

Thermodynamical model of the Dark Energy (DE)

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An interacting 2-component model

- ▶ Spatially flat FLRW metrics ($a(t)$ the scale factor)

$$ds^2 = -dt^2 + a(t)^2(dr^2 + r^2d\Omega^2), \quad (1)$$

J. Gariel, G. Le Denmat, CQG 16 (1999), 14

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$$ds^2 = -dt^2 + a(t)^2(dr^2 + r^2d\Omega^2), \quad (1)$$

- ▶ Einstein equations:

$$\left(\frac{\dot{a}}{a}\right)^2 = H^2 = \chi \frac{\rho}{3}, \quad (2)$$

$$\frac{\ddot{a}}{a} = -\frac{\chi}{6}(\rho + 3P), \quad \chi = \frac{8\pi G}{c^4} \quad (3)$$

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The model

The Gibbs' eq.

Observational constraints

Corrective Logarithmic term for the entropy

- ▶ 2 perfect fluids in interaction: $\rho = \rho_\Lambda + \rho_m$ and $P_T = P_\Lambda + P_m$

barotropic EOS (Equ. of state):

- the vacuum: $P_\Lambda = \omega \rho_\Lambda$, $\omega = -1$, $\rho_\Lambda = \frac{\Lambda}{\chi}$
- the matter : $P_m = (\gamma - 1)\rho_m$
- global energy conservation
- (*radiation separately conserved*)

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- ▶ complete thermo. description for each fluid (T, n, P) \leftrightarrow Gibbs' eq. (Barbachoux, Gariel, Le Denmat, PLB 658, (2008), 181)

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- ▶ for de Sitter,

$$\Lambda + \chi P_\Lambda = 0 \quad (1)$$

$$\Lambda = 3 \left(\frac{k}{c\hbar} \right)^2 T_\Lambda^2 \quad (2)$$

and

$$T_\Lambda = \frac{\hbar c}{kr}$$

with r the proper radius r of the event horizon

$$r(t) = a(t) \int_t^\infty \frac{cdt'}{a(t')} \quad (\text{Gibbons, Hawking, PRD 15, 1977})$$

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$$\Rightarrow \Lambda = \frac{3}{r^2} .$$



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- ▶ Eq. (1, 2,3) = EOS defining a local equilibrium for the DE compnt.
- ▶ EOS independent of the metrics, \Rightarrow Eq. (1,2,3) still valid for the FLRW ($k=0$) spacetime. $\Lambda = \frac{3}{r^2}$ for any metric.

\Rightarrow

local equilibrium state for the DE component

$$P_\Lambda = -\Lambda/\chi, \quad T_\Lambda = \frac{c}{kr} \quad \text{and} \quad n_\Lambda = \frac{3}{4\pi r^3}$$

Use of a Gibbs' eq.: C. Barbachoux, J. Gariel and G. Le Denmat : Phys.Lett.B 658 (2008) 181

the main problem:

- ▶ the energy flow through a surface of radius L (we consider a DE component without specification of its expression):

$$-\dot{E} = 4\pi L^2 \rho(1 + \omega) = T\dot{S}.$$

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- ▶ a thermo-horizon of radius L ; for BH and de Sitter spacetimes:

$$T = \frac{1}{2\pi L}, \text{ and } S = \pi L^2. \quad (7)$$

- ▶ Einstein eq: $\frac{\ddot{a}}{a} = -\frac{\chi}{6}(\rho + 3P)$ with $P = \omega\rho$, $0 \leq \omega \leq 1$,
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- ▶ solutions: complex solutions for L or $H = 0$ (de Sitter)

solution: use of a Gibbs' equation C. Barbachoux, J. Gariel and G. Le Denmat : Phys.Lett.B 658 (2008) 181

At the local equilibrium, the Gibbs equation written at the specific level

$$T_\Lambda ds_\Lambda = d\varepsilon_\Lambda + P_\Lambda dv_\Lambda$$

with our model

$$\chi\rho_\Lambda = \frac{3c^2}{r^2} , \quad (8)$$

We deduce

$$\chi n_\Lambda T_\Lambda ds_\Lambda = d\Lambda = -n_\Lambda dr$$

that we can integrate

$$s_\Lambda = -\pi r^2 + C$$

C constant.

The Gibb's eq. associated to this component leads to $E_\Lambda = \frac{\dot{r}}{2}$, to compare with the global energy of the DE component inside the horizon $E_\Lambda = \rho_\Lambda \frac{4\pi}{3} r^3 = \frac{r}{2}$.

Main result: use of a Gibbs' eq. to describe the thermodynamics of the DE component instead of the first law

C. Barbachoux, J. Gariel and G. Le Denmat : Phys.Lett.B **658** (2008) 181

Observational constraints- comparison with the SNIa observations for $0 < z < 2$, J.Gariel, G.Le Denmat, C.Barbachoux, Phys.

Lett B 629 (2005) 1

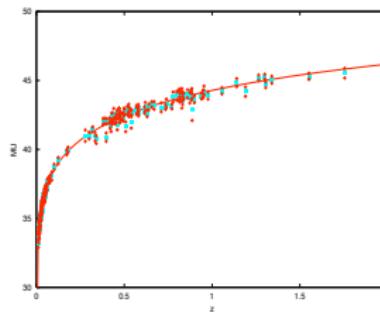
- ▶ assuming: matter = CDM ($P_m = 0, \gamma = 1$)

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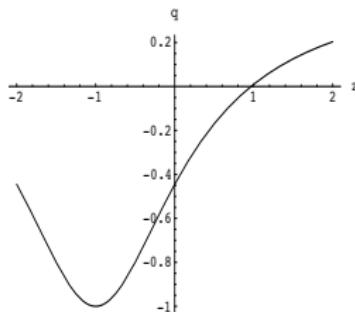
- ▶ assuming: matter = CDM ($P_m = 0, \gamma = 1$)
- ▶ Our results are in excellent agreement with the observations.

The “distance moduli” $\mu(z) = 25 + 5 \log_{10}(d_l)$ (d_l in Mpc) of the ehT model. for $z = 0$, $H_0 = 64 \text{ km s}^{-1} \text{ Mpc}^{-1}$, $\Omega_\Lambda^0 = 0.63^{+0.1}_{-0.01}$



The model solves also the problem of coincidence by the determination of the beginning of the acceleration.

$$q = \frac{1}{2}(1 - 3\Omega_\Lambda)$$



The deceleration parameter $q(z)$ of the ehTmodel $q_0 = -0.445$ and the epoch of the beginning of the acceleration $q = 0$ corresponds to $z_T = 0.965$

(J.Gariel, G.Le Denmat, C.Barbachoux, Phys. Lett B 629 (2005) 1)

Corrective Logarithmic term for the entropy C. Barbachoux, J.

Gariel and G. Le Denmat, submitted PLB

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- ▶ the specific entropy becomes:

$$s_A = -4 \frac{k}{L_P^2} (\pi r^2 + 2\pi\alpha \ln(\pi r^2)) + C$$

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- ▶ Using Gibbs' eq., Λ is modified

$$\Lambda = \frac{3}{r^2} \left(1 + \left(\frac{r_1}{r}\right)^2\right), \quad r_1^2 := \alpha \geq 0$$

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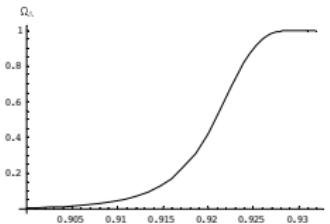
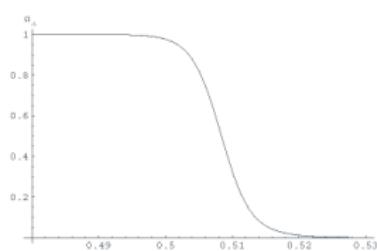
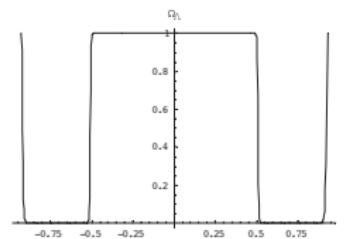
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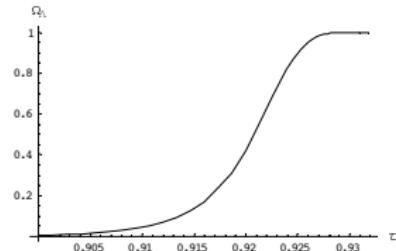
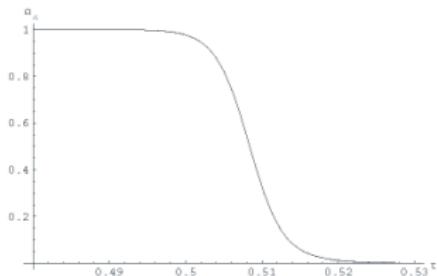
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- ▶ With this expression for Λ and the Einstein eq. in FLRW ($k=0$), we deduce the time evolution for H and r

The energy density $\Omega_\Lambda(\tau)$





epoch	τ	$t(s)$	$H(s^{-1})$	Ω_Λ	$\Lambda(m^{-2})$
beginning	0	0	$4.574 \cdot 10^{38}$	1	$6.98 \cdot 10^{60}$
end acc.	$\tau_2 = 0.50993$	$t_2 = 1.24 \cdot 10^{-39}$	$2.79 \cdot 10^{38}$	0.333	$8.685 \cdot 10^{59}$
min. Ω_Λ	$\tau_m = 0.6862$	$t_m = 2.90 \cdot 10^{-11}$	$2.29 \cdot 10^{10}$	$5 \cdot 10^{-20}$	$8.795 \cdot 10^{-16}$
start recomb.	$\tau_* = 0.8786$	$t_* = 1.17 \cdot 10^{13}$	$5.68 \cdot 10^{-14}$	$1.43 \cdot 10^{-4}$	$1.542 \cdot 10^{-47}$
end recomb.	$\tau'_* = 0.8796$	$t'_* = 1.537 \cdot 10^{13}$	$4.34 \cdot 10^{-14}$	$1.72 \cdot 10^{-4}$	$1.082 \cdot 10^{-47}$
start acc.	$\tau_3 = 0.9189$	$t_3 = 2.29 \cdot 10^{17}$	$3.52 \cdot 10^{-18}$	0.333	$1.378 \cdot 10^{-52}$
$\Omega_m = \Omega_\Lambda$	$\tau_4 = 0.92081$	$t_4 = 3.615 \cdot 10^{17}$	$3.15 \cdot 10^{-18}$	0.5	$1.655 \cdot 10^{-52}$
now	$\tau_0 = 0.92207$	$t_0 = 4.88 \cdot 10^{17}$	$2.07 \cdot 10^{-18}$	0.63	$9.046 \cdot 10^{-53}$

Comparison with the holographic model

ehT model	holographic DE
Gariel (99)	Li (2004)
$\Omega_\Lambda = -1$	$\Omega_\Lambda(t)$
global energy conservation	separated energy conservation
thermodynamics	holographic principle

- ▶ The same expression for the density of energy of the DE $\rho_\Lambda = \frac{3}{r^2}$ obtained by M. Li from the holographic principle.

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- ▶ Possibility of different choices for the horizons with / the IR cut off

$$\rho_\Lambda = \frac{3}{l^2}$$

/ = apparent horizon (or Hubble horizon) = H^{-1} ,
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Two main differences:

- 1) Holographic model :
 - a) separated conservation of the energy of each component
 - b) a variable EoS ω_Λ
- 2) Thermodynamical model :
 - a) interaction between the two components (and conservation of the total energy)
 - b) the EoS is constant : $\omega_\Lambda = -1$ (EoS of the vacuum)

The observational predictions are very similar.

Refs.

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