A Solar System Constant and Uniform External Field Effect (EFE) in MOND? Observational Constraints

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Outline

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The impact of a constant and uniform field

- MOND does *not* fulfill the *Strong Equivalence Principle* in the sense that the internal dynamics of a gravitating system s of bodies *does depend*, in principle, on the *external background gravitational field* $\vec{E} = \vec{g}_{\rm ext}$ of a larger system S in which s is embedded, even if $\vec{E}$ is *constant* and *uniform*; it is the so-called External Field Effect (EFE)
- Our Solar System revolves through the Milky Way at about 8.5 kpc from its center; the Galactic gravitational attraction can be evaluated from the magnitude of the centrifugal acceleration $A_{\rm cen} \approx A_0$, where $A_0 = 1.27 \times 10^{-10}$ m s^{-2} is the MOND characteristic acceleration scale. Thus, such an *EFE should affect the inner dynamics of the Solar System* as well

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Common statements on EFE in the Solar System

- "MOND breaks down the Strong Equivalence Principle. This means that the acceleration of solar system's bodies depends indeed on the background gravitational field and not only on the tidal field. As shown by Milgrom, even if the external field was constant (and the tidal force vanishes), the internal acceleration would depend on the external field. Claiming that $A_{\rm cen}$ is irrelevant is only valid if the field equation were linear. [...] *for trans-Neptunian objects and planets, one can ignore the* $A_{\rm cen}$."
- "For the *main planets*, the acceleration is much larger than A_0 (the order of magnitude of the EFE), and *the effect is negligible* [...] The EFE maintains a *constant direction in the planet revolution*, and its effect *cancels out*. "

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Our goals

We will address the following points:

- Does a *constant* and *uniform* acceleration $\vec{E}$ having a *generic spatial direction* $\hat{\nu}$ and a magnitude of the order of $E = A_{\rm cen} \approx 10^{-10}$ m s^{-2} affect the motion of the *major bodies* of the Solar System?
- Are the effects of such an acceleration *negligible* for the Sun's *planets*, although they are, in principle, present?
- Since $\vec{E}$ is *constant and uniform*, are its effects *canceled out over planetary revolutions?*

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The gravitational acceleration of a planet

Let us start from

$$\mu\,(x)\,(\vec{A}+\vec{E})=\vec{N},$$

where $\vec{A}$ is the total gravitational acceleration felt by a planet, $\vec{N}$ is its Newtonian part, and $\mu(x)$ is the MOND interpolating function. Since in the planetary regions $\mu \approx 1$, we can use

$$\vec{A}\approx\vec{N}+\vec{E},\ \ \vec{E}=E_x\vec{i}+E_y\vec{j}+E_z\vec{k},\ \ E=10^{-10}\ \mathrm{m\ s}^{-2},$$

and treat $\vec{E}$ perturbatively with the standard Gauss approach

$R-T-N$ decomposition of $\vec{E}$

Let us project $\vec{E}$ onto the co-moving planetary frame with radial $\hat{r}$, transverse $\hat{t}$ and normal $\hat{n}$ orthogonal unit vectors

$$\begin{aligned}
\hat{r} &= (\cos\Omega\cos u - \cos I\sin\Omega\sin u)\,\vec{i} + (\sin\Omega\cos u + \cos I\cos\Omega\sin u)\,\vec{j} + \sin I\sin u\,\vec{k},\\
\hat{t} &= (-\sin u\cos\Omega - \cos I\sin\Omega\cos u)\,\vec{i} + (-\sin\Omega\sin u + \cos I\cos\Omega\cos u)\,\vec{j} + \sin I\cos u\,\vec{k},\\
\hat{n} &= \sin I\sin\Omega\,\vec{i} - \sin I\cos\Omega\,\vec{j} + \cos I\,\vec{k}.
\end{aligned}$$

Ω, I, u are the longitude of the ascending node, the inclination of the orbital plane and the argument of latitude: $u = \omega + f$, where ω is the argument of perihelion and f is the true anomaly, all referred to an *inertial Solar System Barycenter (SSB) frame*. We, thus, obtain the radial, transverse and normal components $E_r = \vec{E}\cdot\hat{r},\ E_t = \vec{E}\cdot\hat{t},\ E_n = \vec{E}\cdot\hat{n}$ of EFE

Averaged orbital effects

Let us insert E_r, E_t, E_n into the Gauss variational equations

$$\frac{d\Omega}{dt} = \frac{1}{na\sin I\sqrt{1-e^2}} E_n \left(\frac{r}{a}\right)\sin u,$$

$$\frac{d\varpi}{dt} = \frac{\sqrt{1-e^2}}{nae}\left[-E_r\cos f + E_t\left(1+\frac{r}{p}\right)\sin f\right] + 2\sin^2\left(\frac{I}{2}\right)\frac{d\Omega}{dt};$$

a is the semimajor axis, e is the eccentricity, $p = a(1-e^2)$ is the *semilatus rectum*, $n = \sqrt{GM/a^3}$ is the unperturbed Keplerian mean motion. Then, evaluate the right-hand-sides onto the unperturbed Keplerian ellipse given by

$$r = \frac{a(1-e^2)}{1+e\cos f},$$

and take the average over one orbital revolution with

$$dt = \frac{(1-e^2)^{3/2}}{2\pi(1+e\cos f)^2}df.$$

The secular perihelion precessions

The net, averaged perihelion precession of a planet p is a *non-zero, linear combination* of E_x, E_y, E_z

$$\left\langle \dot{\varpi}^{(p)} \right\rangle = \mathcal{C}_x^{(p)} E_x + \mathcal{C}_y^{(p)} E_y + \mathcal{C}_z^{(p)} E_z,$$

with coefficients

$$\mathcal{C}_j^{(p)} = \frac{1}{n_p a_p} \sum_k F_{jk}(e_p) \cos \xi_{jk}^{(p)}, \ j = x, y, z$$

where $F_{jk}(e_p)$ are complicated functions of the planet's eccentricity and $\xi_{jk}^{(p)}$ are linear combinations of the planet's Keplerian orbital elements ϖ_p, Ω_p and I_p.

Computation of the coefficients $\mathcal{C}_j^{(p)}$

Below we compute the coefficients $\mathcal{C}_x^{(p)}, \mathcal{C}_y^{(p)}, \mathcal{C}_z^{(p)}$ for p=Venus, Earth, Mars

Table: Computed values of the coefficients $\mathcal{C}_j^{(p)}$, in s m^{-1}, for p=Venus, Earth, Mars. For $e_p, \varpi_p, \Omega_p, I_p$ entering $F_{jk}(e_p)$ and $\cos\xi_{jk}^{(p)}$ the values at the reference epoch (J2000) have been used (http://ssd.jpl.nasa.gov/txt/p_elem_t1.txt).

p	$\mathcal{C}_x$	$\mathcal{C}_y$	$\mathcal{C}_z$
Venus	0.0023	-0.0052	-0.0002
Earth	-0.0005	-0.0019	5×10^{-10}
Mars	-0.0005	9×10^{-6}	-1×10^{-10}

Confrontation with the observations

From the corrections $\Delta\dot{\varpi}$ to the standard Newtonian/Einsteinian perihelion precessions of the inner planets, in 10^{-4} arcsec cty^{-1} (1 arcsec cty^{-1} $= 1.5 \times 10^{-15}$ s^{-1}), estimated by E.V. Pitjeva with the EPM ephemerides,

Mercury	Venus	Earth	Mars
-36 ± 50	-4 ± 5	-2 ± 4	1 ± 5

it is possible to obtain the following constraints on E_x, E_y, E_z, in m s^{-2}:

E_x $(\times 10^{-15})$	E_y $(\times 10^{-16})$	E_z $(\times 10^{-14})$
-0.3 ± 1	2 ± 5	-0.6 ± 3

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Our findings

We have shown that:

- A *constant* and *uniform*, *non-radial* acceleration $\vec{E}$ small enough to be treated perturbatively, as in this case, *does induce non-zero long-period*, i.e. averaged over one orbital revolution, *effects* on the Keplerian orbital elements of a planet in an *inertial Solar System Barycentric frame*
- By assuming $E \approx A_0$, the resulting *perihelion precessions* of the inner planets are $4 - 6$ orders of magnitude *larger* than the *present-day limits on the recently estimated non-standard perihelion rates*

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Another form of violation of the Strong Equivalence Principle?

The standard picture of EFE in MOND refers to an *inertial Galactocentric inertial frame*, while we have used an *inertial Solar System barycentric frame*; the dynamics of our gravitating system turned out to be *different* in such two local inertial frames freely falling with *different velocities at different spatial locations*. After all, this should *not* surprise too much, since *MOND does violate the Strong Equivalence Principle, and the Local Lorentz Invariance and Local Position Invariance, violated by our analysis, are just part of it*.

References I

L. Iorio
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