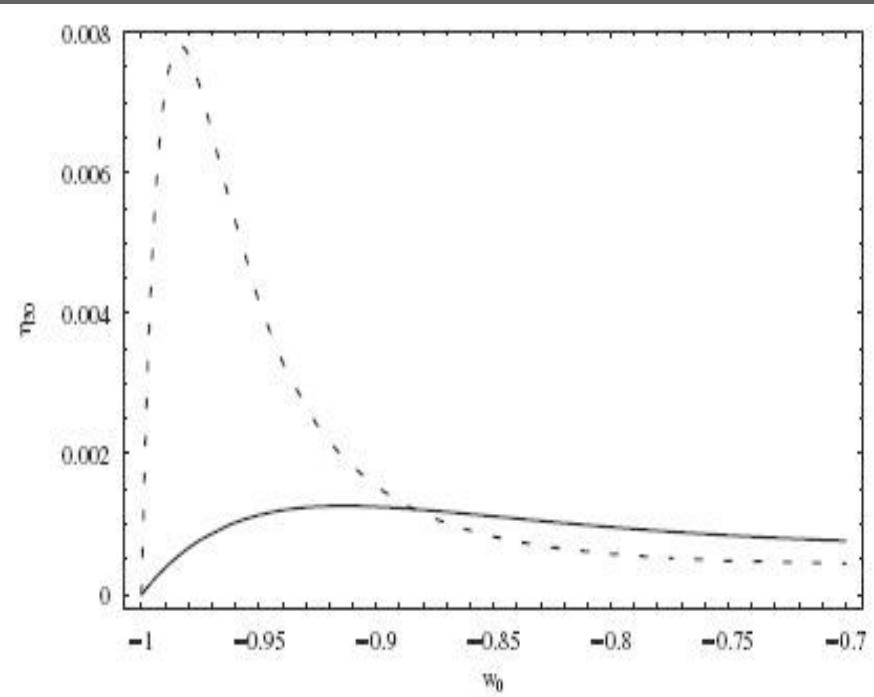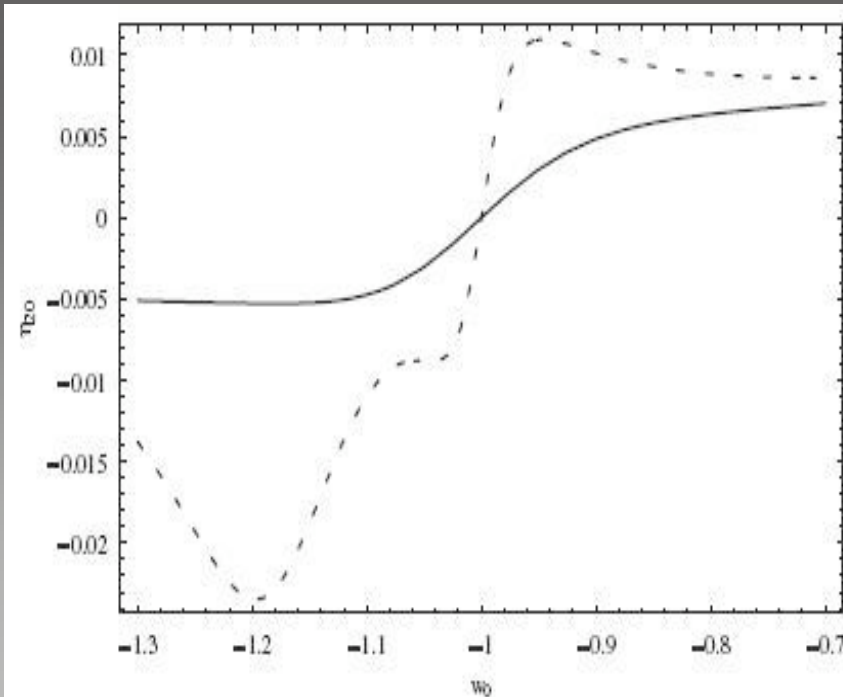
CPL Cosmography and f(R) : varying EoS case

- General case: $w_a \neq 0; \ w_0 = -1$

- Beware of divergences in the *f(R)* derivatives

- Small deviations from GR

- Large deviations for baryonic dominated universe

Constraining f(R) models by Cosmography

- Procedure:

1. Estimate (q(0), j(0), s(0), l(0)) observationally

2. Compute f(R0), f '(R0), f " (R0), f '''(R0)

3. Solve for f(R) parameters from derivatives

4. Constraint f(R) models

- e.g. Double Power-Law:

$$f(R) = R(1 + \alpha R^n + \beta R^{-m})$$

$$
\begin{cases}
f(R_0) = R_0(1 + \alpha R_0^n + \beta R_0^{-m}) \\
f'(R_0) = 1 + \alpha(n+1)R_0^n - \beta(m-1)R_0^{-m} \\
f''(R_0) = \alpha n(n+1)R_0^{n-1} + \beta m(m-1)R_0^{-(1+m)} \\
f'''(R_0) = \alpha n(n+1)(n-1)R_0^{n-2} \\
\qquad - \beta m(m+1)(m-1)R_0^{-(2+m)}.
\end{cases}
$$

$$
\begin{cases}
\alpha = \frac{1-m}{n+m}\left(1 - \frac{\phi_0}{R_0}\right)R_0^{-n} \\
\beta = -\frac{1+n}{n+m}\left(1 - \frac{\phi_0}{R_0}\right)R_0^m,
\end{cases}
$$

$$
\begin{cases}
\alpha = \frac{\phi_2 R_0^{1-n}[1+m+(\phi_3/\phi_2)R_0]}{n(n+1)(n+m)} \\
\beta = \frac{\phi_2 R_0^{1+n}[1-n+(\phi_3/\phi_2)R_0]}{m(1-m)(n+m)}.
\end{cases}
$$

$$
\begin{cases}
\frac{n(n+1)(1-m)(1-\phi_0/R_0)}{\phi_2 R_0[1+m+(\phi_3/\phi_2)R_0]} = 1 \\
\frac{m(n+1)(m-1)(1-\phi_0/R_0)}{\phi_2 R_0[1-n+(\phi_3/\phi_2)R_0]} = 1.
\end{cases}
$$

$$
m = -[1 - n + (\phi_3/\phi_2)R_0]
$$

$$
n = \frac{1}{2}\left[1 + \frac{\phi_3}{\phi_2}R_0 \pm \frac{\sqrt{\mathcal{N}(\phi_0, \phi_2, \phi_3)}}{\phi_2 R_0(1 + \phi_0/R_0)} \right]
$$

Cosmography and data

- Cosmographic parameter from SNeIa:

  What we have to expect from data

$$q_0 = -0.90 \pm 0.65, \qquad j_0 = 2.7 \pm 6.7,$$
$$s_0 = 36.5 \pm 52.9, \qquad l_0 = 142.7 \pm 320.$$

- Fisher information matrix method:

$$F_{ij} = \left\langle \frac{\partial^2 L}{\partial \theta_i \partial \theta_j} \right\rangle$$

- FM ingredients :

$$\chi^2(H_0, \mathbf{p}) = \sum_{n=1}^{\mathcal{N}_{SNeIa}} \left[\frac{\mu_{obs}(z_i) - \mu_{th}(z_n, H_0, \mathbf{p})}{\sigma_i(z_i)} \right]^2$$

$$d_L(z) = \mathcal{D}_L^0 z + \mathcal{D}_L^1 z^2 + \mathcal{D}_L^2 z^3 + \mathcal{D}_L^3 z^4 + \mathcal{D}_L^4 z^5$$

$$\sigma(z) = \sqrt{\sigma_{\text{sys}}^2 + \left(\frac{z}{z_{\max}}\right)^2 \sigma_m^2}$$

- Estimating error on g:

$$\sigma_g^2 = \left| \frac{\partial g}{\partial \Omega_M} \right|^2 \sigma_M^2 + \sum_{i=1}^{i=4} \left| \frac{\partial g}{\partial p_i} \right|^2 \sigma_{p_i}^2 + \sum_{i \neq j} 2 \frac{\partial g}{\partial p_i} \frac{\partial g}{\partial p_j} C_{ij}$$

- Survey: Davis (2007)

$\sigma_M/\Omega_M = 10\%$; $\sigma_{sys} = 0.15$

$N_{SNeIa} = 2000$; $\sigma_m = 0.33$

$z_{max} = 1.7$

$\sigma_1 = 0.38$

$\sigma_2 = 5.4$

$\sigma_3 = 28.1$

$\sigma_4 = 74.0$

$\sigma_{20} = 0.04$

$\sigma_{30} = 0.04$

- Snap like survey:

$\sigma_M/\Omega_M = 1\%$; $\sigma_{sys} = 0.15$

$N_{SNeIa} = 2000$; $\sigma_m = 0.02$

$z_{max} = 1.7$

$\sigma_1 = 0.08$

$\sigma_2 = 1.0$

$\sigma_3 = 4.8$

$\sigma_4 = 13.7$

$\sigma_{20} = 0.007$

$\sigma_{30} = 0.008$

- Ideal PanSTARRS survey:

$\sigma_M/\Omega_M = 0.1\%$; $\sigma_{sys} = 0.15$

$N_{SNeIa} = 60000$; $\sigma_m = 0.02$

$z_{max} = 1.7$

$\sigma_1 = 0.02$

$\sigma_2 = 0.2$

$\sigma_3 = 0.9$

$\sigma_4 = 2.7$

$\sigma_{20} = 0.0015$

$\sigma_{30} = 0.0016$

Constraining Extended Theories of Gravity

by Large Scale Structure

• Constraining f(R) -gravity by Cosmography

Capozziello, S., Cardone, V., **Salzano**, **V.**, PRD 78 (2008) 063504

• e.g. **Constraining f(R) - by Clusters of Galaxies**

Capozziello, S., De Filippis, E., **Salzano**, **V.** MNRAS 394 (2009) 947

f(R) gravity motivations

- Gravity action : $\mathcal{A} = \int d^4x \sqrt{-g}\, [f(R) + \mathcal{X}\mathcal{L}_m]$

- General requirement: Taylor expandable

  Lagrangian $\qquad f(R) \sim a_1 R + a_2 R^2 + ...$

Point like potential:

$$\phi(r) = -\frac{3GM}{4a_1 r}\left(1 + \frac{1}{3}e^{-\frac{r}{L}}\right)$$

Interaction length:

$$L \equiv L(a_1, a_2) = \left(-\frac{6a_2}{a_1}\right)^{1/2}$$

Purpose: Fit clusters mass profiles ➡ **Build in a Self consistent theory**

If: $\mathcal{L} = f(R, \Box R, ..\Box^k R, ..\Box^n R)\sqrt{-g}$ ➡ Effective actions from quantum field theory on curved space-time

$$\phi(r) = -\frac{GM}{r}\left[1 + \sum_{k=1}^{n}\alpha_k e^{-r/L_k}\right]$$

If: $\begin{cases} r \ \text{À} \ L \\ r \ \text{¿} \ L \end{cases}$

Gravitational coupling G

Gravitational coupling $G(1+\alpha_1)$

Our potential: $\phi(r) = -\dfrac{3GM}{4a_1 r}\left(1 + \dfrac{1}{3}e^{-\frac{r}{L}}\right)$ ⟷ $\phi(r) = -\dfrac{GM}{r}\left[1 + \alpha_1 e^{-r/L_1}\right]$

Clusters of galaxies dynamics

Cluster model: spherical mass distribution in hydrostatic equilibrium

- Boltzmann equation:
$$-\frac{d\Phi}{dr} = \frac{kT(r)}{\mu m_p r}\left[\frac{d\ln\rho_{gas}(r)}{d\ln r} + \frac{d\ln T(r)}{d\ln r}\right]$$

- Newton *classical* approach:
$$\phi(r) = -\frac{GM}{r}$$
$$\rho_{cl,EC}(r) = \rho_{dark} + \rho_{gas}(r) + \rho_{gal}(r) + \rho_{CDgal}(r)$$

- f(R) approach:
$$\phi(r) = -\frac{3GM}{4a_1 r}\left(1 + \frac{1}{3}e^{-\frac{r}{L}}\right)$$
$$\rho_{cl,EC}(r) = \rho_{gas}(r) + \rho_{gal}(r) + \rho_{CDgal}(r)$$

- Rearranging the Boltzmann equation:
$$\phi_N(r) = -\frac{3GM}{4a_1 r}$$
$$\phi_C(r) = -\frac{GM}{4a_1}\frac{e^{-\frac{r}{L}}}{r}$$

$$M_{bar,th}(r) = \frac{4a_1}{3}\left[-\frac{kT(r)}{\mu m_p G}r\left(\frac{d\ln\rho_{gas}(r)}{d\ln r} + \frac{d\ln T(r)}{d\ln r}\right)\right] - \frac{4a_1}{3G}r^2\frac{d\Phi_C}{dr}(r)$$
$$M_{bar,obs}(r) = M_{gas}(r) + M_{gal}(r) + M_{CDgal}(r)$$

Fitting Mass Profiles

DATA:

- Sample: 12 clusters from Chandra (Vikhlinin 2005, 2006)

- Temperature profile from spectroscopy

- Gas density: modified beta-model

$$n_p n_e = n_0^2 \cdot \frac{(r/r_c)^{-\alpha}}{(1 + r^2/r_c^2)^{3\beta - \alpha/2}} \cdot \frac{1}{(1 + r^\gamma/r_s^\gamma)^{\epsilon/\gamma}} + \frac{n_{02}^2}{(1 + r^2/r_{c2}^2)^{3\beta_2}}$$

- Galaxy density:

$$\rho_{gal}(r) = \begin{cases} \rho_{gal,1} \cdot \left[1 + \left(\frac{r}{R_c}\right)^2\right]^{-\frac{3}{2}} & r < R_c \\ \rho_{gal,2} \cdot \left[1 + \left(\frac{r}{R_c}\right)^2\right]^{-\frac{2.6}{2}} & r > R_c \end{cases}$$

$$\rho_{CDgal} = \frac{\rho_{0,J}}{\left(\frac{r}{r_c}\right)^2 \left(1 + \frac{r}{r_c}\right)^2}$$

Table 1. Column 1: Cluster name. Column2: Richness. Column 2: cluster total mass. Column 3: gas mass. Column 4: galaxy mass. Column 5: cD-galaxy mass. All mass values are estimated at $r = r_{max}$. Column 6: ratio of total galaxy mass to gas mass. Column 7: minimum radius. Column 8: maximum radius.

name	R	$M_{cl,N}$ ($M_\odot$)	M_{gas} ($M_\odot$)	M_{gal} ($M_\odot$)	M_{cDgal} ($M_\odot$)	$\frac{gal}{gas}$	r_{min} (kpc)	r_{max} (kpc)
A133	0	$4.35874 \cdot 10^{14}$	$2.73866 \cdot 10^{13}$	$5.20269 \cdot 10^{12}$	$1.10568 \cdot 10^{12}$	0.23	86	1060
A262	0	$4.45081 \cdot 10^{13}$	$2.76659 \cdot 10^{12}$	$1.71305 \cdot 10^{11}$	$5.16382 \cdot 10^{12}$	0.25	61	316
A383	2	$2.79785 \cdot 10^{14}$	$2.82467 \cdot 10^{13}$	$5.88048 \cdot 10^{12}$	$1.09217 \cdot 10^{12}$	0.25	52	751
A478	2	$8.51832 \cdot 10^{14}$	$1.05583 \cdot 10^{14}$	$2.15567 \cdot 10^{13}$	$1.67513 \cdot 10^{12}$	0.22	59	1580
A907	1	$4.87657 \cdot 10^{14}$	$6.38070 \cdot 10^{13}$	$1.34129 \cdot 10^{13}$	$1.66533 \cdot 10^{12}$	0.24	563	1226
A1413	3	$1.09598 \cdot 10^{15}$	$9.32466 \cdot 10^{13}$	$2.30728 \cdot 10^{13}$	$1.67345 \cdot 10^{12}$	0.26	57	1506
A1795	2	$1.24313 \cdot 10^{14}$	$1.00530 \cdot 10^{13}$	$4.23211 \cdot 10^{12}$	$1.93957 \cdot 10^{12}$	0.11	79	1151
A1991	1	$1.24313 \cdot 10^{14}$	$1.00530 \cdot 10^{13}$	$1.24608 \cdot 10^{12}$	$1.08241 \cdot 10^{12}$	0.23	55	618
A2029	2	$8.92392 \cdot 10^{14}$	$1.24129 \cdot 10^{14}$	$3.21543 \cdot 10^{13}$	$1.11921 \cdot 10^{12}$	0.27	62	1771
A2390	1	$2.09710 \cdot 10^{15}$	$2.15726 \cdot 10^{14}$	$4.91580 \cdot 10^{13}$	$1.12141 \cdot 10^{12}$	0.23	83	1984
MKW4	-	$4.69503 \cdot 10^{13}$	$2.83207 \cdot 10^{12}$	$1.71153 \cdot 10^{11}$	$5.29855 \cdot 10^{11}$	0.25	60	434
RXJ1159	-	$8.97997 \cdot 10^{13}$	$4.33256 \cdot 10^{12}$	$7.34414 \cdot 10^{11}$	$5.38799 \cdot 10^{11}$	0.29	64	568

Fitting Mass Profiles

METHOD:

- Minimization of chi-square:

$$\chi^2 = \frac{1}{N - n_p - 1} \cdot \sum_{i=1}^{N} \frac{(M_{bar,obs} - M_{bar,theo})^2}{M_{bar,theo}}$$

- Markov Chain Monte Carlo:

$$\alpha(\mathbf{p}, \mathbf{p}') = min\left\{1, \frac{L(\mathbf{d}|\mathbf{p}')P(\mathbf{p}')q(\mathbf{p}', \mathbf{p})}{L(\mathbf{d}|\mathbf{p})P(\mathbf{p})q(\mathbf{p}, \mathbf{p}')}\right\}$$
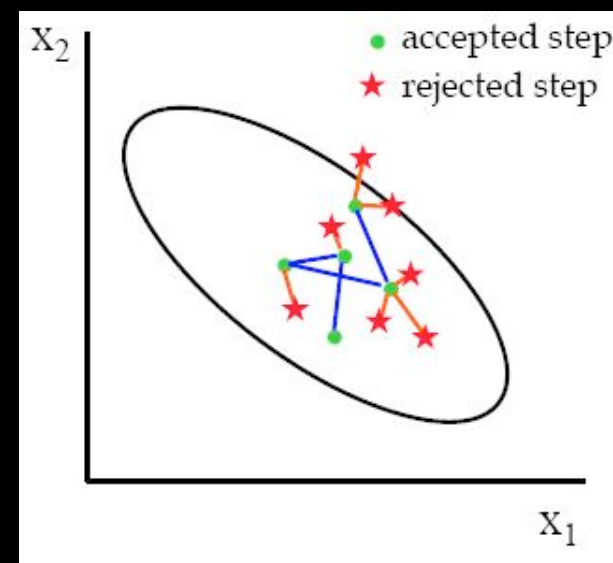

Reject min < 1:
- new point out of prior
- new point with greater chi-square
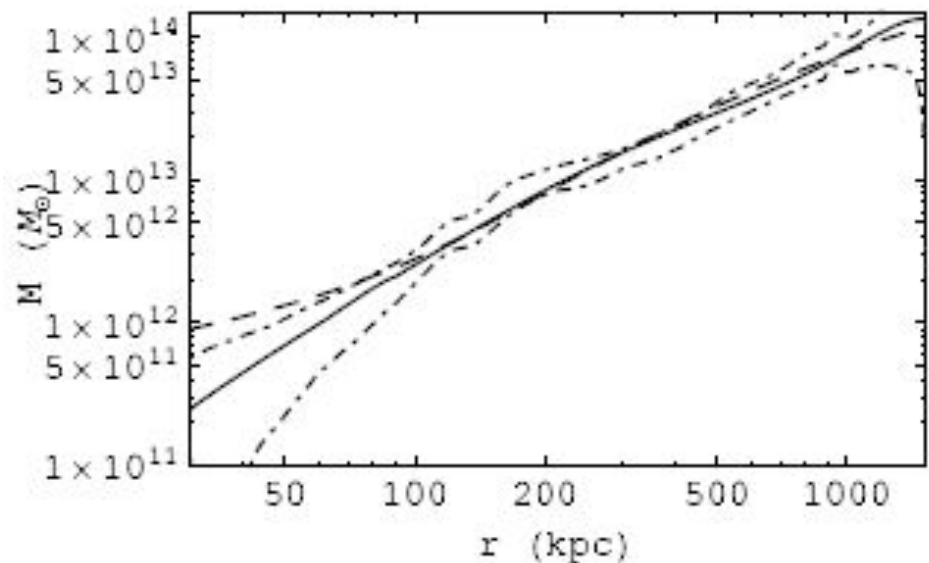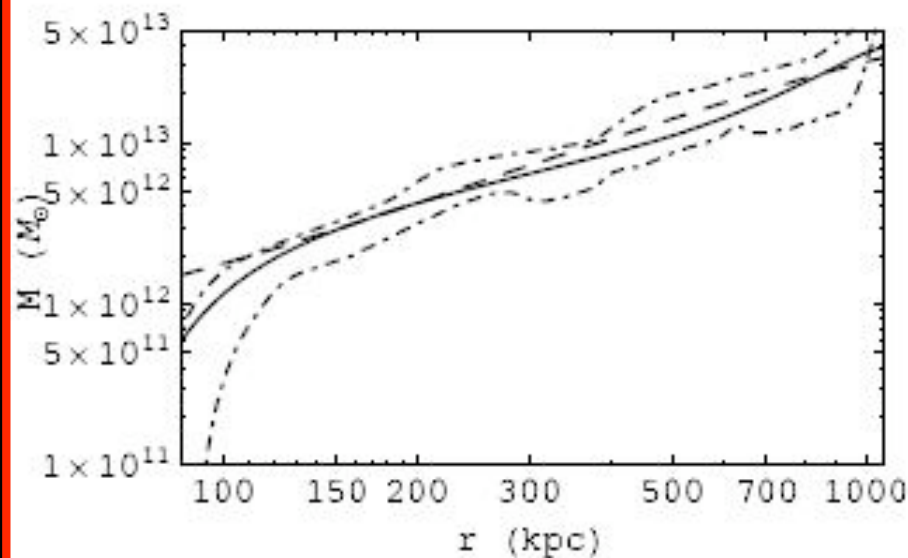
Accept min = 1: new point in prior and less chi-square

Sample of accepted points ⟹ Sampling from underlying probability distribution

- Power spectrum test convergence:

Discrete power spectrum from samples ⟹ Convergence = flat spectrum

Results: gravitational length

- Differences between theoretical and observed fit *less than 5%*

- *Typical scale* in [100; 150] kpc range where is a turning-point:

  • Break in the hydrostatic equilibrium

  • Limits in the expansion series of f(R): $R - R_0 << \frac{a_1}{a_2}$ in the range [19;200] kpc

  Proper gravitational scale (as for galaxies, see Capozziello et al MNRAS 2007)

  • Similar issues in Metric-Skew-Tensor-Gravity (Brownstein, 2006): we have better and more detailed approach

Results

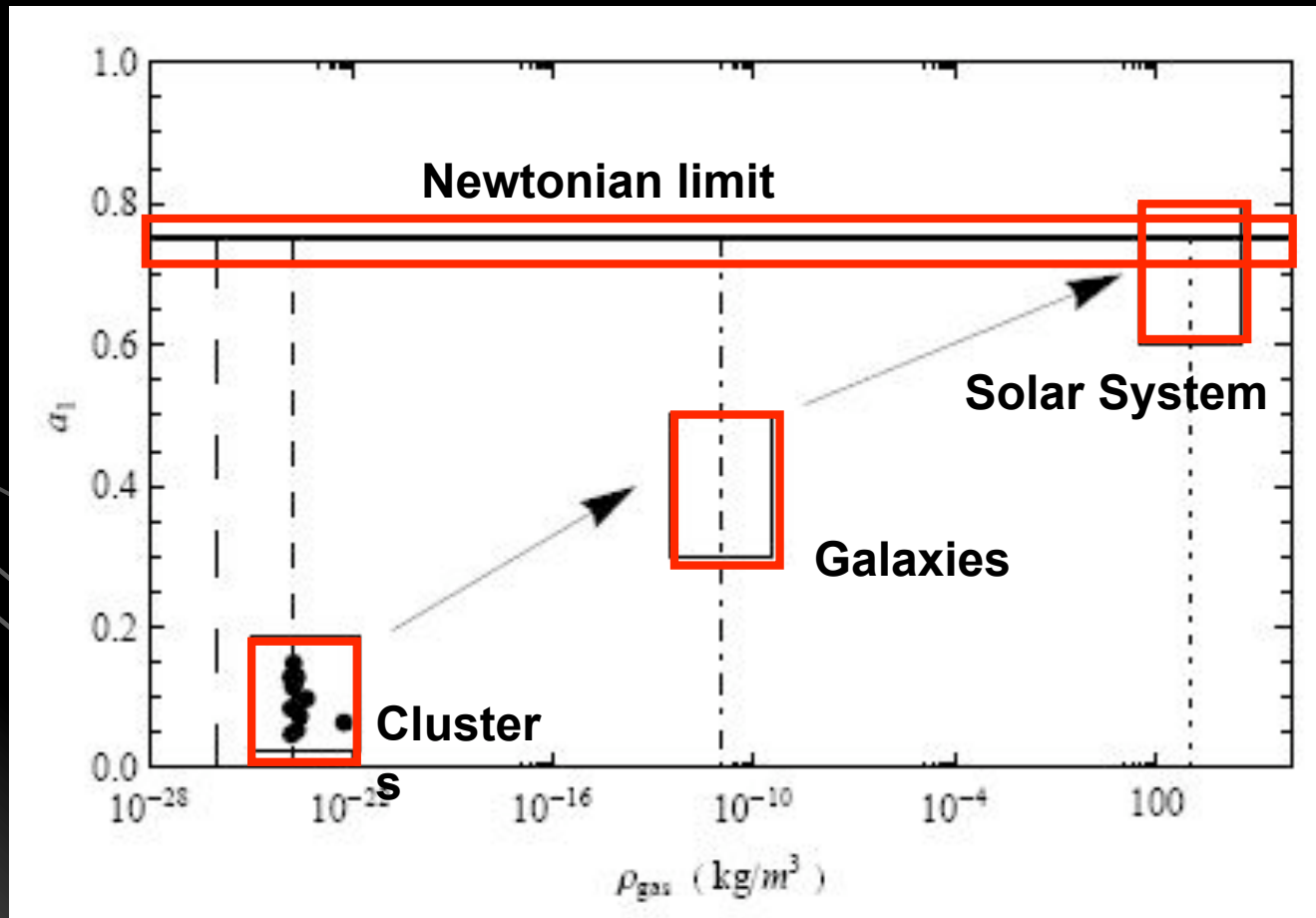| name	a_1	$[a_1 - 1\sigma, a_1 + 1\sigma]$	a_2 (kpc^2)	$[a_2 - 1\sigma, a_2 + 1\sigma]$ (kpc^2)	L (kpc)	$[L - 1\sigma, L + 1\sigma]$ (kpc)
A 133	0.085	[0.078, 0.091]	$-4.98 \cdot 10^3$	$[-2.38 \cdot 10^4, -1.38 \cdot 10^3]$	591.78	[323.34, 1259.50]
A 262	0.065	[0.061, 0.071]	-10.63	$[-57.65, -3.17]$	31.40	[17.28, 71.10]
A 383	0.099	[0.093, 0.108]	$-9.01 \cdot 10^2$	$[-4.10 \cdot 10^3, -3.14 \cdot 10^2]$	234.13	[142.10, 478.06]
A 478	0.117	[0.114, 0.122]	$-4.61 \cdot 10^3$	$[-1.01 \cdot 10^4, -2.51 \cdot 10^3]$	484.83	[363.29, 707.73]
A 907	0.129	[0.125, 0.136]	$-5.77 \cdot 10^3$	$[-1.54 \cdot 10^4, -2.83 \cdot 10^3]$	517.30	[368.84, 825.00]
A 1413	0.115	[0.110, 0.119]	$-9.45 \cdot 10^4$	$[-4.26 \cdot 10^5, -3.46 \cdot 10^4]$	2224.57	[1365.40, 4681.21]
A 1795	0.093	[0.084, 0.103]	$-1.54 \cdot 10^3$	$[-1.01 \cdot 10^4, -2.49 \cdot 10^2]$	315.44	[133.31, 769.17]
A 1991	0.074	[0.072, 0.081]	-50.69	$[-3.42 \cdot 10^2, -13]$	64.00	[32.63, 159.40]
A 2029	0.129	[0.123, 0.134]	$-2.10 \cdot 10^4$	$[-7.95 \cdot 10^4, -8.44 \cdot 10^3]$	988.85	[637.71, 1890.07]
A 2390	0.149	[0.146, 0.152]	$-1.40 \cdot 10^6$	$[-5.71 \cdot 10^6, -4.46 \cdot 10^5]$	7490.80	[4245.74, 15715.60]
MKW4	0.054	[0.049, 0.060]	-23.63	$[-1.15 \cdot 10^2, -8.13]$	51.31	[30.44, 110.68]
RXJ1159	0.048	[0.047, 0.052]	-18.33	$[-1.35 \cdot 10^2, -4.18]$	47.72	[22.86, 125.96]

Results

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Results: expectations

- First derivative, a_1 : very well constrained ⟹ *It scales with the system size*

$a_1 \sim 3\text{-}4$

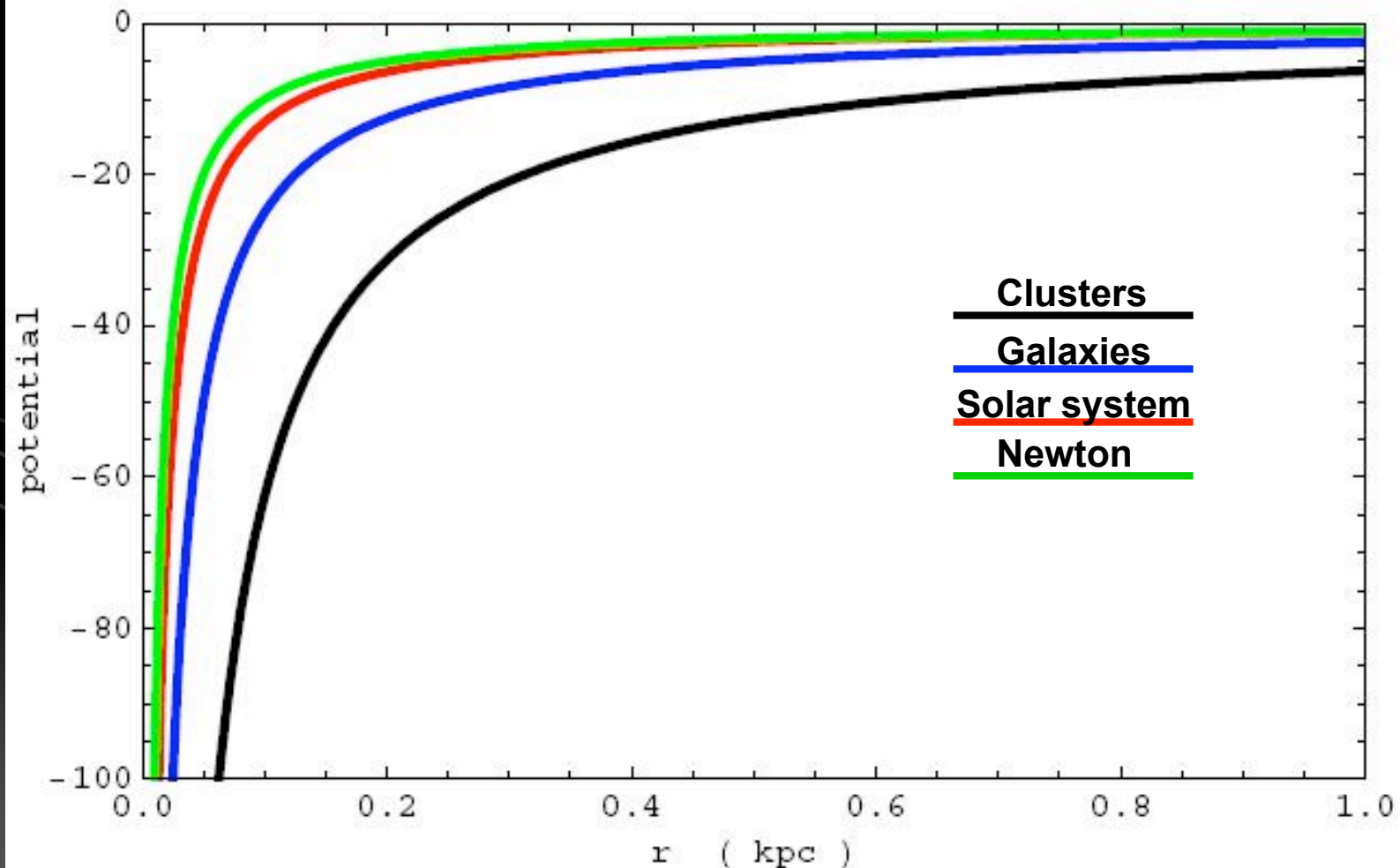- Newtonian limit: $\phi(r) = -\dfrac{3GM}{4a_1 r}\left(1 + \dfrac{1}{3}e^{-\frac{r}{L}}\right)$ ⟹

Point like potential:

Cluster of Galaxies: $a_1 = 0.16 - L = 1000$ kpc

Galaxies: $a_1 = 0.4 - L = 100$ kpc

Solar System: $a_1 = 0.75 - L = 1$ kpc

Newton Limit: $a_1 = 0.75 - L = 0$ kpc

Results

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Results: expectations

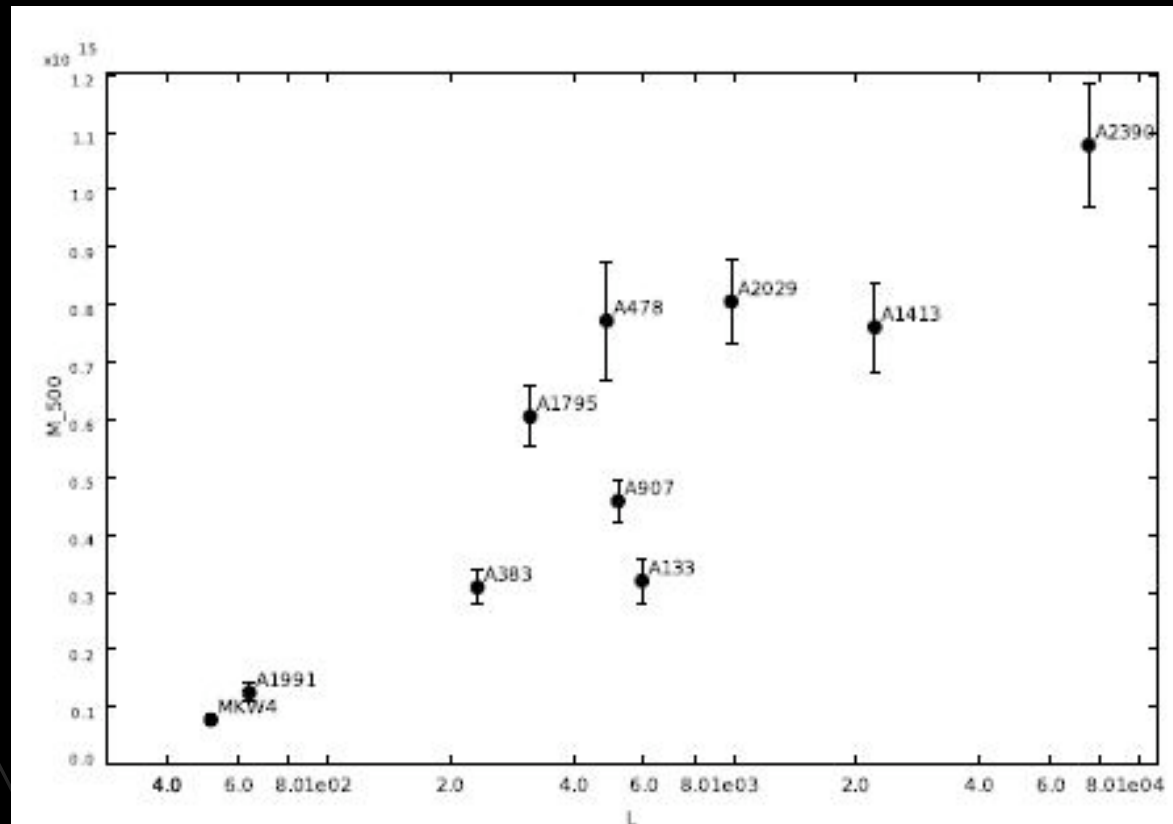- Gravitational length: $L \equiv L(a_1, a_2) = \left(-\dfrac{6a_2}{a_1}\right)^{1/2}$ ⟷ **Strong characterization of Gravitational potential**

- Mean length:
$$< L >_\rho = 318 \text{ kpc} \qquad < a_2 >_\rho = -3.40 \cdot 10^4$$
$$< L >_M = 2738 \text{ kpc} \qquad < a_2 >_M = -4.15 \cdot 10^5$$

- Strongly related to virial mass (the same for gas mass):

- Strongly related to average temperature:

Results: expectations

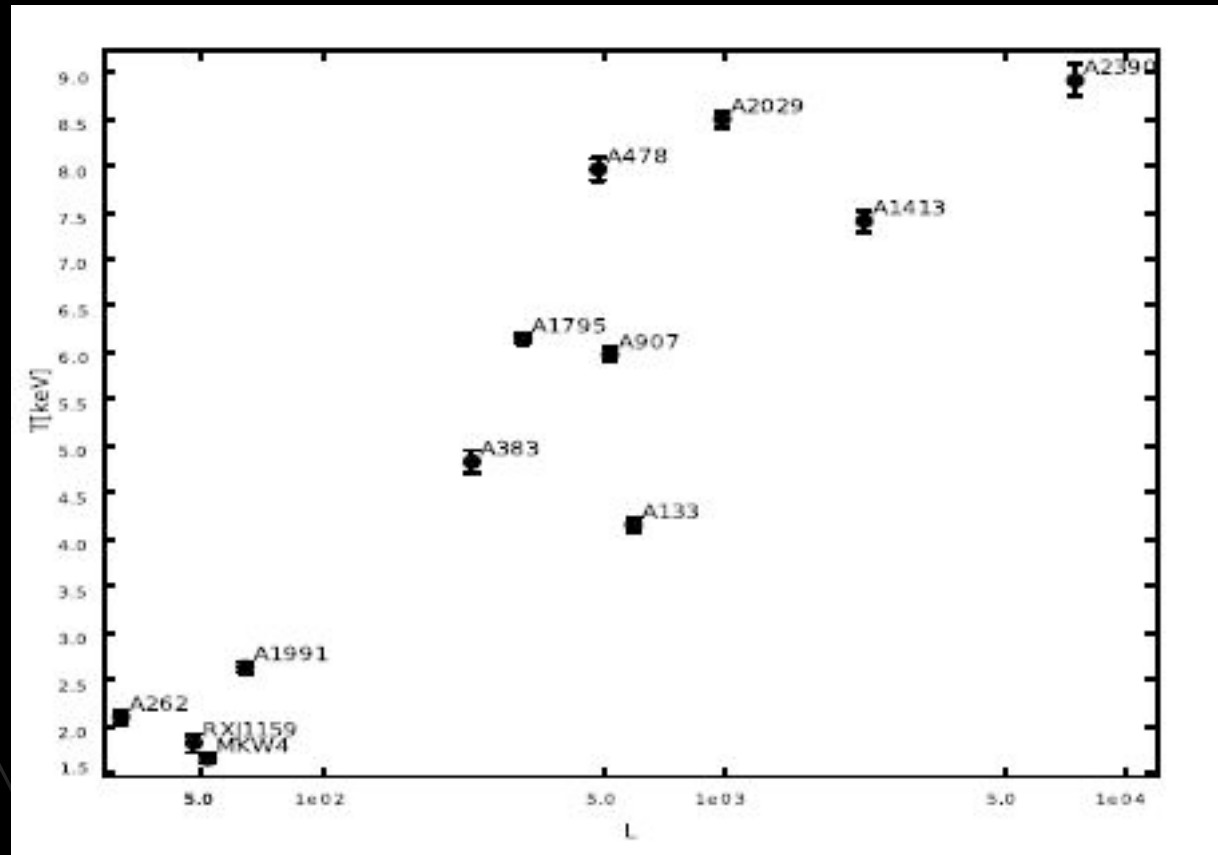- Gravitational length: $L \equiv L(a_1, a_2) = \left(-\dfrac{6a_2}{a_1}\right)^{1/2}$ ⟷ **Strong characterization of Gravitational potential**

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- Strongly related
to virial mass
(the same for gas mass):

- Strongly related
to average temperature:

Conclusions

- Cosmography: model independent approach to f(R) -gravity

- Cosmographic parameters to constraint f(R) - gravity models

- Cosmography to "discriminate" between Dark Energy and f(R)

Perspectives:
- Montecarlo simulations to assess precision on cosmography
- Combine different datasets to strengthen the constraints
- Introduce theoretically motivated priors on cosmography

- Fitting Large Scale Structure with f(R) gravity (Clusters of Galaxies)

- Well motivated f(R) models (in agreement with observations)

- f(R) parameters strongly characterize gravitational systems

Perspectives:
- Extending to any self-gravitating systems
- Recover Newtonian limit and evade Solar System tests
- Understand physical meaning or dependency of parameters