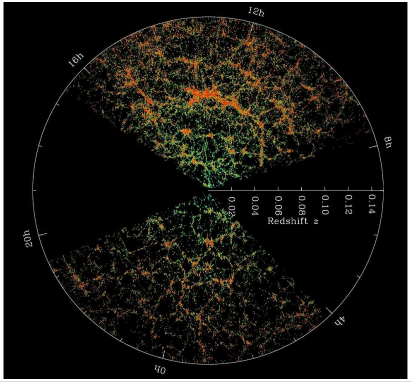

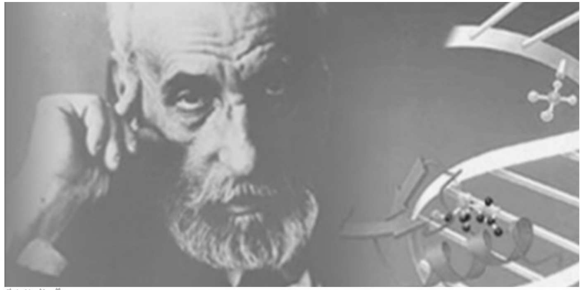

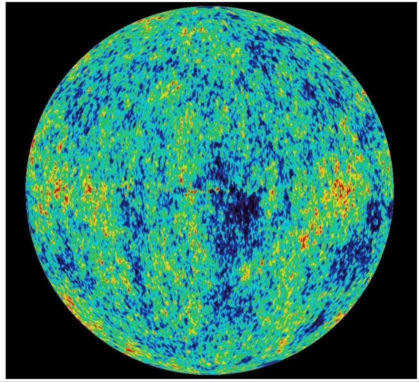

INSTITUTO DE ESTRUCTURA DE LA MATERIA
CONSEJO SUPERIOR DE INVESTIGACIONES CIENTÍFICAS
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New Phenomenology for Palatini f(R): Non-singular Universes

Gonzalo J. Olmo

Instituto de Estructura de la Materia - CSIC (Spain)

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- Structure of the talk:
  - Basics of the Palatini $f(R)$ field equations.
  - What is new in these theories?
  - Infrared and Ultraviolet corrected models.
  - Applications to the early Universe and Quantum Gravity.

Basics of Palatini and IR-Corrected Models

Basics of Palatini

- The action is defined as $S[g,\Gamma,\psi_m] = \frac{1}{2\kappa^2}\int d^4x\sqrt{-g}f(R) + S_m[g_{\mu\nu},\psi_m]$

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  Metric variation: $f_R R_{\mu\nu}(\Gamma) - \frac{f}{2} g_{\mu\nu} = \kappa^2 T_{\mu\nu}$ where $f_R = df/dR$

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- The metric equations can be written in two alternative ways. Using $g_{\mu\nu}$:

$$G_{\mu\nu}(g)=\frac{\kappa^2}{f_R}T_{\mu\nu}-\frac{Rf_R-f}{2f_R}g_{\mu\nu}+\frac{1}{f_R}\left(\nabla_\mu\nabla_\nu f_R-g_{\mu\nu}\Box f_R\right)-\frac{3}{2f_R^2}\left(\partial_\mu f_R\partial_\nu f_R-\frac{1}{2}g_{\mu\nu}(\partial f_R)^2\right)$$

Or using $h_{\mu\nu}$:

$$G_\mu{}^\nu(h)=\frac{\kappa^2}{f_R^2(T)}\left[T_\mu{}^\nu-\left(\frac{Rf_R-f}{2\kappa^2}\right)\delta_\mu{}^\nu\right]$$

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- In vacuum, both reduce to $G_{\mu\nu} = -\Lambda_{eff}g_{\mu\nu}$,

  where $\Lambda_{eff} = \left.\frac{Rf_R - f}{2f_R}\right|_0$.

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- UV-corrected models may be designed to avoid the Big Bang singularity.
  - Bouncing and cyclic cosmologies are very common in Palatini $f(R)$.
  - There exist important links between Palatini and the effective dynamics of Loop Quantum Cosmology.

Hydrogen and Palatini Gravity

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- The positive energy spinor η in the non-relativistic limit satisfies:

$$\mathcal{E}\eta = \left\{ \frac{1}{\tilde{m}+m_0} [(\vec{p} - e\vec{A})^2 - e\vec{\sigma} \cdot \vec{B}] + eA_0 + V(\Omega) + \Delta m \right\} \eta$$

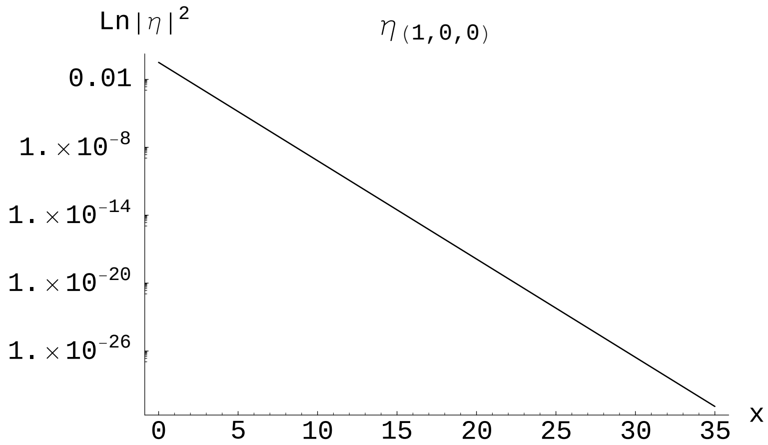
where $E = m_0 + \mathcal{E}$, $T \approx -m\eta^\dagger\eta$, $\Delta m = \tilde{m} - m_0$, and

$$V(\Omega) = \frac{1}{\tilde{m}+m_0} \left[i\vec{\sigma}(\vec{\nabla}\Omega \times \vec{\nabla}) - 2ie(\vec{A} \cdot \vec{\nabla}\Omega) + 2(\vec{\nabla}\Omega \cdot \vec{\nabla}) + \vec{\nabla}^2\Omega - |\vec{\nabla}\Omega|^2 \right]$$

The gravity lagrangian induces new interaction terms $V(\Omega)$ and Δm

Ground state $\eta_{(1,0,0)}$ for $f(R) = R - \mu^4/R$

$$\boxed{\text{Wavefunction} \sim e^{-r/a_0}}$$



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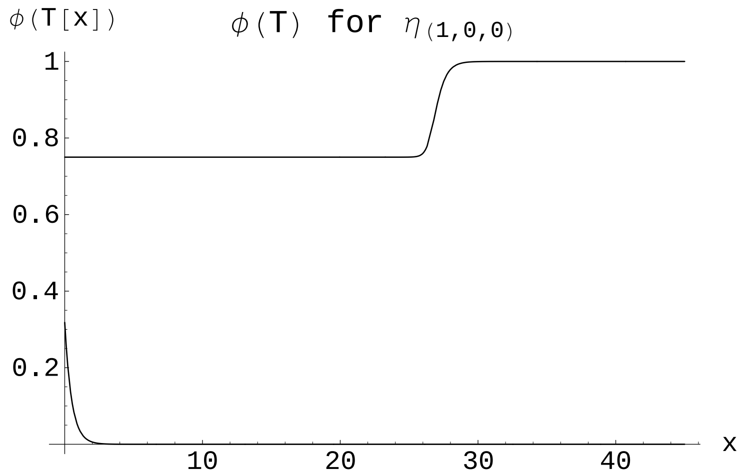

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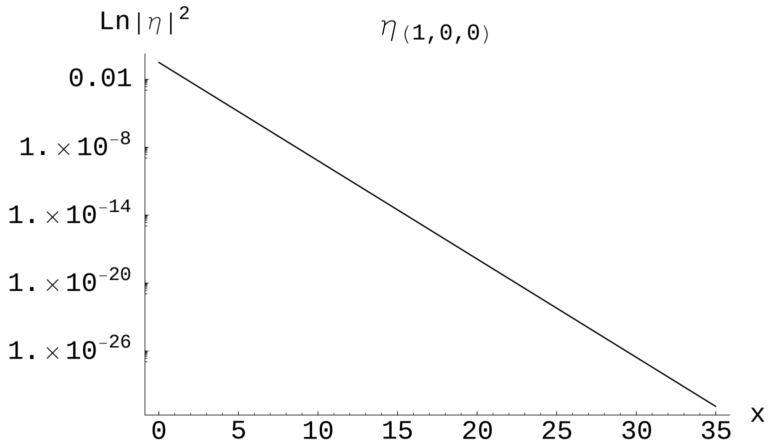

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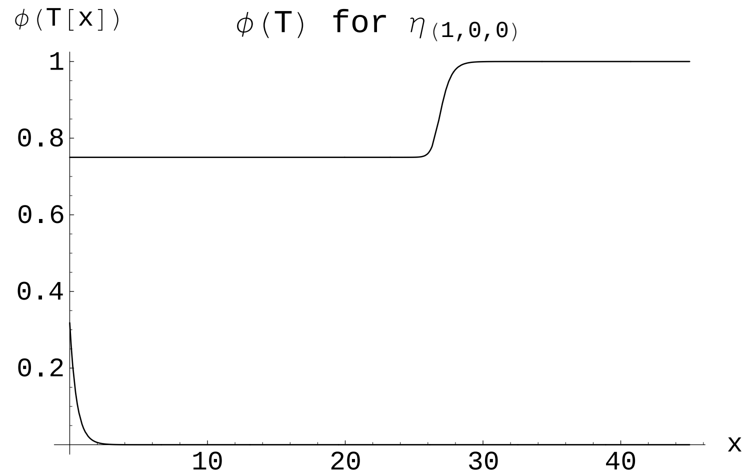
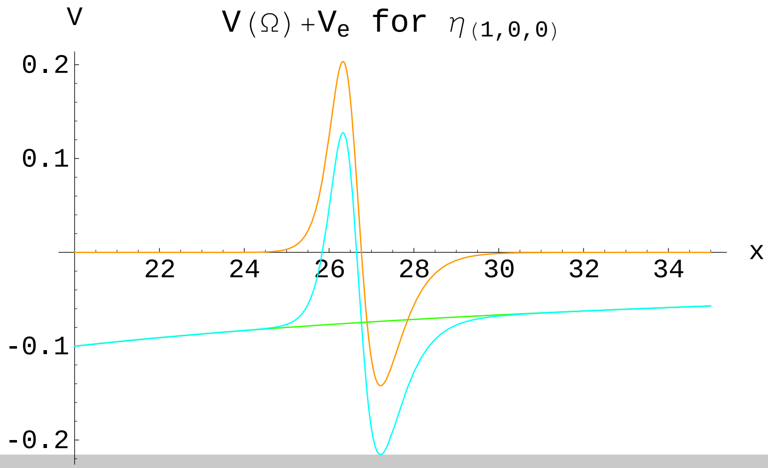
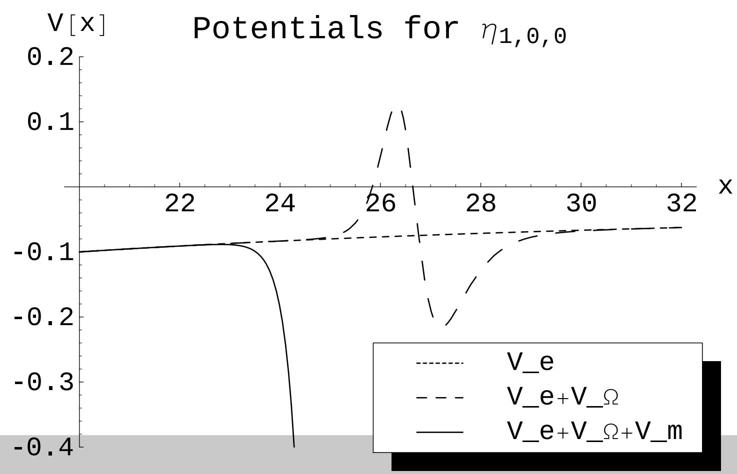


$$V(\Omega) + V_{electric}$$



$$\Delta m + V(\Omega) + V_{electric}$$



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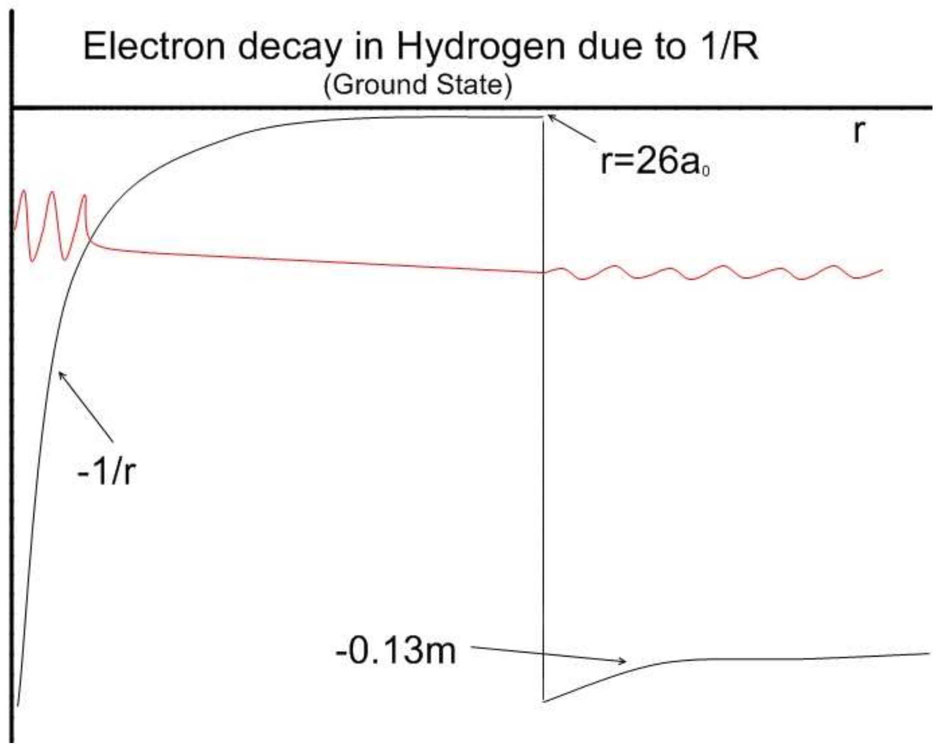
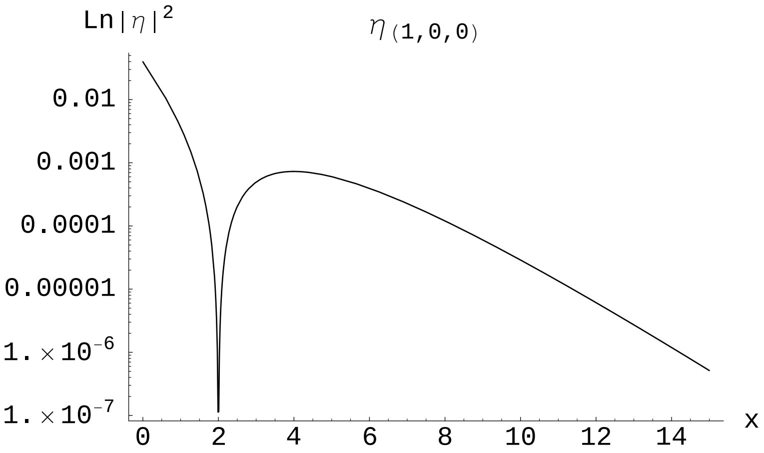


- The potential well induced by Δm tends to $\Delta m \approx -0.13m \approx -5000 \times 13'6$ eV
- This well triggers the flux of probability density to the exterior of the atom.
- The electron escapes similarly as in nuclear alpha decay.
- In the $1/R$ model, the half life of the electron is $\tau_e \sim 6 \times 10^3$ s.

Non-perturbative effects in $\eta_{(2,0,0)}$

Wavefunction



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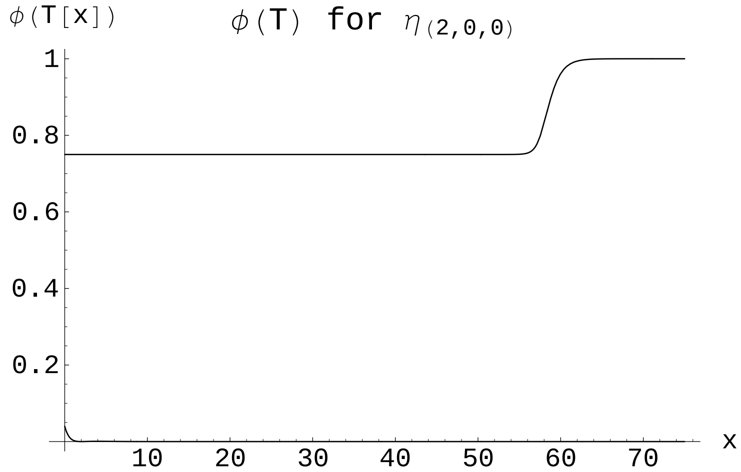

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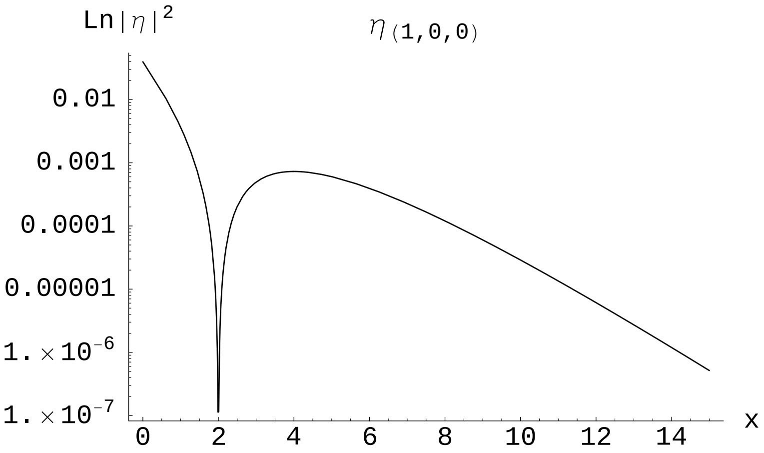

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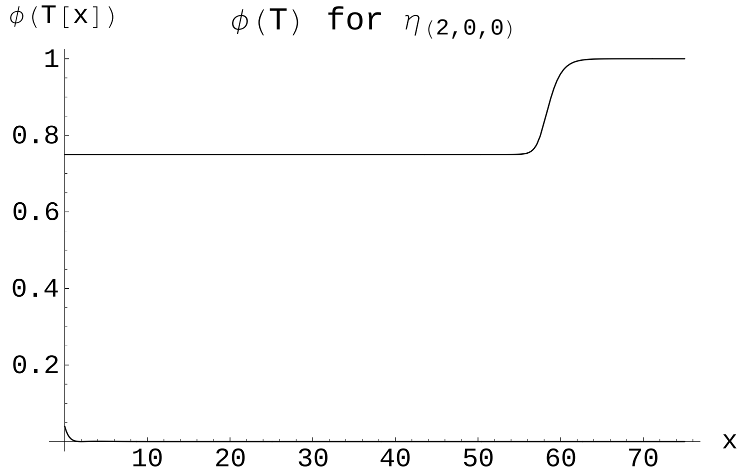
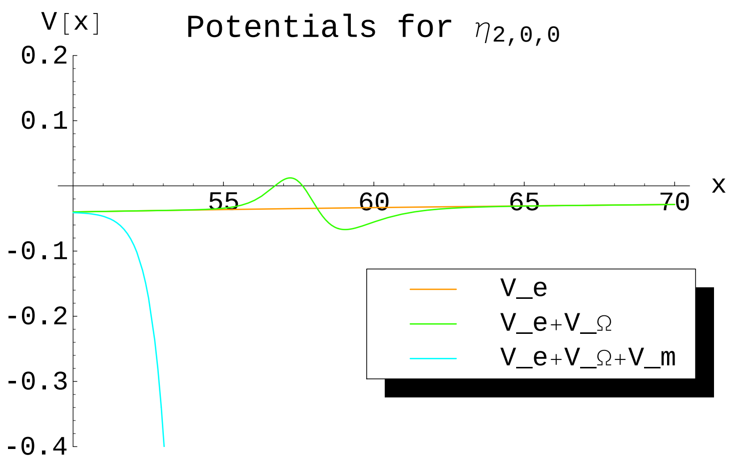
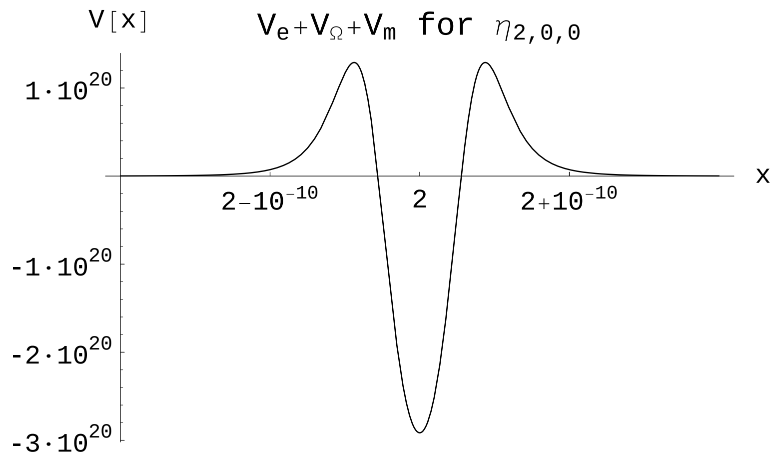


Outer V_{Total}



V_{Total} at $r = 2a_0$



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- Strong non-perturbative gravitational effects arise near the zeros (radial and angular!) of the wave functions.

> Strong gravitational effects occur in IR models when their characteristic density scale is reached.
> Are there similar effects in UV models?

UV models and Non-singular Universes

A Non-singular Universe

- In the simplest UV model, $f(R) = R \pm \frac{R^2}{R_P}$, we find

$$H^2 = \frac{\kappa^2\rho}{3} \frac{\left(1+\frac{2(1-3w)\kappa^2\rho}{R_P}\right)\left(1+\frac{(1-3w)^2}{2}\frac{\kappa^2\rho}{R_P}\right)}{\left[1-(1-9w^2)\frac{\kappa^2\rho}{R_P}\right]^2} - \frac{K}{a^2}$$

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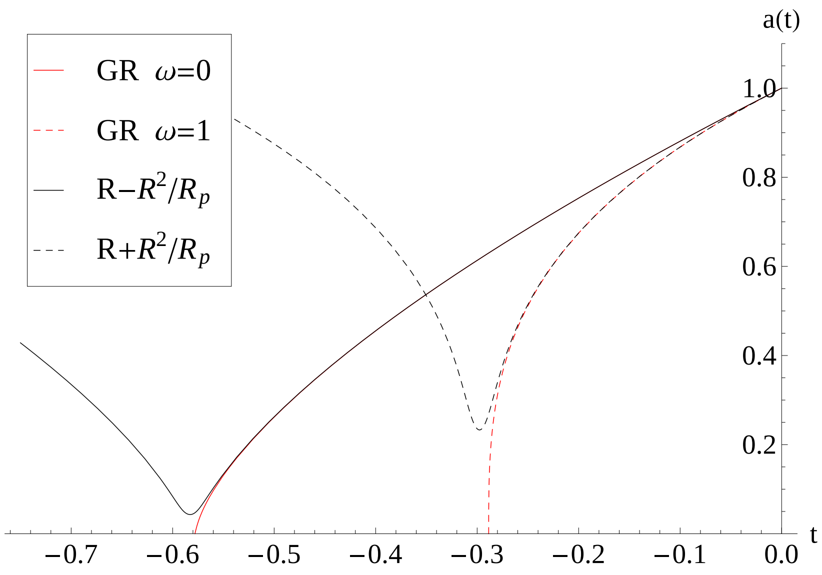
Flat Universe: $K = 0$.



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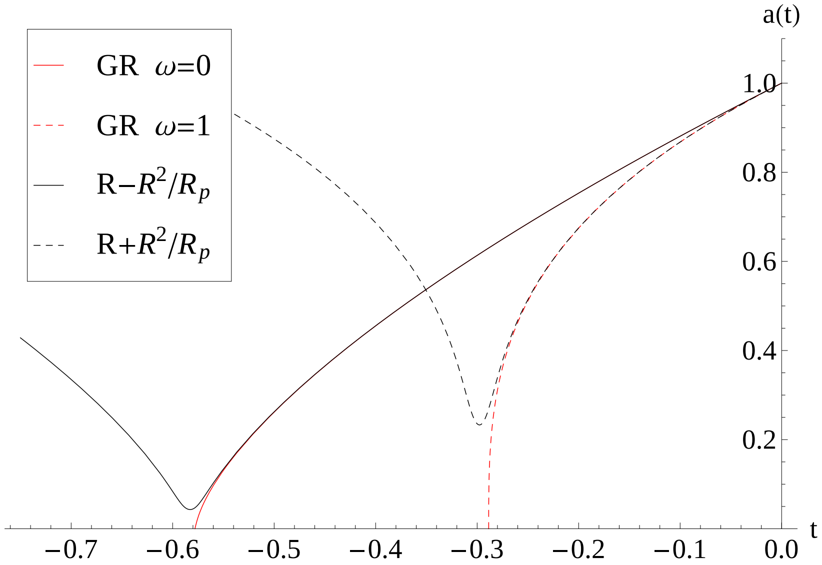
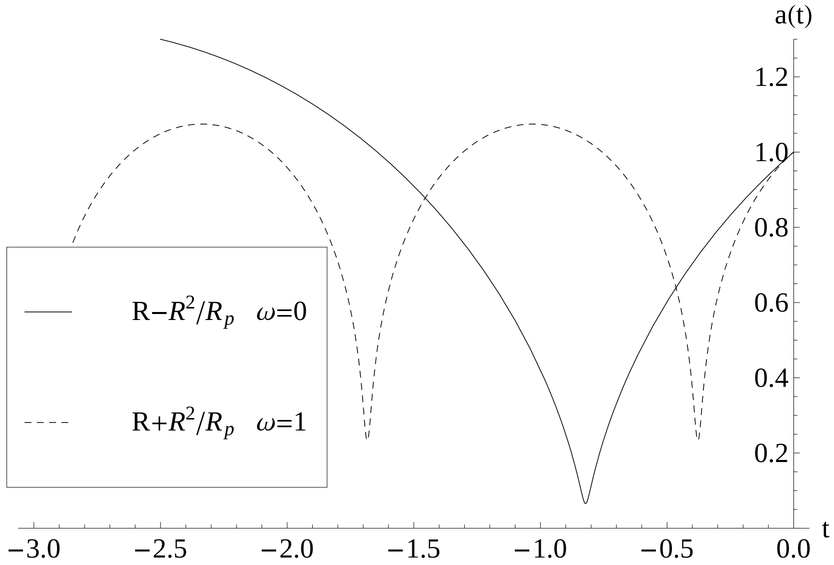


Closed Universe: $K > 0.$



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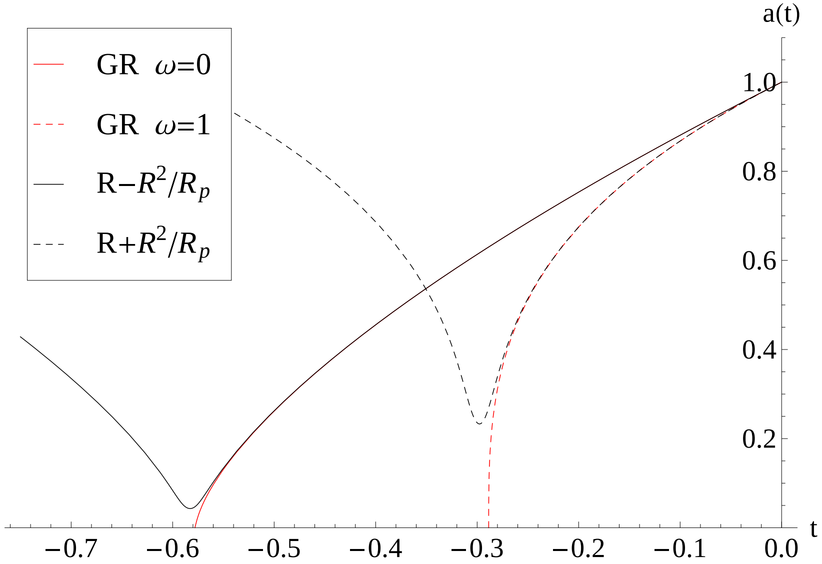

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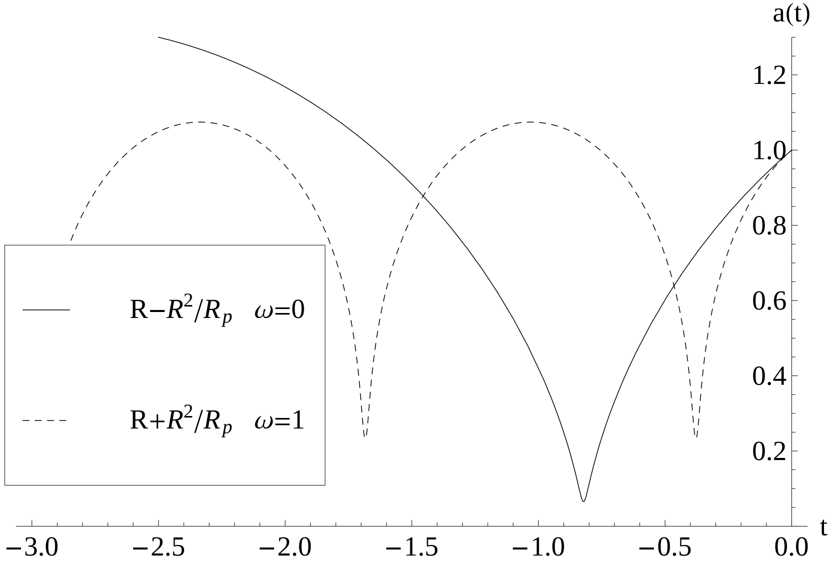

- In metric formalism there are no bouncing solutions of this type.

Characterizing the Bounce.

- For a general $f(R)$ theory, the Hubble function is given by ($P = w\rho$)

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- Cosmic bounce occurs whenever $H^2 = 0$. When $K = 0$, this may happen if [C.Barragán, G.J.O, and H.Sanchis-Alepuz , To appear]
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> It is possible to construct Palatini $f(R)$ theories as successful as GR at low energies but free from high curvature singularities.
> Is this just an accident or is there anything deeper?

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- No new degrees of freedom in LQC. Matter alone can cure the singularity.
- The non-perturbative dynamics of LQC can be captured by a Palatini $f(R)$.
  - Use the trace equation $(3w-1)\kappa^2\rho = R f_R - 2f$ to find $\rho = \rho(R, f, f_R)$.
  - For $K = 0$ and $w = 1$: $8\pi G\rho\left(1-\frac{\rho}{\rho_{crit}}\right) = \frac{f_R\left(\kappa^2\rho+(\mathcal{R} f_R - f)/2\right)}{\left(f_R - \frac{12\kappa^2\rho f_{RR}}{2(\mathcal{R} f_{RR} - f_R)}\right)^2}$.
  - Simple manipulations lead to the following o.d.e.:

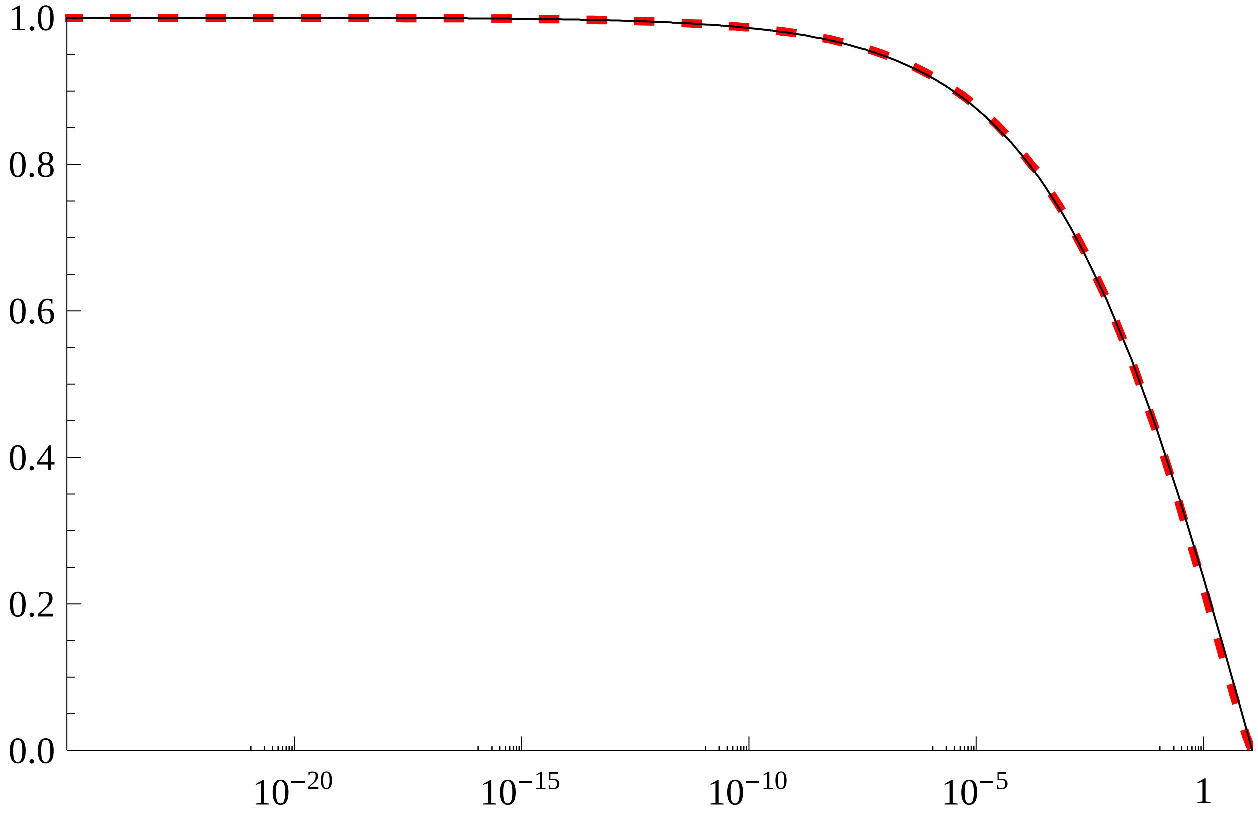
$$f_{RR} = -f_R\left(\frac{A f_R - B}{2(\mathcal{R} f_R - 3f)A + \mathcal{R} B}\right)$$

where

$$A = \sqrt{2(\mathcal{R} f_R - 2f)(2\mathcal{R}_c - [\mathcal{R} f_R - 2f])}\,,$$
$$B = 2\sqrt{\mathcal{R}_c f_R(2\mathcal{R}_c f_R - 3f)}, \text{ and } \mathcal{R}_c \equiv \kappa^2\rho_c.$$

Numerics and Functional Fits

- Dashed red line: Numerical Curve. [G.J.O and P.Singh, JCAP030(2009)]
- Solid line: $\frac{df}{dR} = -\tanh\left(\frac{5}{103}\ln\left[\left(\frac{R}{12\mathcal{R}_c}\right)^2\right]\right)$



Vertical axis: df/dR ; Horizontal axis: R/R_c

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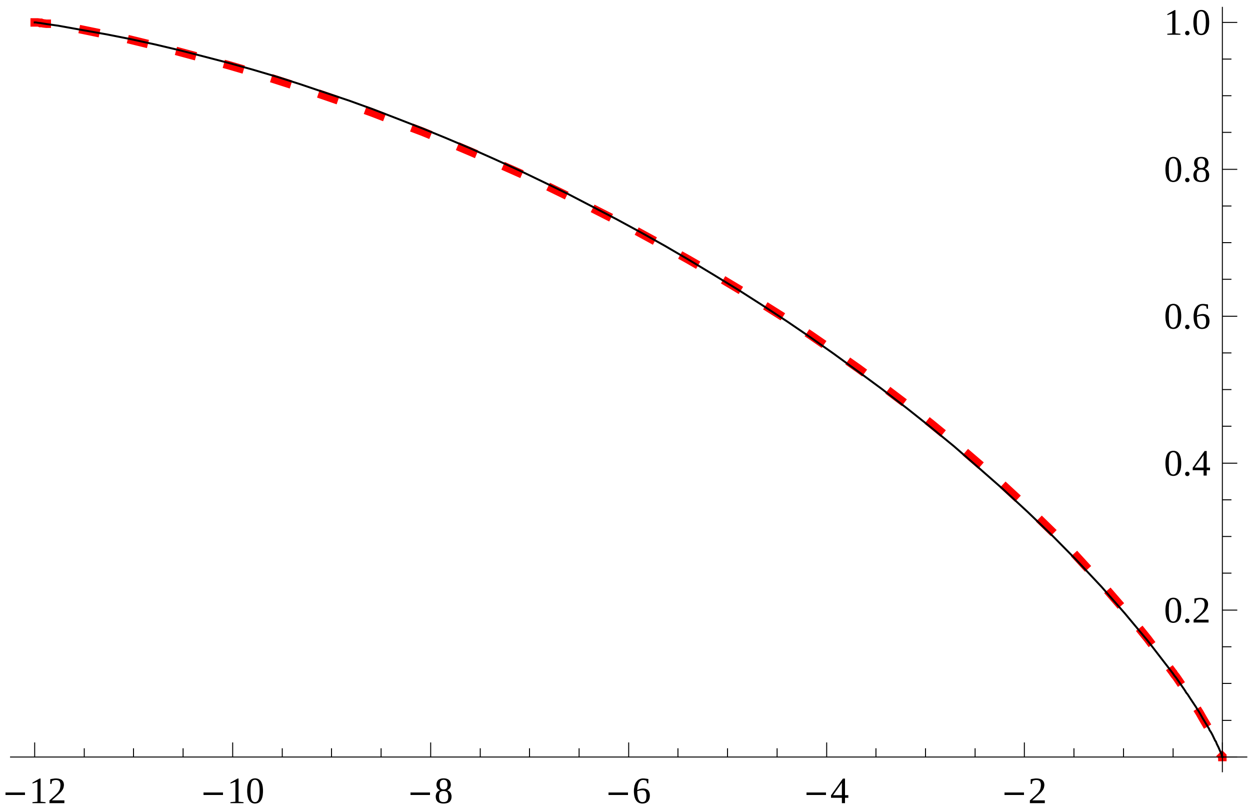

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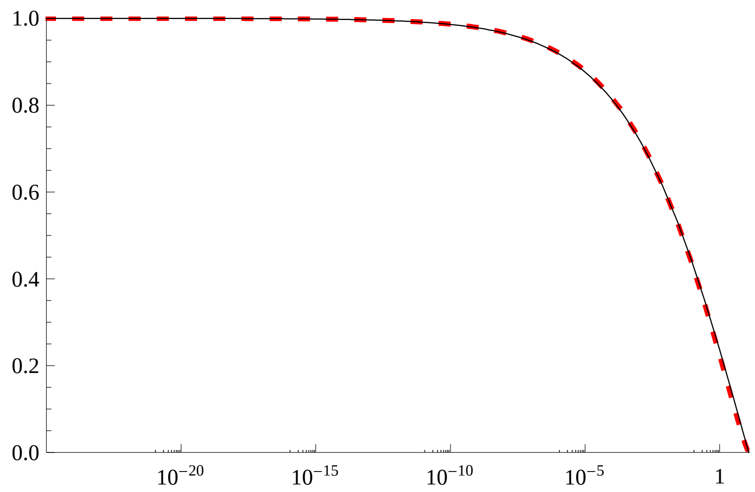

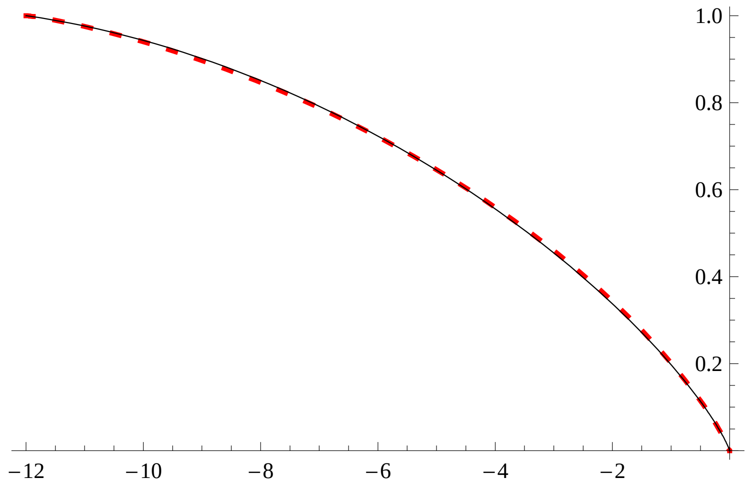

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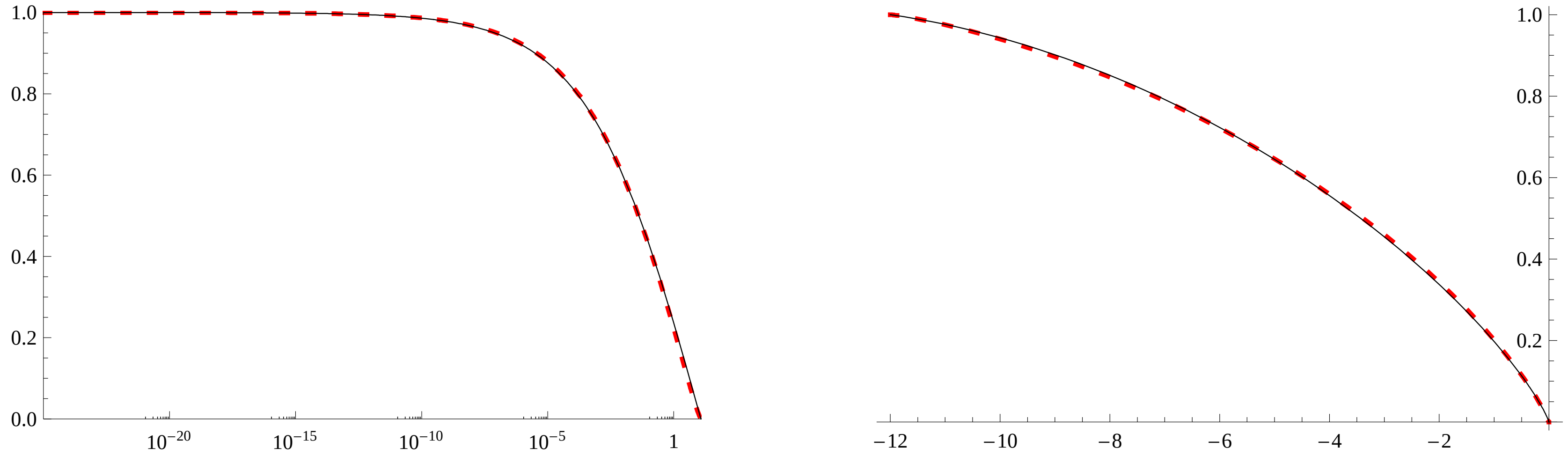

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> Palatini $f(R)$ is able to fully capture the dynamics of *LQC* even though the form of the lagrangian near the bounce cannot be estimated by perturbative methods.

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- About this talk ...

Basics of Palatini

UV models
- A Non-singular Universe
- Characterizing the Bounce
- Palatini $f(R)$ and LQC
- Numerics and Fits
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The End

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- LQC can be exactly reproduced by a Palatini $f(R)$ action. Palatini theories could play an important role in phenomenological descriptions of quantum gravity.
- Future work:
  - Are black holes non-singular in Palatini $f(R)$?
  - How is the dynamics of extended Palatini theories $f(R, R_{\mu\nu}R^{\mu\nu}, \ldots)$?

Thanks !!!