Dark Metric in the Non-Linear Palatini Formalism

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Torino, 1987

Torino, 2008

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The need for dark matter (and dark energy) is - among other things - also based upon the following statement: *"Among the different Theories of Gravitation we should prefer the simplest (*in the sense of *the one with the simplest Lagrangian)".* Dark matter (and dark energy) seem in fact to be necessary to explain observational data, interpreted inside the paradigm of General Relativity (**GR**). GR is based on a very simple variational principle, constructed assuming the metric g of SpaceTime as the fundamental object.

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Question 2: *Was Einstein right in making this assumption?*

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Question 1: *Well… really should we?*
Question 2: *Was Einstein right in making this assumption?*

Our answer to both questions is: *probably not!*

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either:

1) **we do not yet understand well the matter content of the Universe** (practically 95% of the estimated matter is “dark” and yet invisible if not indirectly);

or:

2) **we do not yet understand well the gravitational aspects of our Universe** (there is no theoretical evidence that Gravity is obliged to obey Einstein Equations)

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3) **we do not understand both aspects….!**

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2) **we do not yet understand well the gravitational aspects of our Universe** (Einstein Equations follow from Hilbert Lagrangian, that is linear in curvature but rather singular from the Hamiltonian viewpoint);

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because of this we argue that:

3) **to better understand both aspects one should renounce to use Einstein Equations “rigidly” and allow a broader framework for gravity**

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… but unfortunately field equations turn to be of fourth order….!

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… but also to separate metric effects (measurements) from gravitational effects (free fall)

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4) **allow a framework “à la Palatini”;**
5) **avoid any kind of primordial prejudice against the need of revisiting Einstein Theory in both its mathematical and physical structure**

In so-called "Palatini" (or "first-order") approach to Gravity, one assumes that the properties of the gravitational field are mediated by two independent fields:

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Γ defines the **curvature** of SpaceTime (**free-fall**) through its Riemann and Ricci tensors

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In so-called **nonlinear** “Palatini” (or “first-order”) approach to Gravity, one assumes that the properties of the gravitational field are mediated by two independent fields (g and Γ) and **the Lagrangian is assumed to be a non-linear function of the curvature of Γ.**

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Γ by itself has no "scalar" curvature; it can only be defined by using the metric g as an instrument to measure curvature, by setting:

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$$L_{grav} = f(R) = f(R(\Gamma,g))$$

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Again the simplest choice is to assume that **the gravitational part of the Lagrangian is a non-linear arbitrary function of the “scalar” curvature**, while the full action contains also a (minimally coupled) matter Lagrangian:

$$L_{tot} = L_{grav} + L_{mat} = f(R) + L_{mat}(g,\partial g,\phi,\nabla^{g}\phi)$$

In **nonlinear f(R) “Palatini” approach** to Gravity based on the Lagrangian:

$$L_{tot} = L_{grav} + L_{mat} = f(R) + L_{mat}(g, \partial g, \phi, \nabla^{g} \phi)$$

we have two gravitational field equations, one for g (obtained by $\delta/\delta g$) and one for Γ (obtained by $\delta/\delta\Gamma$), both of order two - besides the specific field equation for matter that follows from varying with respect to ϕ.

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These two "gravitational" equations are (in dim=4):

$$f'(R)R_{\mu\nu} - 1/2\, f(R)g_{\mu\nu} = T_{\mu\nu}$$

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The trace of the first dynamical equation ($\delta/\delta g$) gives the **master equation**:

$$f'(R)R - 2f(R) = \tau$$

$$\tau = g^{\mu\nu}\, T_{\mu\nu}$$

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The **master equation** ensures in turn that the scalar curvature $R(g,\Gamma)$ can be expressed - via its analytic inversion (implicit function theory) - as a function of the trace τ of the energy-momentum tensor of ordinary matter:

$$R = R(\tau)$$

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Inserting this back into f’(R) - i.e., the first derivative of f(R) with respect to R - gives finally the factor f’(R) as a function of τ:

$$f'(\tau) = f'(R(\tau))$$

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Inserting this into the other field equation (obtained by $\delta/\delta\Gamma$), field equations imply that Γ is the Levi-Civita connection of a new metric h related to g by the conformal transformation

$$h^{\mu\nu} = f'(R)g^{\mu\nu}$$

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The second field equation (obtained by $\delta/\delta\Gamma$), in fact, under the conformal transformation $h^{\mu\nu} = f'(R)g^{\mu\nu}$ is turned into the following:

$$\nabla^{\Gamma}_{\lambda} (\sqrt{h}\, h^{\mu\nu}) = 0$$

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and by a well known result of Differential Geometry (Levi-Civita Theorem) this in turn entails for a torsionless Γ:

$$\Gamma = \Gamma_{LC}(h)$$

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and obliges us to see **extra matter as an induced curvature effect** (**dark matter and dark energy**)

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and obliges us to see **extra matter as an induced curvature effect** (**dark matter and dark energy**);

5) **an additional term in the stress appears due to nonlinearity** and Γ

In **nonlinear f(R) “Palatini” approach** to Gravity based on the Lagrangian:

$$L_{tot} = L_{grav} + L_{mat} = f(R) + L_{mat}(g, \partial g, \phi, \nabla^{g} \phi)$$

then the following conclusions can be drawn:
1) **the main dynamical field (gravitational field) is** Γ, while the metric g is at the beginning **a field with “induced dynamics”**;
2) field equations tell us that the connection Γ is a posteriori a metric connection, the new metric h being conformally related with the original g by a factor that depends on the choice of the function f and on matter;
3) this new metric is the **“gravitational metric”** (also **“dark metric”**) - it **tells how matter falls under the gravitational field** Γ and **deforms locally, point by point, the true metric g that we use to set up measures**;
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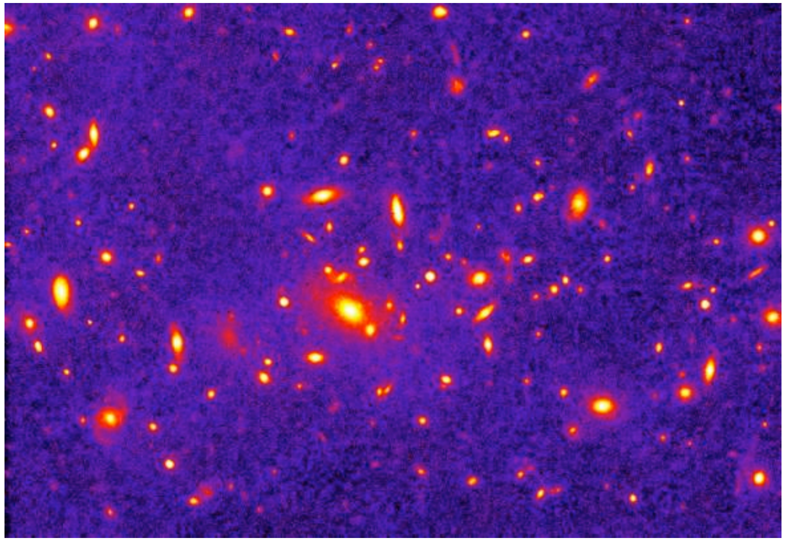
7) **when** (and only when) f(R) = R **the theory reduces to Einstein gravity**.

CONCLUSION: Is Dark Matter really “matter” or is it rather a (non-linear) effect of curvature……?

What we have seen until now tells us that we do not understand Gravity at a full scale if we insist in pretending that: 1) the metric is the fundamental field; 2) the Lagrangian is linear in the curvature



Assuming non-linear functions of the curvature allows one to find “exotic effects” in Einstein Equations at different scales (Solar System, Galaxy, Extra Galactic, Cosmological) as curvature effects due to non-linearity of the Lagrangian rather that effects due to unseen matter or energy (“the dark side of the Universe”).

Palatini approach - moreover - tell us why the metric we see generating Gravity is different, point by point, by the metric we use to make experiments. **Gravity drives the “gauge” of our instruments**. Clocks tick slower in a Gravitational field. Curvature tunes up Gravity and forces us to redefine rods and clocks….!

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