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Tully-Fisher relation, key to dark matter in galaxies

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In its simplified and heuristic form, the Tully-Fisher relation (TF) states: The orbital speeds of outlying objects in spiral galaxies are almost constants proportional to the fourth root of the mass of the galaxies.

$$v^2 = r\frac{d\phi}{dr} \propto M^{1/2} \qquad (1)$$

There are two points to be noted:

a) Dependence of the gravitational force on its source, *M*, is nonlinear, and

b) *M* is an integral feature of the gravitating system. It indicates non-locality in the law of gravitation.

Any formulation of the law of gravitation should take these two features into account.

In the weak field regime, we propose a nonlinear and non-local action integral and attempt to deduce the law of gravitation through a variational principle.

$$I = \int \left[|\nabla \phi|^2 + \rho\phi \right] d^3x + \Lambda \int \frac{\rho(x')\phi(x)}{|x - x'|^\beta} d^3x' d^3x, \quad (2)$$

where ϕ and ρ are the gravitational potential and mass density, respectively, and Λ and β are constants to be discussed. Setting $\delta I / \delta\phi = 0$ gives

$$\nabla^2\phi + \rho + \Lambda \int \frac{\rho(x')}{|x - x'|^\beta} d^3x' = 0. \quad (3)$$

Eq. (3) is expandable in spherical harmonics. Its spherically symmetric component in the exterior of the gravitating system gives

$$r\frac{d\phi^{(e)}}{dr}=\frac{M}{r}+\frac{\Lambda}{(3\text{-}\beta)}\frac{M}{r^{\beta\text{-}2}}, \quad r\geq R,\ \text{system size} \quad (4)$$

From the Tully-Fisher relation

a) $\beta-2=\gamma\ll 1,$ probably $\prec 0.001,$

based on our examination of a good number of rotation curves.

b) Λ is an integral of the system, closely mimicking $M^{-1/2}$.

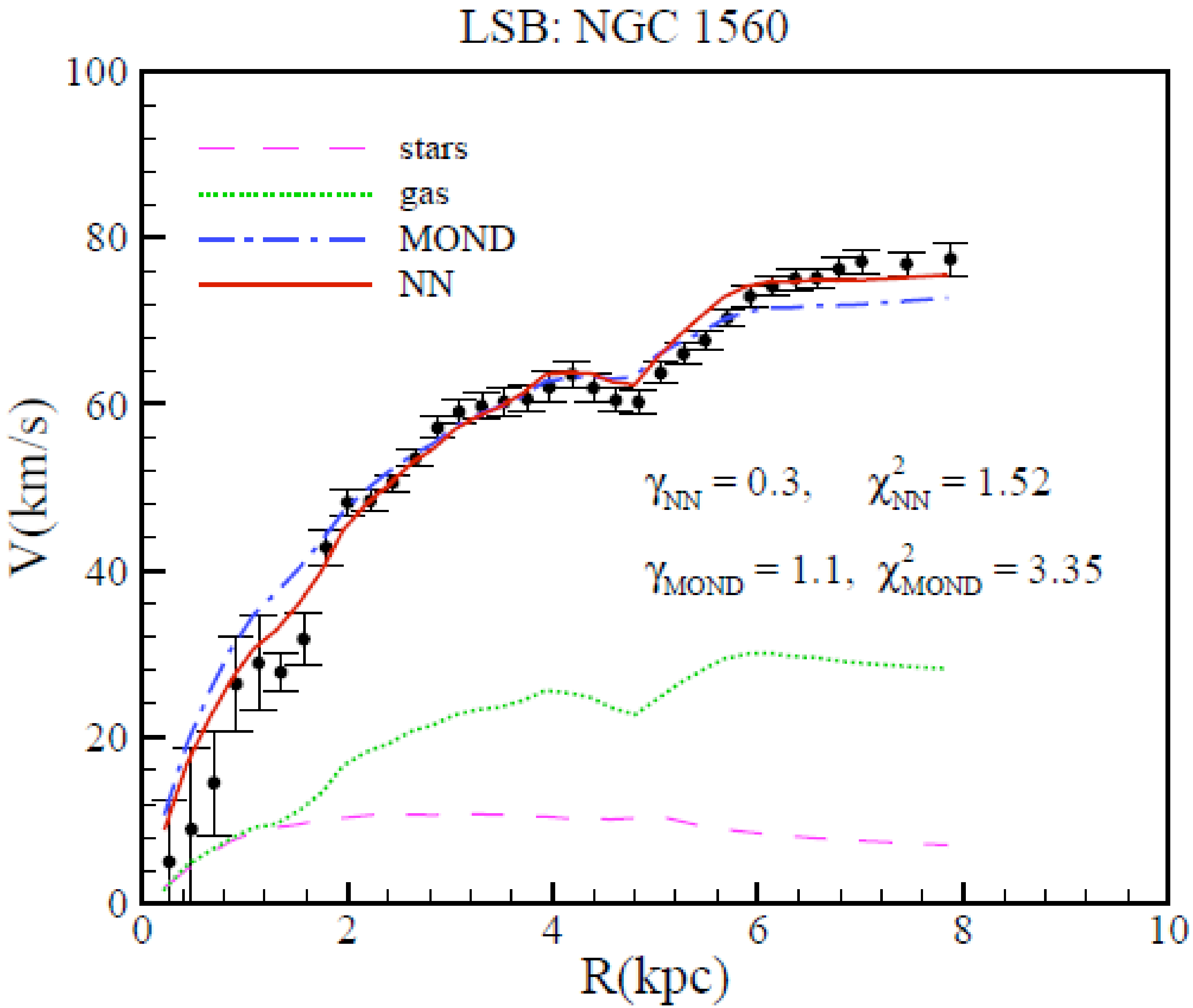
Observed data is not extensive and accurate enough to judge deviations from $M^{-1/2}$.

Hereafter, for pedagogical reasons, we let $\beta \approx 2$, and $\Lambda = \lambda M^{-1/2}$. The internal solution is

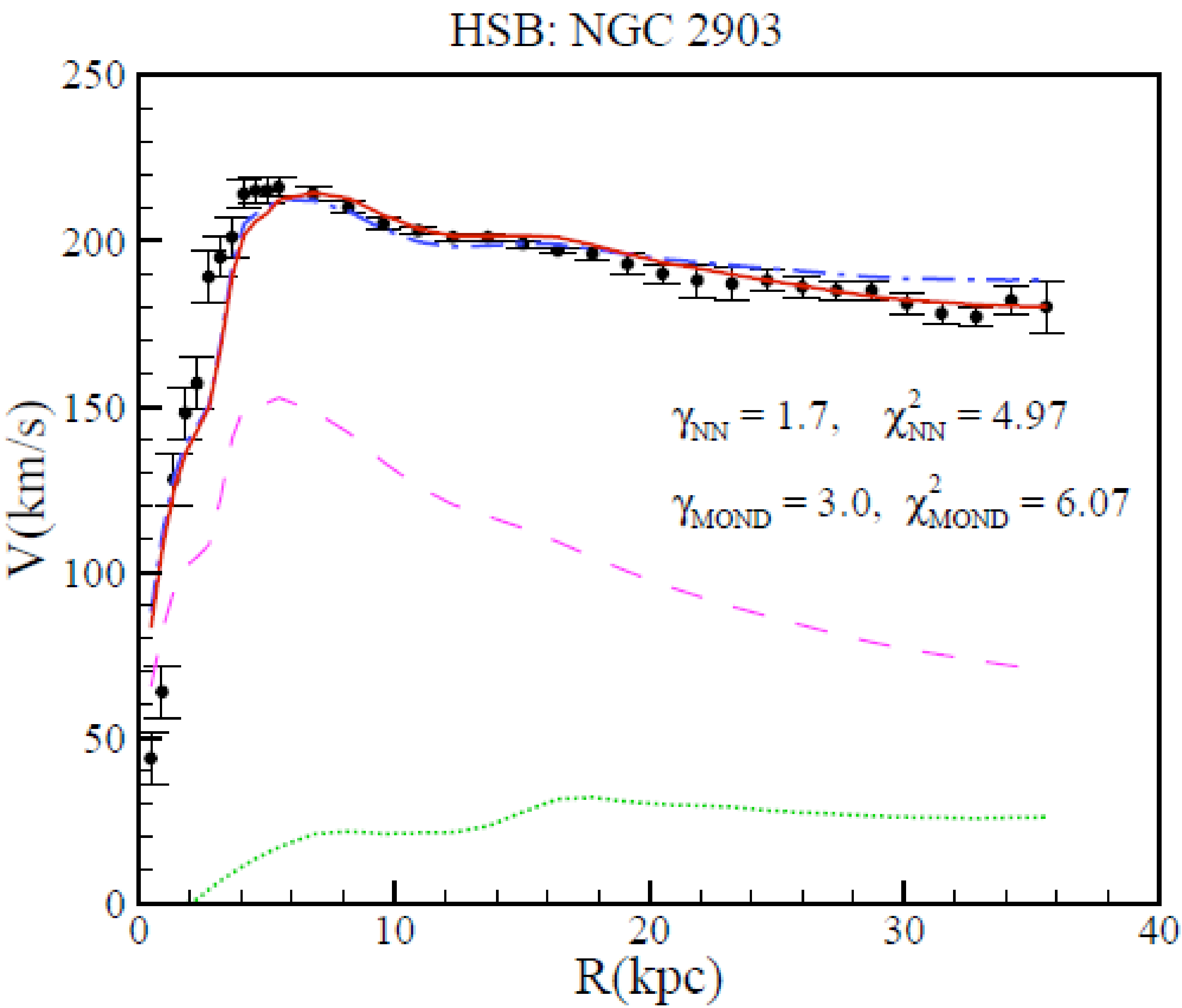
$$
\begin{aligned}
r\frac{d\phi^{(i)}}{dr} = \frac{M(r)}{r^2} &+ \Lambda\left\{\frac{1}{r^2}\int_0^r M(r')dr' - \frac{1}{R^2}\int_0^R M(r')dr\right\} \\
&+ \frac{4\pi}{3}\Lambda\left\{\frac{1}{r^2}\int_0^r \rho(r')r'^3 dr' - \frac{1}{R^2}\int_0^R \rho(r')r'^3 dr'\right\} \\
&+ \frac{4\pi}{3}\Lambda r\int_r^R \rho(r')dr' + \Lambda\frac{M}{R} \qquad (5)
\end{aligned}
$$

Continuity of $d\phi / dr$ at the boundary is guaranteed.

Based on Eqs. (4) and (5), we have constructed the rotation curves of NGC 1560 and NGC 2903, two LSB and HSB galaxies, respectively, and compared results with the observed data and with those obtained through MOND



Legend: Points with error bars are observed data. Dotted and dashed lines are the stellar and gaseous contributions to the rotation curves. Solid line is our rotation curve. Dashed-dotted is that of MOND. The free parameter in matching to data points, is the stellar mass-to-light ratio.



Legend: The same as in previous page

If one wishes to adopt a dark matter language, one may consider the integral in Eq. (3) as the density of the dark matter. Its spherical term outside and inside of the baryonic system is

$$\rho_{\text{dark}}^{(e)} = \lambda \frac{M^{1/2}}{r^2} \qquad r \geq R \qquad (6)$$

$$\rho_{\text{dark}}^{(\text{i})} = \frac{\lambda}{M^{1/2}} \frac{M(r)}{r^2} + 4\pi \frac{\lambda}{M^{1/2}} \int_{\text{r}}^{\text{R}} \rho(r')dr', \quad r << R \qquad (7)$$

ρ_{dark} varies continuously across the boundary.

Implications of the dark matter scenario are to be noticed:

Any baryonic matter has an ever-attendant dark companion and there rules to this companionship, namely Eqs. (6) and (7)

Dark matter does not exist independently from the baryonic matter.

The difference between the dark matter scenarios and alternative theory-paradigms are semantic. One language can be translated into the other.

Further on non-locality and non-linearity:

One heuristic way of incorporating the TF nonlinearity into the variational principle is to let $\Lambda = \lambda\, M^{-1/2}$, and find λ from the observed data. This is what we did above. Somewhat esthetic alternative, however, is to replace the non-local term in Eq.(2) by some power of it:

$$\lambda\, J^{\alpha}, \quad J = \int \frac{\rho(x')\phi(x)}{|x - x'|^{\beta}} d^3x'\, d^3x\,. \tag{8}$$

Then the capital Λ in the equation of motion, Eq.(3), gets replaced by $\alpha\lambda J^{\alpha-1}$. We have calculated J in Eq.(9) below. It is almost proportional to $M^{3/2}$ with some weak

M^2

M^2

dependence on the internal structure of the matter. To recover the asymptotically square-root dependence of the gravity force on M, one finds $\alpha = 2/3$.

$$J = \int \frac{\phi(r)\rho(r')}{|r - r'|} d^3 r d^3 r'$$

$$= 4\pi \int_0^R \phi(r) M(r) dr + 4\pi M \int_R^\infty \phi(r) dr$$

$$+16\pi^2 \int_0^R \phi(r) r^2 dr \int_r^R \rho(r') dr' \qquad (9)$$

The second term in Eq.(9) is by far the largest and is proportional to $M^{3/2}$.

Concluding remarks

In the weak field regime we have formulated a non-local and nonlinear law of gravitation.

Its spherical terms describe the rotation curves of spirals as good as, if not better than, MOND or CDM simulations.

The formalism is capable of treating non-spherical cases. We are pursuing this issue.

In order to clarify certain concepts, a general relativistic formulation of the problem is desirable. For example, in the weak field regime there has no provision and no logic to talk about the pressure of the dark matter. While, in a relativistic version one has to invite in a dark pressure as a requirement of the Biancci identities.