Dipolar dark matter and dark energy

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Motivation

Recover the relevant physics at the relevant scale:

- ΛCDM at cosmological scales
- Phenomenology of MOND at galactic scales
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Assume interpretation of MOND as resulting from the physical mechanism of gravitational polarization is deep [Blanchet 07]

Build a modified matter model which:

- Is fully relativistic and therefore predictive in cosmology
- Recovers the phenomenology of MOND in the non-relativistic limit *via* this interpretation

Outline

A. Model of dipolar dark matter and dark energy

B. First-order perturbations at cosmological scales

C. Recovering the phenomenology of MOND at galactic scales

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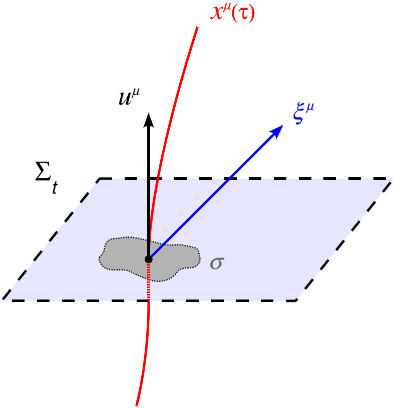
Action of the dipolar fluid

$$S = \int \mathrm{d}^4 x \sqrt{-g}\, L\big[J^\mu, \xi^\mu, g_{\mu\nu}\big]$$



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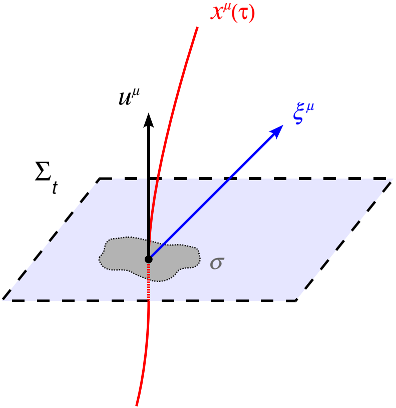

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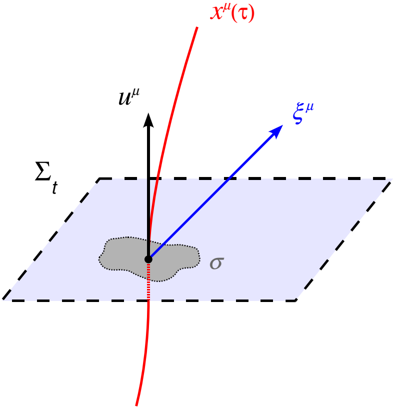
$$\nabla_\mu J^\mu = 0$$



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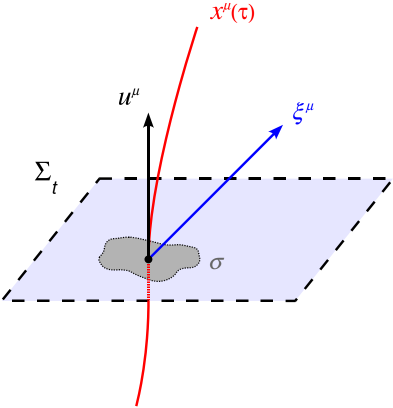
- Dipole moment variable ξ^μ



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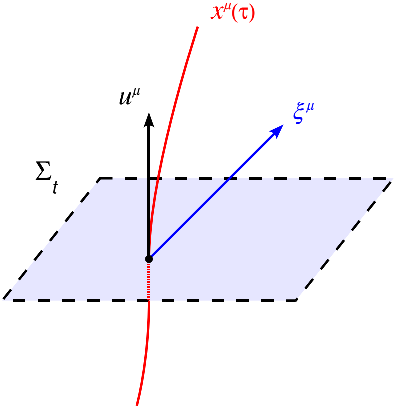
Second term can be replaced by $J_\mu \dot{\xi}^\mu_\perp$ so that the model only involves 3 extra degrees of freedom in the space-like $\xi^\mu_\perp = \perp^\mu_\nu \xi^\nu$

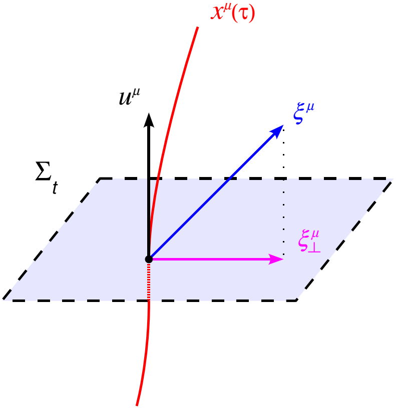
The polarization field $\Pi_\perp$



The polarization field $\Pi_\perp$

Project the dipole moment

$$\xi_\perp^\mu = \perp^\mu_\nu \, \xi^\nu$$



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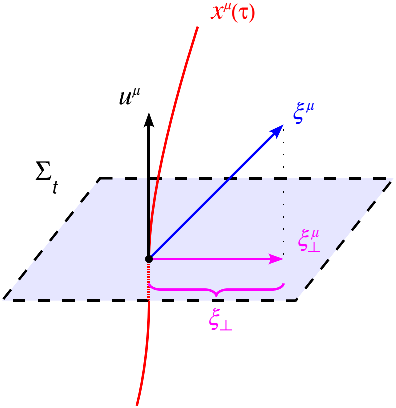
$$\xi_\perp = \sqrt{g_{\mu\nu}\, \xi_\perp^\mu \xi_\perp^\nu}$$



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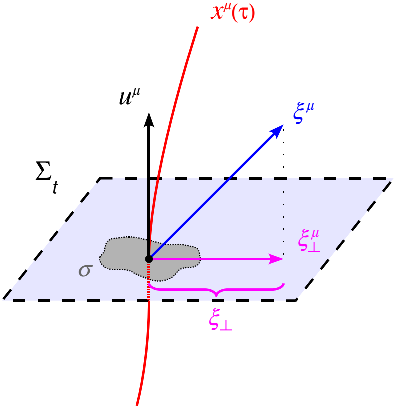

Equations of motion and stress-energy tensor

Equation of motion for u^μ

$$\dot{u}^\mu = -\mathcal{F}^\mu \quad \text{where} \quad \underbrace{\mathcal{F}^\mu = \hat{\xi}_\perp^\mu \mathcal{W}'}_{\text{internal force}}$$

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Equation of evolution for $\xi_\perp^\mu$

$$\dot{\Omega}^\mu = \underbrace{\frac{1}{\sigma}\nabla^\mu\left(\mathcal{W} - \Pi_\perp \mathcal{W}'\right)}_{\text{“pressure” term}} + \underbrace{\xi_\perp^\nu R^\mu{}_{\rho\lambda\nu} u^\rho u^\lambda}_{\text{Riemann coupling}} \quad \text{where} \quad \Omega^\mu = \dot{\xi}_\perp^\mu + \cdots$$

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Stress-energy tensor

$$T^{\mu\nu} = \underbrace{\Omega^{(\mu} J^{\nu)}}_{\text{monopolar}} - \underbrace{\nabla_\rho\left(\left[\Pi^\rho_\perp u^{(\mu} - u^\rho \Pi^{(\mu}_\perp\right] u^{\nu)}\right)}_{\text{dipolar term}} - \underbrace{\left(\mathcal{W} - \Pi_\perp \mathcal{W}'\right) g^{\mu\nu}}_{\text{“dark energy fluid”}}$$

Link between dark energy and MOND

In the weak-field regime $g \ll a_0 \;\Leftrightarrow\; \Pi_\perp \ll a_0$

$$\mathcal{W}(\Pi_\perp) = \frac{\Lambda}{8\pi} + 2\pi\,\Pi_\perp^2 + \frac{16\pi^2}{3a_0}\,\Pi_\perp^3 + \mathcal{O}(\Pi_\perp^4)$$

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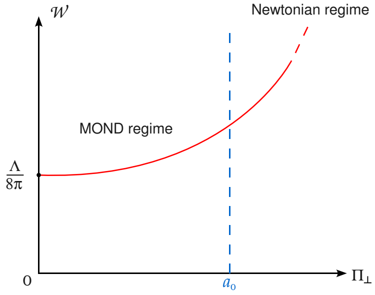

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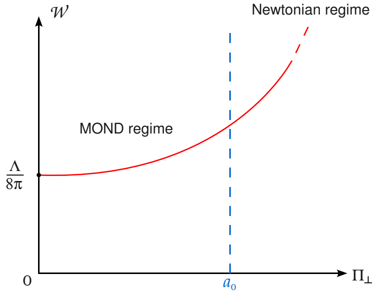

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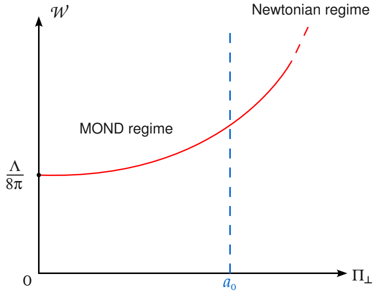
"Cosmic coincidence" that $\Lambda \sim a_0^2$ comes out naturally



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Perturbation around a FLRW background

Gravitational field

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Standard SVT gauge-invariant formalism

$$\delta u^i = D^i v + v^i$$
$$\delta\xi_\perp^i = D^i y + y^i$$
$$\delta g_{ij} = \cdots$$

Dipolar fluid stress-energy tensor

At first perturbation order

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With DE and DM components

$$\begin{aligned} T^{\mu\nu}_{\mathrm{de}} &= -\frac{\Lambda}{8\pi}\, g^{\mu\nu} \\ T^{\mu\nu}_{\mathrm{dm}} &= \rho\, \widetilde{u}^{\mu}\, \widetilde{u}^{\nu} \quad \text{where} \quad \rho = \sigma - D_i \Pi^i_{\perp} \end{aligned}$$

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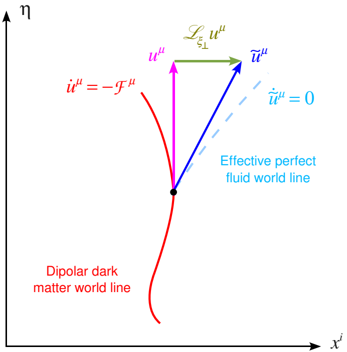
This is the stress-energy tensor of a perfect fluid with vanishing pressure, i.e. cold dark matter

The effective four-velocity

In terms of the variable

$$\widetilde{u}^\mu = u^\mu + \mathscr{L}_{\xi_\perp} u^\mu$$



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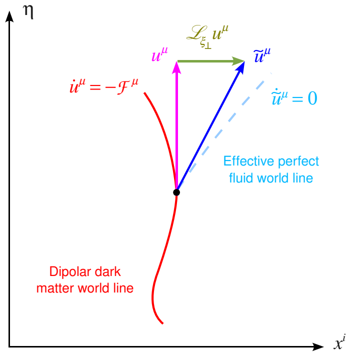

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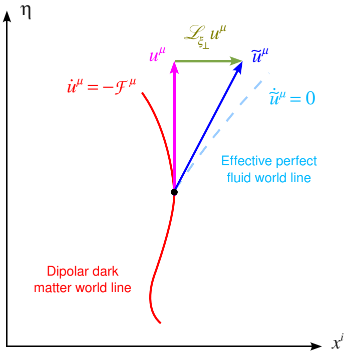

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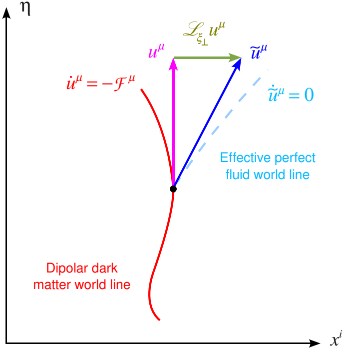

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- Adjusting $\bar{\sigma}$ so that $\Omega_{\mathrm{dm}} \simeq 0.23$ and Λ so that $\Omega_{\mathrm{de}} \simeq 0.73$ the model predicts exactly the same CMB fluctuations as ΛCDM

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Non-relativistic limit of the Lagrangian

Lagrangian density of the dipolar fluid

$$L = -\sigma - \mathcal{W}(\Pi_\perp) + J_\mu \dot{\xi}^\mu$$

Non-relativistic limit of $\mathcal{L} = \sqrt{-g}L$

$$\mathcal{L}_{\mathsf{NR}} = \frac{1}{2}\sigma_* \mathbf{v}^2 + \sigma_* U - \mathcal{W}(\Pi_\perp) + \mathbf{\Pi}_\perp \cdot \mathbf{g} + \sigma_* \mathbf{v} \cdot \frac{\mathrm{d}\boldsymbol{\xi}_\perp}{\mathrm{d}t}$$

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- Standard coupling to gravity
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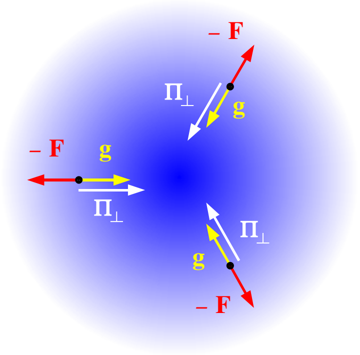
- Coupling of polarization $\boldsymbol{\Pi}_\perp$ to gravitational field $\mathbf{g}$

An exact solution in spherical symmetry

If baryonic matter is modeled by a mass distribution $\rho_b(r)$, there is a solution where the dipolar dark matter distribution is:

- Spherical: $\sigma = \sigma_0(r)$
- At rest: $\mathbf{v} = \mathbf{0}$
- In equilibrium: $\mathbf{g} = \mathbf{F}$
- Stationary: $\mathbf{\Pi}_\perp = \mathbf{cst.}$
- Polarized: $\mathbf{\Pi}_\perp \parallel \mathbf{g}$



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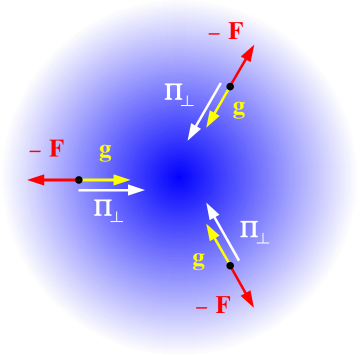

This motivates the "weak clustering hypothesis": $\sigma \simeq \bar{\sigma} \ll \rho_{\rm b}$

From the Poisson equation to the MOND equation

The Poisson equation reads

$$\nabla \cdot \mathbf{g} = -4\pi \Big(\rho_\mathsf{b} + \underbrace{\sigma - \nabla \cdot \mathbf{\Pi}_\perp}_{\text{mass density } \rho_\mathsf{dm}} \Big)$$

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$$\boldsymbol{\nabla}\cdot\big(\mu\,\mathbf{g}\big) = -4\pi\,\rho_\mathsf{b} \qquad \text{with} \qquad \mu = 1 + \chi$$

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- It provides a simple explanation for this phenomenology through the physical mechanism of polarization

Conclusion

Main results:

- ✔ Relativistic model of dark matter and dark energy based on a simple and physically meaningful matter action in GR
- ✔ Undistinguishable from ΛCDM at 1st order in cosmological perturbations $\Longrightarrow$ Consistent with observed CMB fluctuations
- ✔ The "cosmic coincidence" that $\Lambda \sim a_0^2$ is a natural outcome
- ✔ Recovers the phenomenology of MOND from the mechanism of gravitational polarization (under WCH)

Conclusion

Directions for future work:

- Relate this phenomenological model to more fundamental physics valid at microscopic scales
- Departure from ΛCDM expected at 2$^{\text{nd}}$ order in cosmological perturbations (e.g. non-gaussianity in CMB spectrum)
- Numerical computation of the non-linear growth of pert. and formation of large scale structures (test of WCH)
- Intermediate scale of galaxy clusters