

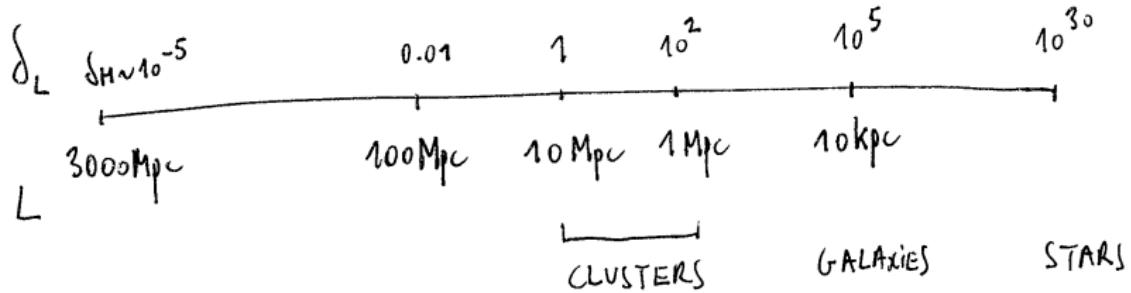
A **possible** catch 22 for large distance modifications of gravity

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the (inhomogeneous) Universe



backreaction for Einstein gravity

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$$\approx 8\pi G_N \bar{\rho}_{tot}(t) + 8\pi G_N \bar{\rho}_M(t)\delta(x, t)$$

$$\begin{aligned} G_i^i &\approx -(3H^2 + 6\frac{\ddot{a}}{a}) + 2a^{-2}\nabla^2(\phi + \psi) + a^{-2}(-3\partial_i\phi\partial_i\phi + \dots) + \dots \\ &\approx 24\pi G_N \bar{\rho}_{tot}(t) \end{aligned}$$

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first order: $\phi^{(1)} = -4\pi G_N \bar{\rho}_M \frac{a^2}{\nabla^2} \delta$ $\psi^{(1)} = -\phi^{(1)}$

local compact system: $\psi \approx -\phi \approx \frac{G_N M}{ra}$

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higher order: $\psi = -\frac{G_N M}{ra}(1 - \frac{G_N M}{ra} + \dots)$

nearly everywhere in the in the linear Newtonian regime → only a small backreaction (Ishibashi and Wald 2006)

backreaction for Einstein gravity

estimating the backreaction: (Kolb et al 2005): $\delta_L \sim \delta_H \frac{a}{(LH_0)^2}$ for $\delta_L \leq 1$

$$\langle \phi \frac{\nabla^2}{a^2} \phi \rangle \approx (4\pi G_N)^2 \bar{\rho}_M^2 \langle \delta \frac{a^2}{\nabla^2} \delta \rangle \sim H^4 L^2 a^2 \delta_L^2 \sim \left(\frac{H}{H_0}\right)^4 \frac{a^4 \delta_H^2}{L^2} \sim \delta_H H^2$$

large distance modifications of gravity

alternative to dark energy

$$\begin{aligned} G_{\mu\nu}(g) + H_{\mu\nu}(g, \pi) &= 8\pi G_N T_{\mu\nu} & \left(H_{00}^{(0)} \sim H^2 \left(\frac{H_0}{H}\right)^n \right) \\ E_\pi(g, \pi) &= 0 \end{aligned}$$

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- homogeneous Universe: dark energy=modified gravity
- gravity is everywhere: Earth,Solar system,galaxies,...,Universe

large distance modifications of gravity

linear perturbations

Brans-Dicke, DGP, $f(R)$, $f(\mathcal{G})$, ...

$$G_{00}^{(0)} + H_{00}^{(0)} + \frac{1}{a^2} (-2\nabla^2\phi + \nabla^2(c_1\phi + c_2\psi + c_3\pi)) \approx 8\pi G_N \bar{\rho}_M(t)(1 + \delta(x, t))$$

$$G_i^{i(0)} + H_i^{i(0)} + \frac{1}{a^2} (2\nabla^2(\phi + \psi) + \nabla^2(c_4\phi + c_5\psi + c_6\pi)) \approx 0$$

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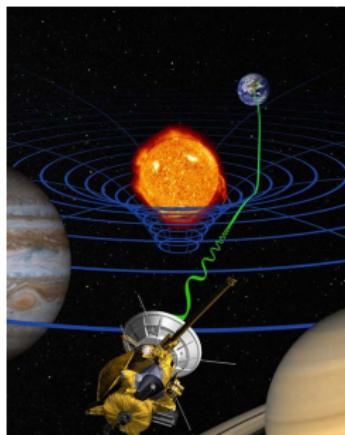
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Cassini: $\frac{\alpha_1}{\alpha_2} = (-1 \pm 10^{-5})$
(Bertotti et al, 2003)

ruled out, if the linearization is valid
inside Solar system

large distance modifications of gravity

nonlinear terms (Vainshtein 1972; Deffayet et al 2001; Navarro, KVA 2006)

$$H_{00}, H_i^i, E_\pi \sim \frac{(\nabla^2)^{m+1} \phi^{n+1}}{a^{2(m+1)} H_0^{2m}}$$

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- $f(R)$, Brans-Dicke: $m = 0$, $R_V = R_S \rightarrow$ ruled out
(Erickcek et al 2006; Navarro and KVA 2006; Faulkner et al 2006)

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(Erickcek et al 2006; Navarro and KVA 2006; Faulkner et al 2006)
- DGP, $f(\mathcal{G})$: $m = 1, n = 1$ $R_V = (G_N M / H_0^2)^{1/3}$
 - ▶ all systems with $\delta \gtrsim 1$, gravitationally nonlinear regime
 - ▶ order one backreaction? $\frac{1}{H_0^2} \langle \nabla^2 \phi \nabla^2 \phi \rangle \sim H_0^2 \delta^2$

$f(\mathcal{G})$ gravity

nonlinear terms (KVA 2009)

$$\begin{aligned} S &= \frac{1}{16\pi G_N} \int d^4x \sqrt{-g} f(R^2 - 4R_{\mu\nu}R^{\mu\nu} + R_{\mu\nu\sigma\tau}R^{\mu\nu\sigma\tau}) \\ &= \frac{1}{16\pi G_N} \int d^4x \sqrt{-g} \left(f_{\mathcal{G}}(\mathcal{G})(R^2 - 4R_{\mu\nu}R^{\mu\nu} + R_{\mu\nu\sigma\tau}R^{\mu\nu\sigma\tau} - \mathcal{G}) + f(\mathcal{G}) \right) \end{aligned}$$

$$\mathcal{L}^{(3)} \approx f_{\mathcal{G}\mathcal{G}}^{(0)} \tilde{\mathcal{G}} \frac{8}{a^4} \left(\nabla^2 \phi \nabla^2 \psi - \partial_i \partial_j \phi \partial_i \partial_j \psi \right) \sim \left(\frac{H_0}{H} \right)^n \frac{\pi (\nabla^2 \phi \nabla^2 \psi - \partial_i \partial_j \phi \partial_i \partial_j \psi)}{H^2}$$

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$$\pi(\nabla^2 \phi \nabla^2 \psi - \partial_i \partial_j \phi \partial_i \partial_j \psi) \stackrel{Pl}{=} \psi(\nabla^2 \phi \nabla^2 \pi - \partial_i \partial_j \phi \partial_i \partial_j \pi) \stackrel{Pl}{=} \phi(\nabla^2 \pi \nabla^2 \psi - \partial_i \partial_j \pi \partial_i \partial_j \psi)$$

'Galileian' symmetry (Nicolis et al 2008):

$$\pi \rightarrow \pi + a_\pi + b_\pi^i x_i \quad \phi \rightarrow \phi + a_\phi + b_\phi^i x_i \quad \psi \rightarrow \psi + a_\psi + b_\psi^i x_i$$

- shift symmetry \rightarrow total derivatives in eqs. \rightarrow averages out to zero!
- extra 'Galileian' symmetry \rightarrow no extra degrees of freedom

summary/conclusions/outlook...

- cosmology takes place in our world
- it is not impossible to modify GR in the infrared
 - ▶ consistent with the SS constraints (nonlinear Vainshtein mechanism)
 - ▶ without abandoning the FLRW framework
- 'gravitational energy' averages out to zero, because of the shift symmetry in the nonlinear term, $f(\mathcal{G})$, DGP (Lue 2005)
- a full matching of the linear and nonlinear regime, N-body simulations: still quite a challenge