

Viable Singularity-Free $f(R)$ Gravity Without a Cosmological Constant

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[Phys. Rev. Lett. 102, 221101 (2009)]

What is causing the cosmic acceleration?

new component ?

$$R_{\mu\nu} - \frac{1}{2}g_{\mu\nu}R = - \left(T_{\mu\nu}^{(m,r)} + \boxed{T_{\mu\nu}^{(x)}} \right)$$

modified gravity ?

$$R_{\mu\nu} - \frac{1}{2}g_{\mu\nu}R + \boxed{\Delta_{\mu\nu}[g_{\alpha\beta}]} = -T_{\mu\nu}^{(m,r)}$$

Constraints

- Stability conditions
- Cosmological:
 - Matter-dominated phase
 - Accelerated phase
 - No past (finite time) singularities
- Local gravity
- Relativistic stars

Notations

$$S = \int f(R) d^4x$$

$$f(R) = R + \Delta(R)$$

$$f_R \equiv \frac{df}{dR} \qquad f_{RR} \equiv \frac{d^2f}{dR^2}$$

$$\Delta_R \equiv \frac{d\Delta}{dR}$$

Pros & Cons

- Stability conditions
 - Cosmological:
 - Matter-dominated phase
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 - No past (finite time) singularities
 - Local gravity
 - Relativistic stars
-

Stability conditions

$$f(R) = R + \Delta(R)$$

$$f_R \equiv 1 + \Delta_R > 0 \quad \text{no ghosts}$$

$$f_{RR} \equiv \Delta_{RR} > 0 \quad \text{no tachyons}$$

$$\left. \begin{array}{l} \lim_{R \rightarrow \infty} \frac{\Delta}{R} = 0 \\ \lim_{R \rightarrow \infty} \Delta_R = 0 \end{array} \right\} \text{GR @ early times}$$

No past (finite time) singularities

Frolov [PRL 101, 061103 (2008)]

$$\delta S = 0$$

$$f_R R_{\mu\nu} - \nabla_\mu \nabla_\nu f_R + \left(\square f_R - \frac{1}{2} f \right) g_{\mu\nu} = \frac{8\pi g}{3} T_{\mu\nu}^{(m,r)}$$

$$\square f_R = \frac{1}{3} (2f - f_R R) + \frac{8\pi G}{3} T$$

$$\chi \equiv f_R$$

$$\square f_R = \frac{1}{3}(2f - f_R R) + \frac{8\pi G}{3}T$$

$$\chi \equiv f_R$$

$$\boxed{\square \chi = \frac{dV}{d\chi} - \mathcal{F}}$$

$$\frac{dV}{d\chi} \equiv \frac{1}{3}(2f - f_R R) \quad \mathcal{F} \equiv -\frac{8\pi G}{3}T$$

homogeneous universe:

$$\chi = \chi[R(t)]$$

$$\square \equiv -\frac{\partial^2}{\partial t^2} - 3H\frac{\partial}{\partial t}$$

$$\ddot{\chi} = -\frac{dV}{d\chi} + \mathcal{F}$$

Hu e Sawicki [Phys. Rev. D 76, 064004 (2007)]

$$f(R) = R - \alpha R_* \left\{ 1 - \frac{1}{\left[1 + \left(\frac{R}{R_*} \right) \right]^n} \right\}$$

Starobinsky [JETP Lett. 86, 157 (2007)]

$$f(R) = R - \alpha R_* \left\{ 1 - \frac{1}{\left[1 - \left(\frac{R}{R_*} \right)^2 \right]^n} \right\}$$

Starobinsky's $f(R)$

$$\phi \equiv \chi - 1$$

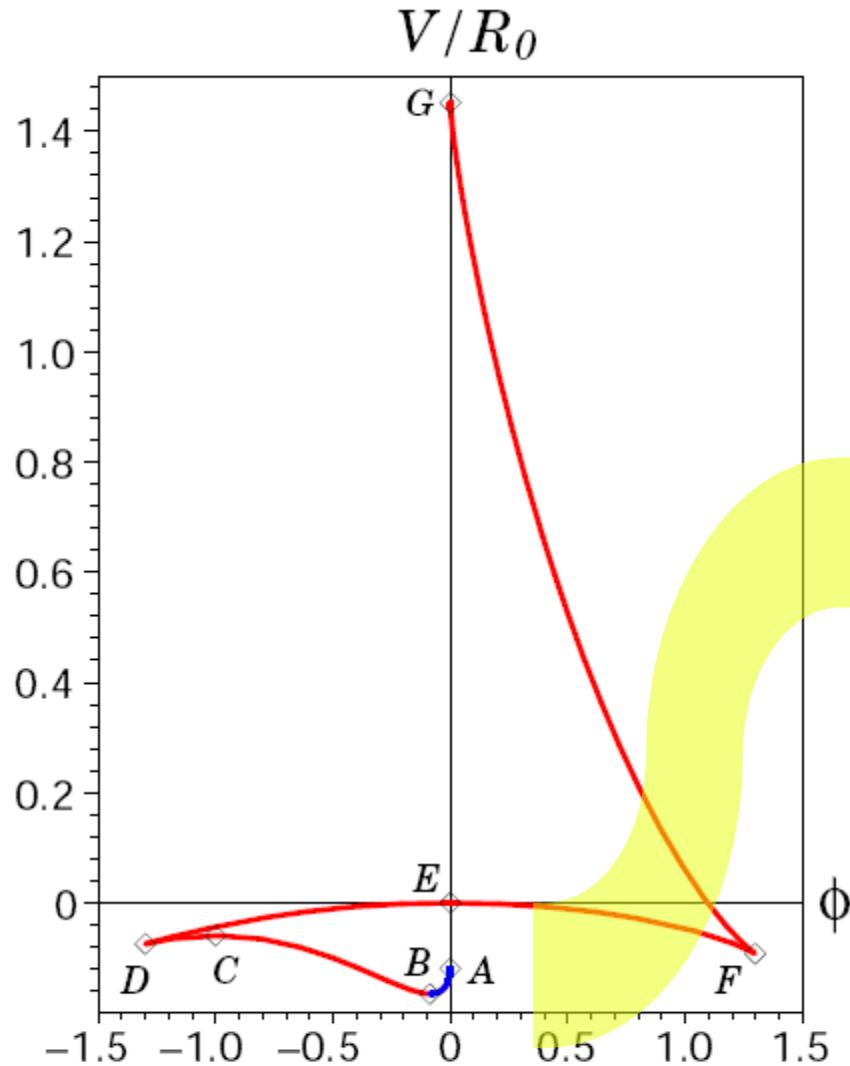


FIG. 1: Effective potential of a scalar degree of freedom in $f(R)$ gravity model (14) with $\lambda = 2$ and $n = 1$. Diamonds mark the location of critical points. The part relevant to cosmological evolution is emphasized by thick blue line.

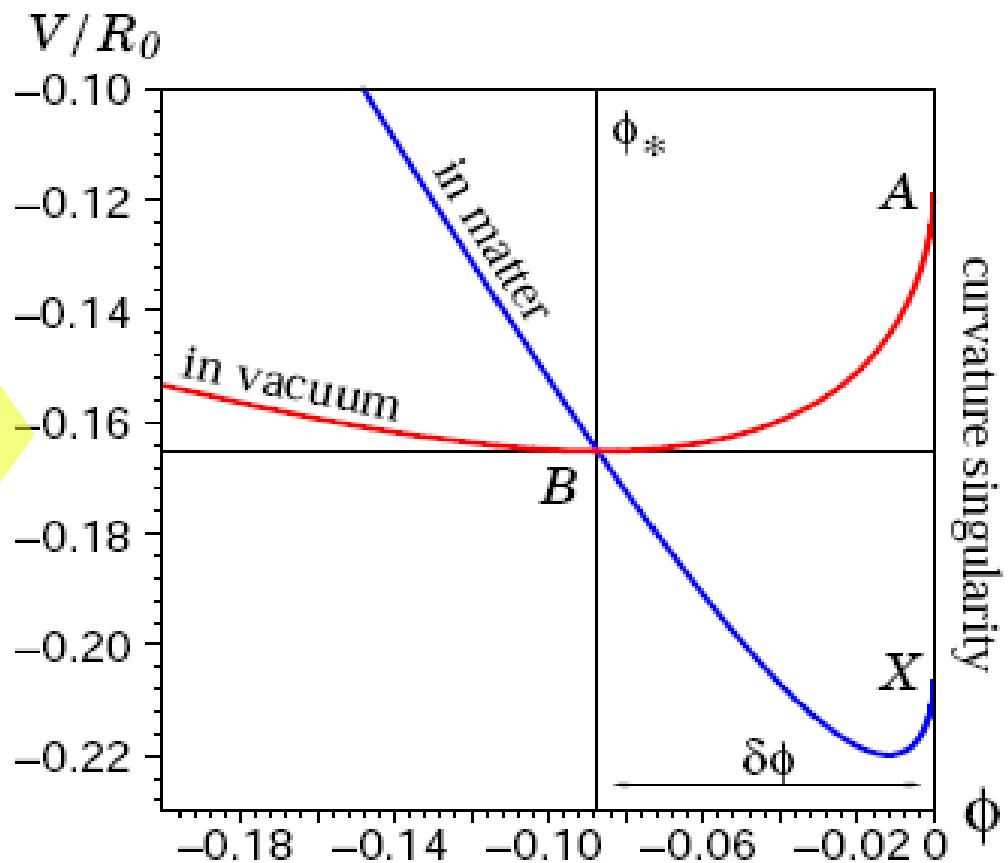


FIG. 2: Adding matter destabilizes the vacuum. Although effective potential inside constant density matter distribution still has a minimum, it is very shallow, and cannot protect the field ϕ from reaching curvature singularity X , which becomes energetically accessible from asymptotic vacuum state B .

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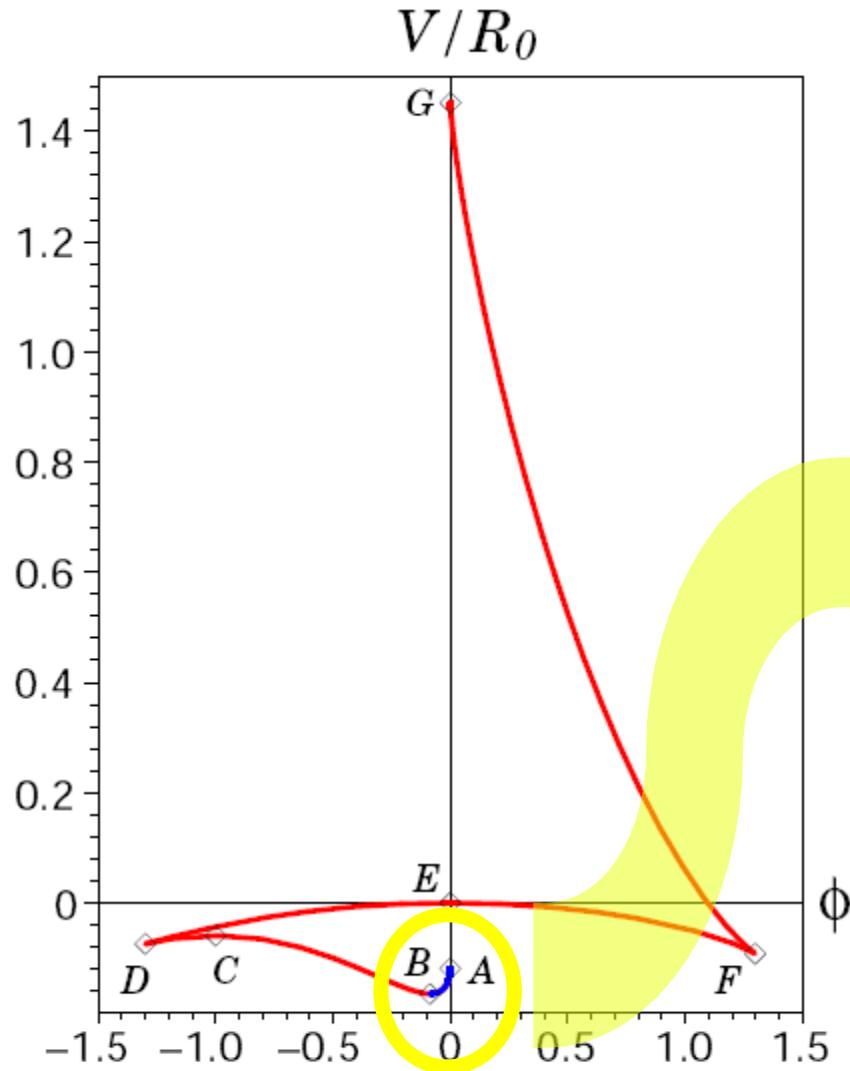


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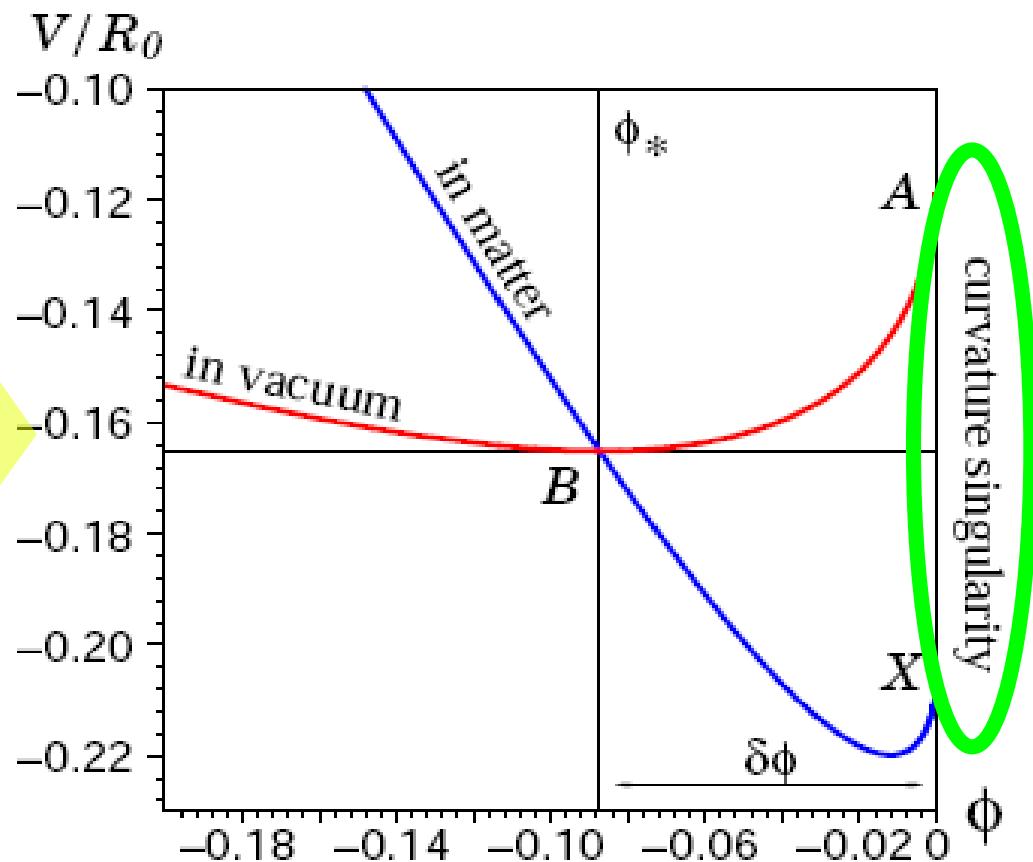
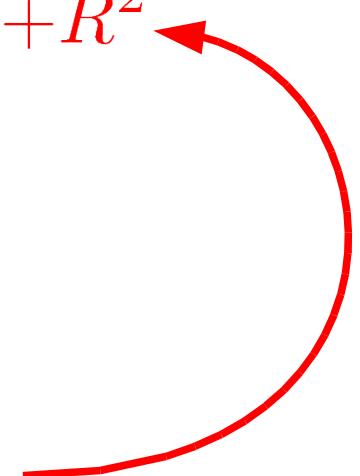


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[Dev *et al.*, PRD 78, 083515 (2008)]

$$f(R) = R - \alpha R_* \left\{ 1 - \frac{1}{\left[1 - \left(\frac{R}{R_*} \right)^2 \right]^n} \right\} + R^2$$

gets rid of this kind singularity



Relativistic stars

Kobayashi & Maeda [PRD 78, 064019 (2008)]

$$\square \chi = \frac{dV}{d\chi} - \mathcal{F}$$

Static and radial configuration:

$$\chi = \chi[R(r)]$$

$$\square \equiv +\frac{\partial^2}{\partial r^2} + \frac{2}{r} \frac{\partial}{\partial r}$$

$$\chi'' + \frac{2}{r}\chi' = -\frac{d(-V)}{d\chi} + (-\mathcal{F})$$

matter

$$U(\chi) \equiv -V(\chi)$$

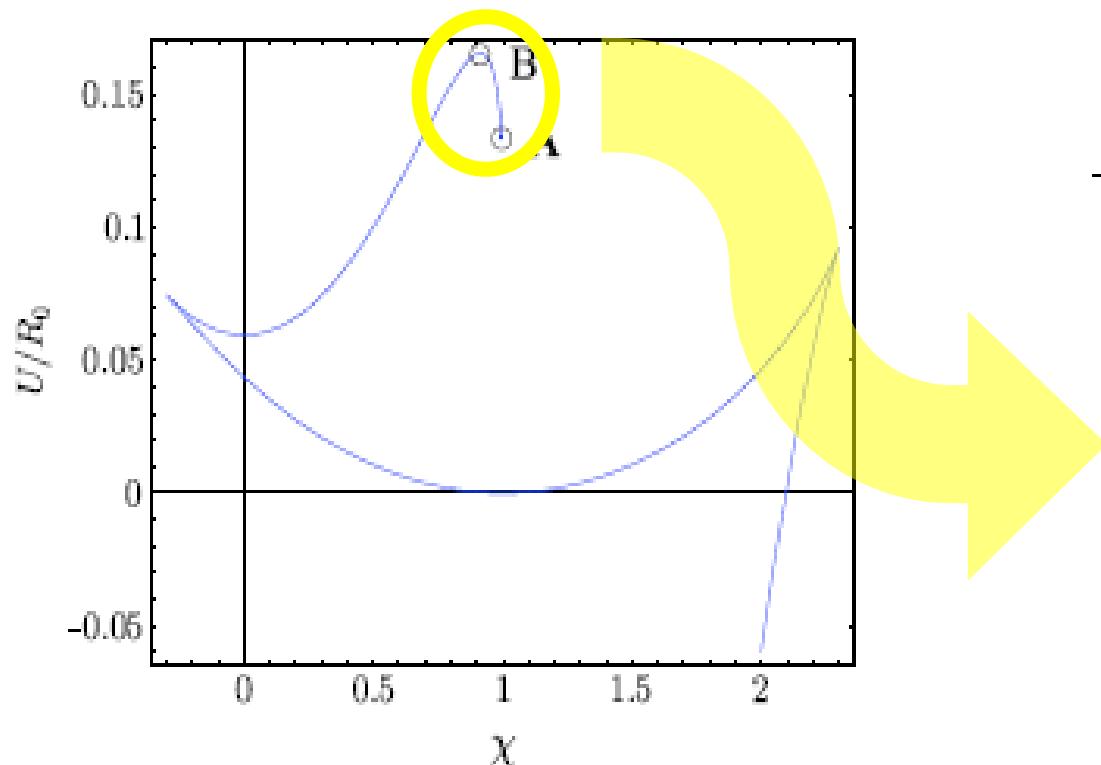


FIG. 2: The (inverted) potential $U(\chi)$ for Starobinsky's $f(R)$ model with $\lambda = 2$ and $n = 1$. The point A corresponds to a curvature singularity ($R = +\infty$), and the point B is the de Sitter extremum. (See also Fig. 1 of Ref. [26].)

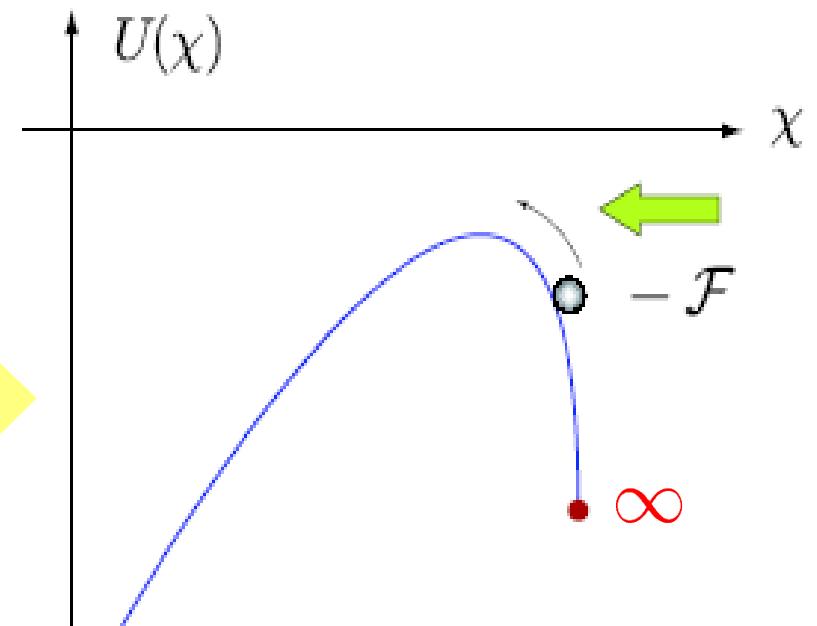


FIG. 3: Motion of a particle near the de Sitter extremum of $U(\chi)$. The particle feels the force \mathcal{F} (< 0) which arises from the trace of the energy-momentum tensor of the matter, $\mathcal{F} \propto T$.

$$U(\chi) \equiv -V(\chi)$$

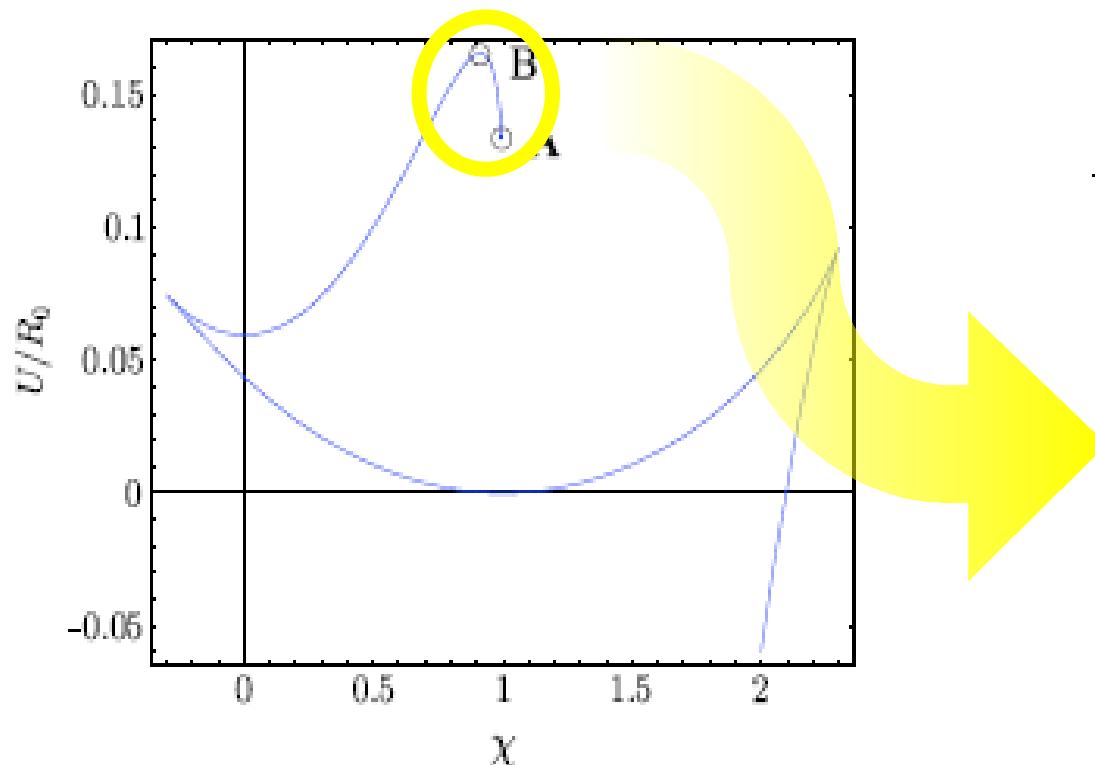


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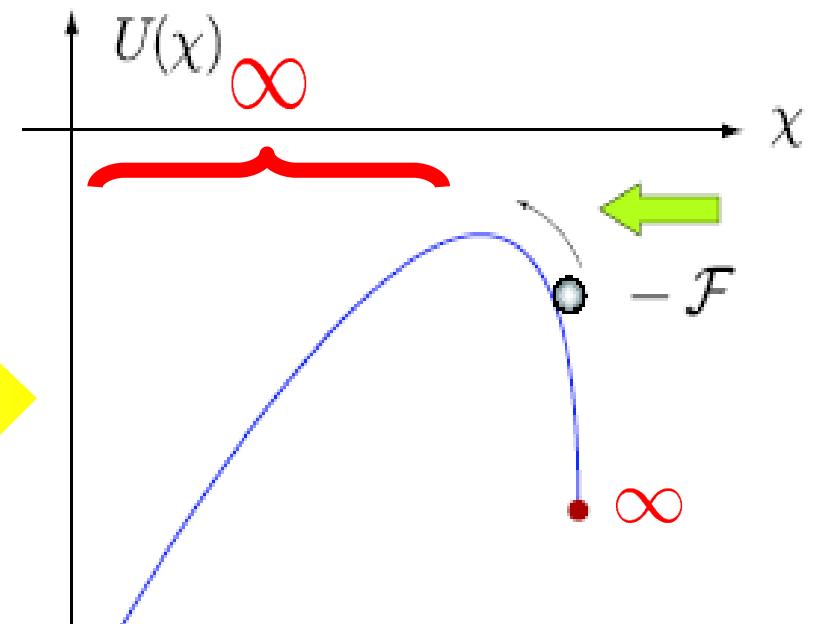


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[A. Upadhye and Wayne Hu, arXiv:0905.4055]

Extremely strong fine-tuning
of the initial (central) conditions:

$$\sim 10^{-60}$$

$$f(R) = R - R_s \beta \left\{ 1 - \left[1 + \left(\frac{R}{R_*} \right)^{\textcolor{blue}{n}} \right] \right\}^{-\frac{1}{\beta}}$$

$$\beta = -1 \implies f(R) = R + aR^m$$

$$\beta = +1 \implies f(R) = R - \alpha R_* \left\{ 1 - \left[1 + \left(\frac{R}{R_*} \right)^n \right]^{-1} \right\}$$

$$n = +2 \implies f(R) = R - \alpha R_* \left\{ 1 - \left[1 + \left(\frac{R}{R_*} \right)^2 \right]^{-\frac{1}{\beta}} \right\}$$

$$n = +1, \beta \rightarrow \infty :$$

$$f(R) = R - \alpha R_* \ln \left(1 + \frac{R}{R_*} \right)$$

Pros & Cons:

- ✓ Stability conditions
- ✓ Cosmological:
 - ✓ Matter-dominated phase
 - ✓ Accelerated phase
 - ✓ No past (finite time) singularities
- ? Local gravity
- ✓ Relativistic stars

Stability conditions

$$f_R \equiv 1 + \Delta_R > 0$$

$$\alpha < 1 + \frac{R_{dS}}{R_*}$$

$$f_{RR} \equiv \Delta_{RR} > 0$$

$$\left. \begin{array}{l} \lim_{R \rightarrow \infty} \frac{\Delta}{R} = 0 \\ \lim_{R \rightarrow \infty} \Delta_R = 0 \end{array} \right\} \text{GR @ early times}$$

$$f(R) = R + \Delta(R)$$

No past (finite time) singularities

Frolov [PRL 101, 061103 (2008)]

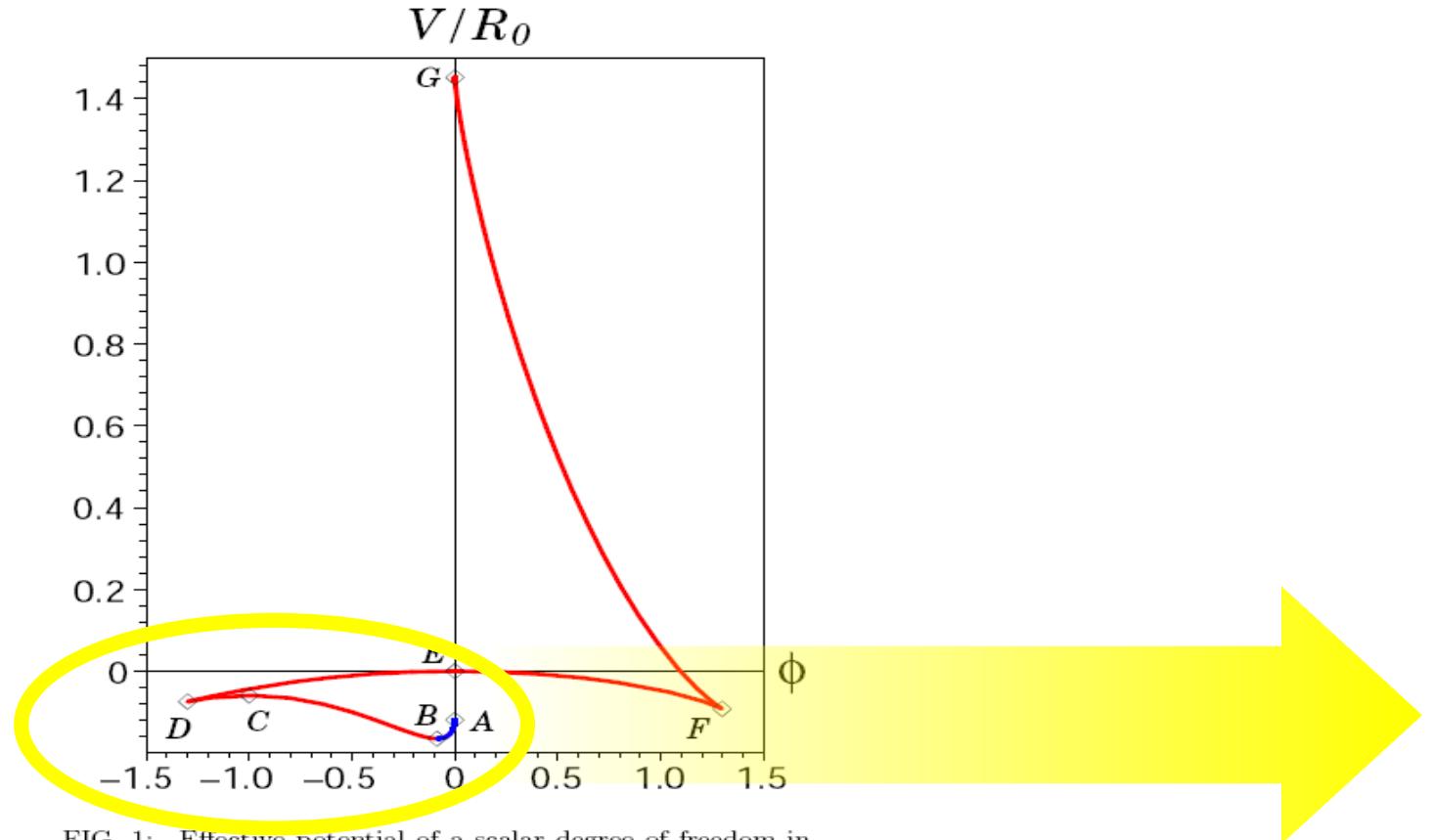
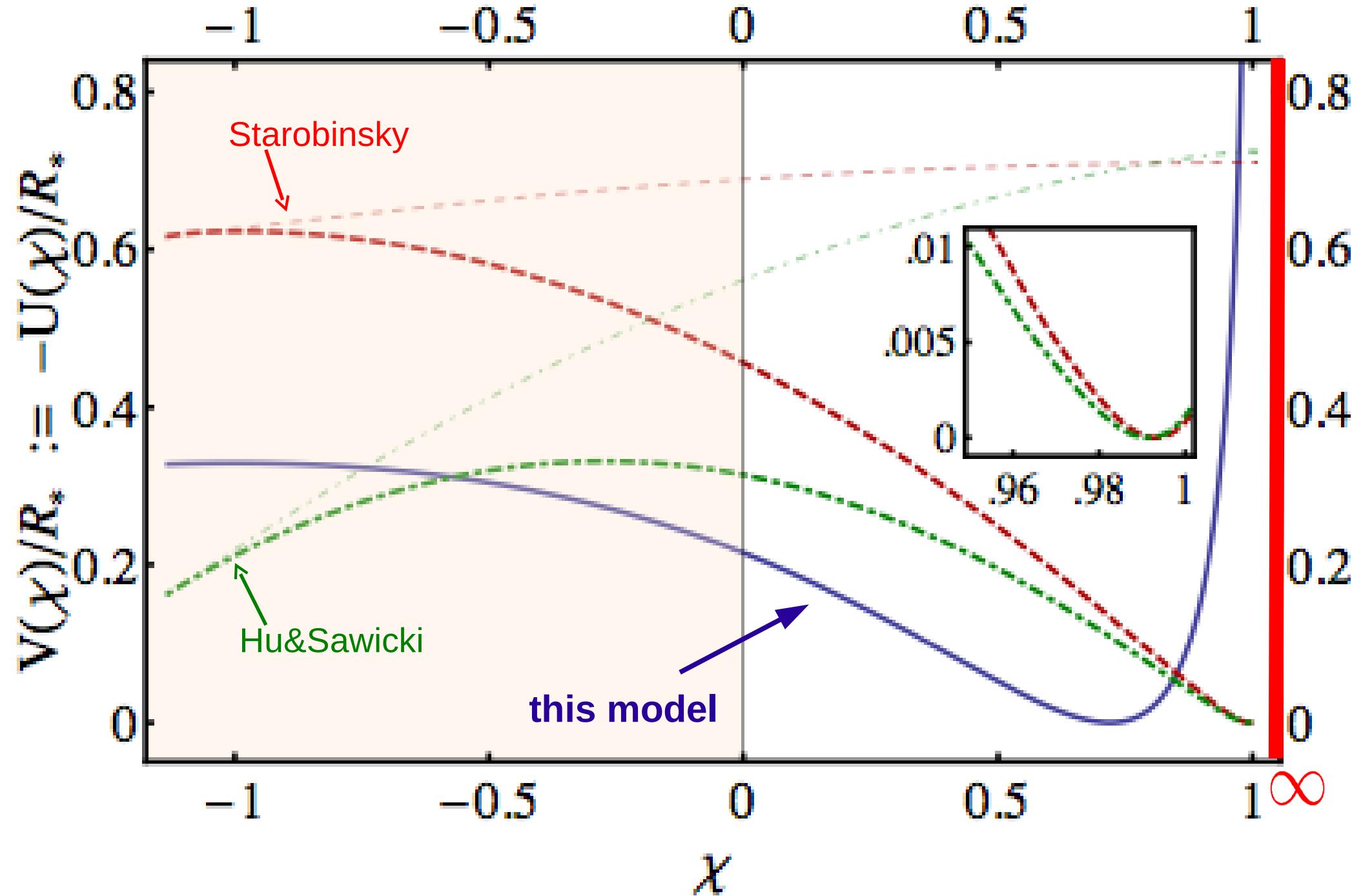


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Einstein's frame:

$$\tilde{\chi} \equiv -\sqrt{\frac{3}{16\pi G}} \ln \chi$$

$$V_E[R(\tilde{\chi})] = \frac{1}{16\pi G} \frac{R\Delta_R - \Delta}{(1 + \Delta_R)^2}$$

$$f(R) = R + \Delta(R)$$

Einstein's frame:

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$$V_E[R(\tilde{\chi})] = \frac{1}{16\pi G} \frac{R\Delta_R - \Delta}{(1 + \Delta_R)^2} \xrightarrow{\chi \rightarrow 1} \infty$$

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Einstein's frame:

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$$V_E[R(\tilde{\chi})] = \frac{1}{16\pi G} \frac{R\Delta_R - \Delta}{(1 + \Delta_R)^2} \xrightarrow{\chi \rightarrow 1} \infty$$
$$> 0$$

$$f(R) = R + \Delta(R)$$

Einstein's frame:

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$\xrightarrow{\chi \rightarrow 1} \infty$

> 0

$$f(R) = R + \Delta(R)$$

Prescription for avoiding Frolov's singularity

$$R\Delta_R \rightarrow \infty$$

or

$$\Delta \rightarrow -\infty$$

Local gravity constraints

[L. Pogosian and A. Silvestri, PRD 77, 023503 (2008)]

$$|f_R - 1| < 10^{-6}$$

[Thongkool *et. al.*, arxiv/09062460]

$$|f_R - 1| \sim \mathcal{O}(1)$$

Relativistic stars

[Kobayashi & Maeda PRD 78, 064019 (2008)]

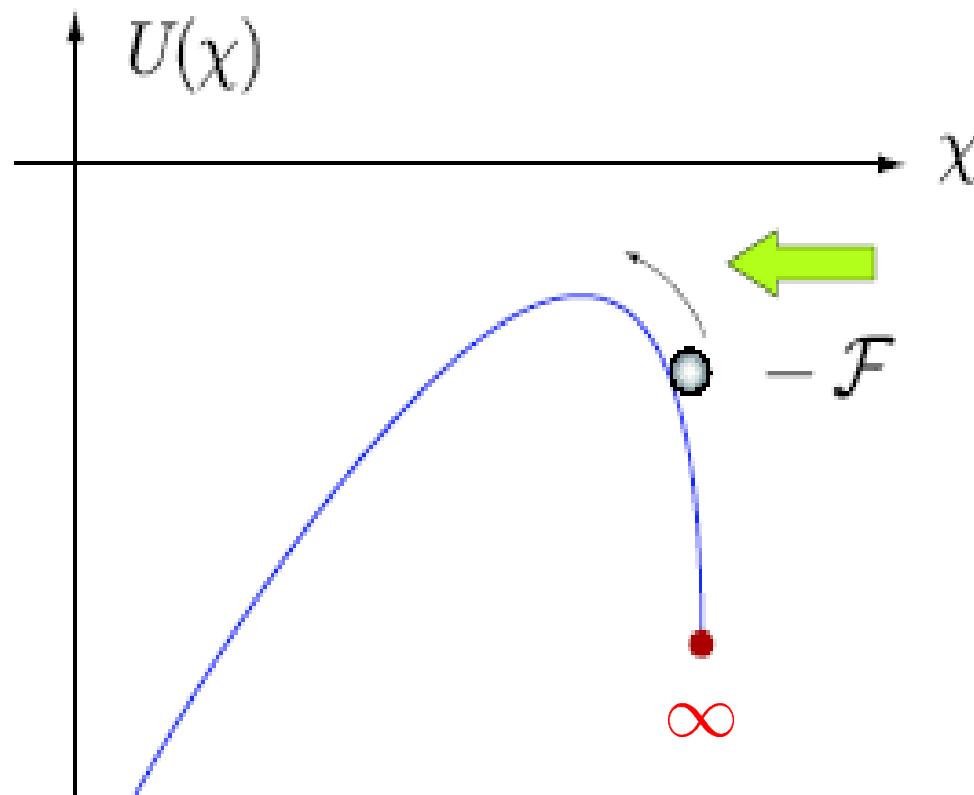


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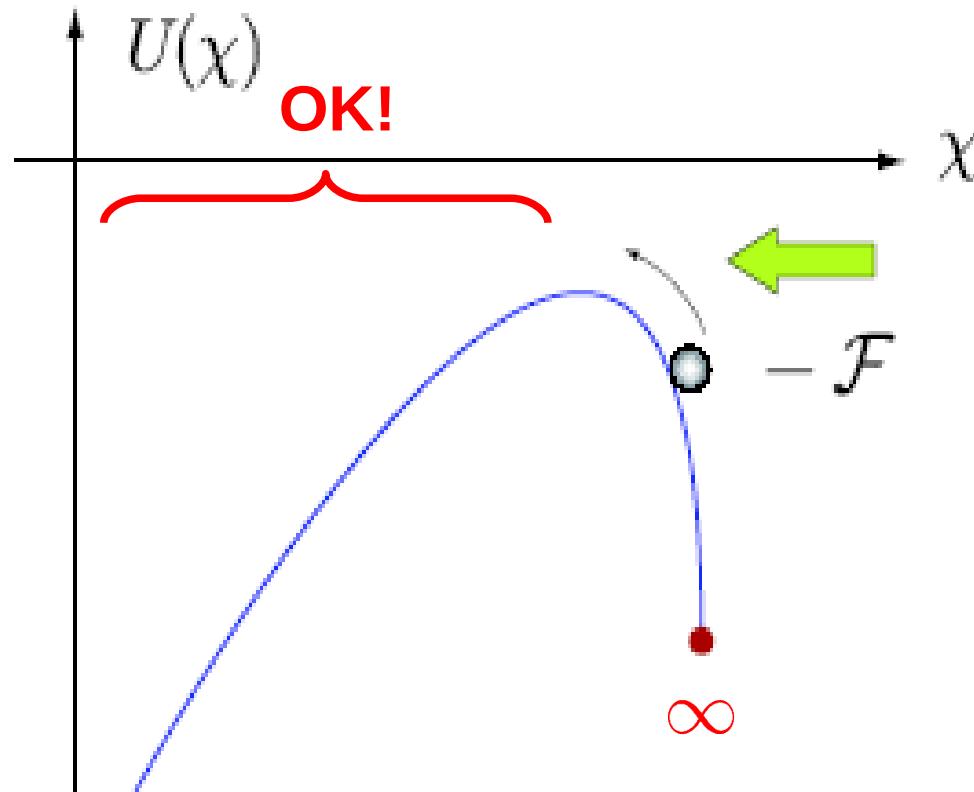
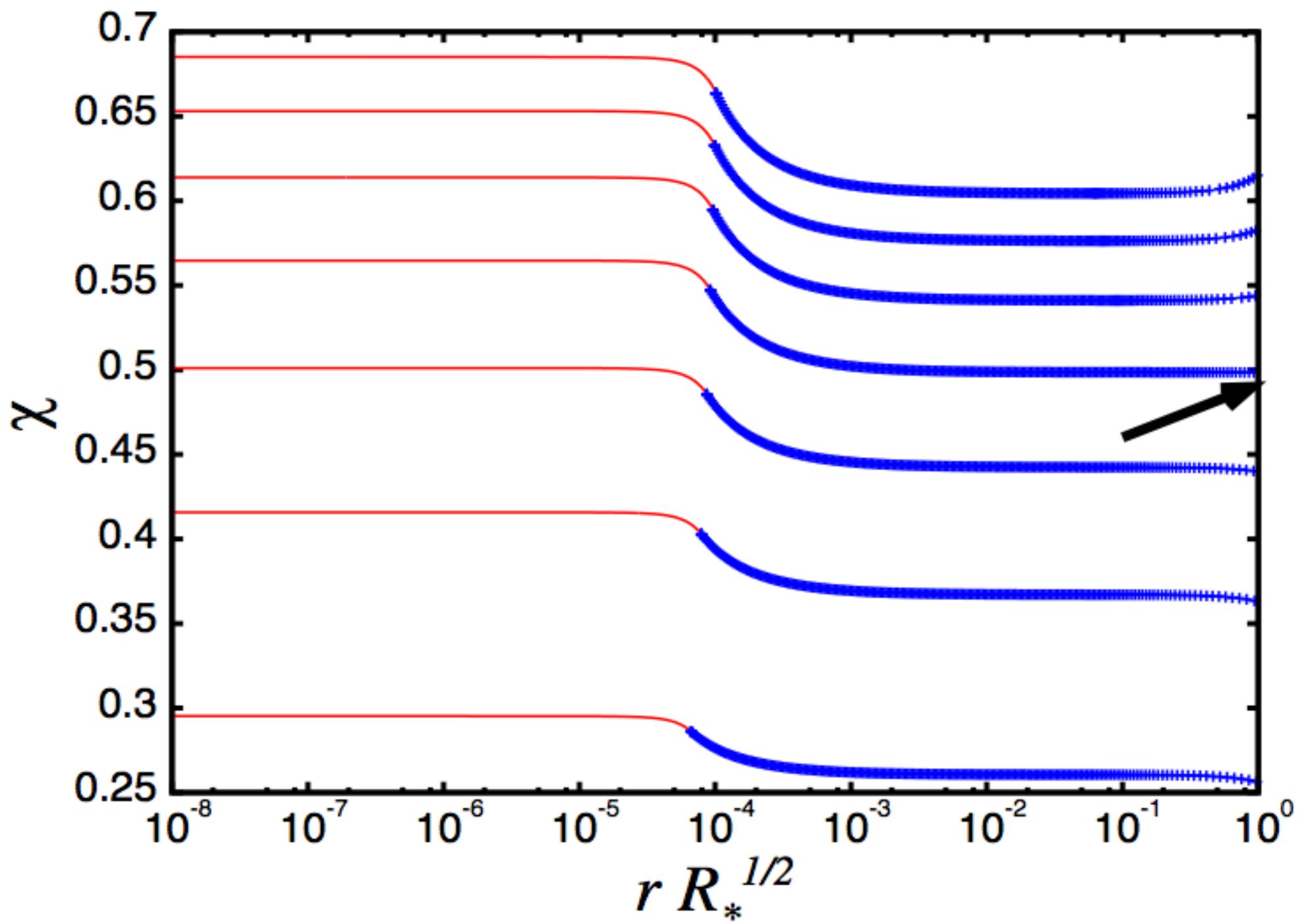


FIG. 3: Motion of a particle near the de Sitter extremum of $U(\chi)$. The particle feels the force $\mathcal{F} (< 0)$ which arises from the trace of the energy-momentum tensor of the matter, $\mathcal{F} \propto T$.

Still strong fine-tuning
of the initial (central) conditions:

$$\sim 10^{-5}$$



Cosmology

Amendola et al PRD **75**, 083504 (2007)

$$m \equiv \frac{R f_{RR}}{f_R}$$

$$r \equiv -\frac{R f_R}{f}$$

$$\Lambda CDM \Leftrightarrow m = 0$$

Matter-dominated era

$$\left\{ \begin{array}{l} m(r \approx -1) \approx 0^+ \\ \frac{dm}{dr} \Big|_{r \approx -1} > -1 \end{array} \right.$$

Final accelerating attractor

$$\left\{ \begin{array}{l} m = -r - 1 \\ (\sqrt{3} - 1)/2 < m \leq 1 \\ \frac{dm}{dr} \Big|_{r \approx -2} < -1 \end{array} \right.$$

$$0 < m(r = -2) \leq 1$$

$$\begin{aligned}
f_R \left(R_{\mu\nu} - \frac{1}{2} g_{\mu\nu} R \right) &= -T_{\mu\nu}^{(m,r)} + \\
&+ \frac{1}{2} g_{\mu\nu} (f - R f_R) + \nabla_\mu \nabla_\nu f_R - \\
&- g_{\mu\nu} \nabla_\alpha \nabla^\alpha f_R
\end{aligned}$$

$$3f_RH^2 = \left(\rho_m + \rho_r\right) + \frac{1}{2}\left(f_RR - f\right) - 3H\dot{f}_R$$

$$3H^2 = \frac{1}{f_R} \big(\rho_m + \rho_r \big) + \frac{1}{f_R} \left[\frac{1}{2} \left(f_R R - f \right) - 3H\dot{f}_R \right]$$

$$3f_R H^2 = (\rho_m + \rho_r) + \frac{1}{2} (f_R R - f) - 3H\dot{f}_R$$

$$\boxed{3H^2 = \frac{1}{f_R}(\rho_m + \rho_r) + \frac{1}{f_R} \left[\frac{1}{2} (f_R R - f) - 3H\dot{f}_R \right]}$$

$$3H^2 = 8\pi G (\rho_m + \rho_r) + \frac{1}{2} (f_R R - f) - 3H\dot{f}_R + 3H^2(1 - f_R)$$

$\underbrace{\hspace{30em}}$

$\equiv 8\pi G \rho_c$

$$-2\dot{H} = 8\pi G \left(\rho_m + \frac{4}{3}\rho_r \right) + \ddot{f}_R - H\dot{f}_R - 2\dot{H}(1 - f_R)$$

$\underbrace{\hspace{20em}}$

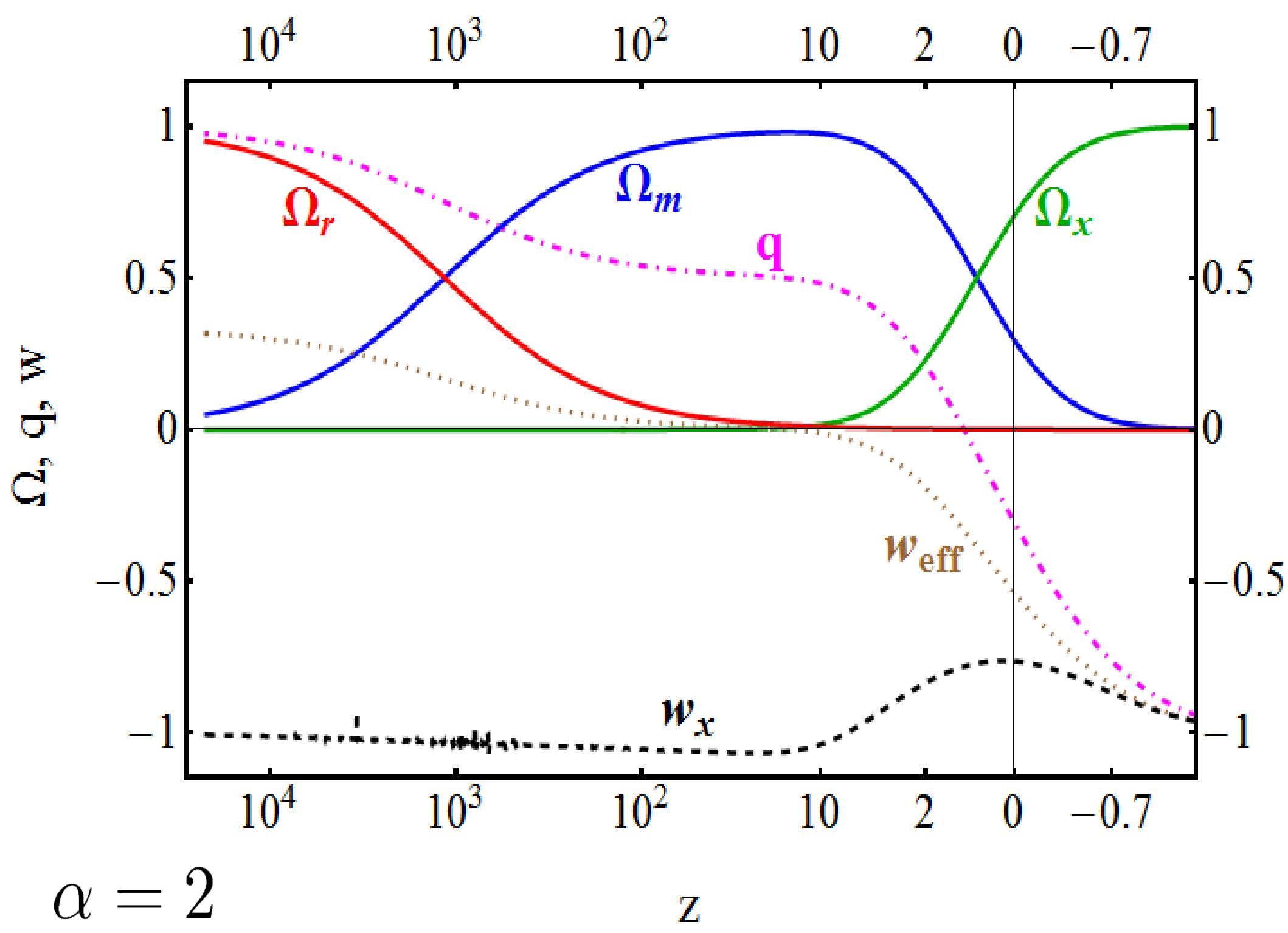
$\equiv 8\pi G(\rho_c + p_c)$

$$8\pi G \rho_x \equiv \frac{1}{2} (f_R R - f) - 3H \dot{f}_R + 3H^2(1 - f_R)$$

$$8\pi G p_x \equiv \ddot{f}_R + 2H \dot{f}_R - (2\dot{H} + 3H^2)(1 - f_R) + \frac{1}{2}(f - f_R R)$$

$$w_x \equiv \frac{p_x}{\rho_x}$$

Not a unique definition!



Conclusions

- Prescription for avoiding cosmological singularity at a finite time in the past:

$$R\Delta_R \rightarrow \infty$$

$$\Delta \rightarrow -\infty$$

- A not-so-fine tuning for relativistic stars
- Local gravity constraints should be carefully investigated!

ACKNOWLEDGEMENTS

ICTP
ICRA-BR