# Non-Minimal Quintessence with a nearly flat potential

**Sudipta Das**

**Harish Chandra Research Institute**
**Allahabad, India**

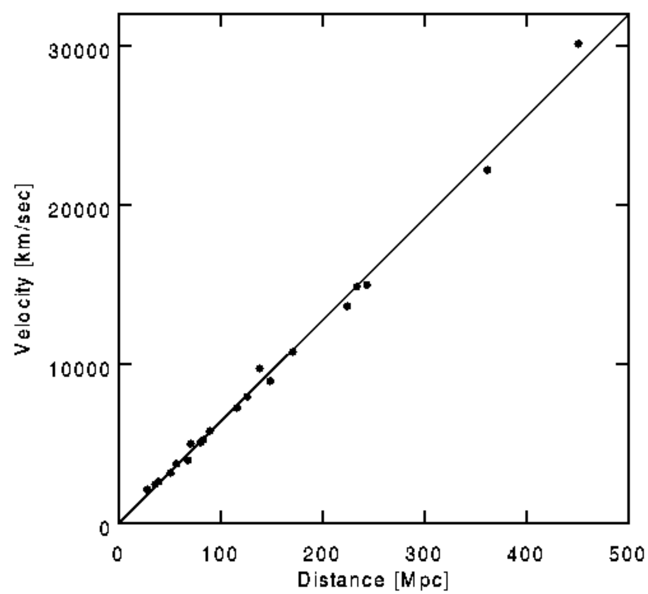
Collaborators : Anjan A Sen, Gaveshna Gupta

# Plan of the talk :

- Introduction
  - Dark energy
  - Motivation of this work
    - Non-minimally coupled scalar fields
    - Why flat potential?
- Brief description of the model
- Results
- Conclusions

# Introduction :

- We live in an expanding universe.
- 1929: Edwin Hubble shows that the further away the galaxy is, the more rapidly it is moving away from us




$$V = H_0 \times D$$

where the slope $H_0$ is the Hubble constant

# Introduction :

- One can use Einstein's equations to relate the expansion of the universe to spacetime curvature and the energy density.

$$G_{\mu\nu} = 8\pi G T_{\mu\nu}$$

- When applied to cosmology, this gives the Friedmann equations :

$$\frac{\dot{a}^2}{a^2} + \frac{\kappa}{a^2} = \frac{8\pi G}{3}\,\rho_{\text{tot}}$$
$$2\frac{\ddot{a}}{a} + \frac{\dot{a}^2}{a^2} + \frac{\kappa}{a^2} = -8\pi G\,p_{\text{tot}}$$

where $H = \frac{\dot{a}}{a}$ is the measure of the expansion rate of the universe.

# Introduction :

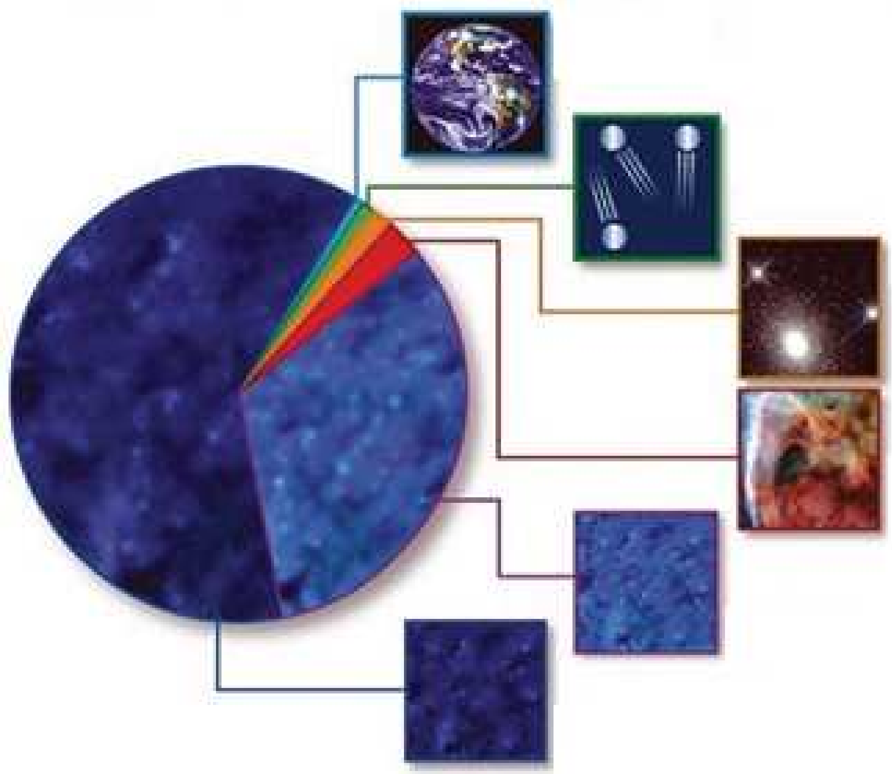
- The expansion was believed to be decelerated (as gravity is attractive).
- Recent observations reveal something completely different
- Observation : Type Ia SNe appear to be dimmer!
- The universe is accelerating!![a]
- This acceleration requires a driving agent $\longrightarrow$ popularly called “dark energy” $\longrightarrow$ overcomes gravitational attraction and generates acceleration.

---

[a] Riess et al., **116**, 1009 (1998)

# Introduction :

- 70% of the total energy density of the universe is made up of *dark energy*.



We know much, but understand very little!

# Introduction :

- Popular dark energy candidates are -
  - Cosmological constant
  - Quintessence scalar fields
  - Non-minimally coupled scalar fields
  - f(R) gravity models
  - and many others......

# Introduction :

- Characterized by equation of state parameter -

$$w_{de} = \frac{p_{de}}{\rho_{de}}$$

- Acceleration requires $w_{\rm tot} < -\frac{1}{3} \Rightarrow w_{de} < -\frac{1}{3}$.
- Recent observations suggest that $w_{de}$ is close to $-1$.
- Approximate bound is $-1.1 \leq w_{de} \leq -0.9$. [a]

[a] Davis et al, Astrophys. J. 666, 716 (2007);

Wood-vasey et al, Astrophys. J. 666, 694 (2007)

# Introduction :

- If $w_{de} = -1$, most reasonable choice is a cosmological constant.
- But if $w_{de} \approx -1 \;\Rightarrow$ dynamical dark energy models may be a good option.
- $$\begin{array}{lll} & & \text{Advantage} \;:\; \text{naturally provide } w_{de} \\ & \nearrow & \qquad\qquad \text{slightly less than } -1 \\ \text{Dynamical models} & & \\ & \searrow & \text{Disadvantage} \;:\; \text{scalar fields do not} \\ & & \qquad\qquad \text{have sound field} \\ & & \qquad\qquad \text{theoretic background} \end{array}$$
- Non-minimally coupled scalar field comes into action.

# Slow roll approximation :

- If the potential is nearly flat, the following slow-roll conditions are satisfied ::

$$\left(\frac{1}{V}\frac{dV}{d\phi}\right)^2 << 1$$

$$\frac{1}{V}\frac{d^2V}{d\phi^2} << 1$$

- This ensures that $w_\phi \equiv w_{de}$ never deviates very far from $-1$.
- With these assumptions, we try to obtain an analytic expression for $w_{de}$ in non-minimally coupled scalar field case.

# Field equations for the system :

- Effective action for Brans-Dicke (BD) theory

$$S = \int \sqrt{-g}\, d^4x \left[\psi R - \frac{\omega}{\psi}\psi^{,\mu}\psi_{,\mu} - 2V(\psi) + L_m\right] ,$$

- For a spatially flat FRW universe ::

$$\left.\begin{aligned}
3\frac{\dot{a}^2}{a^2} &= \frac{\rho_m}{\psi} + \frac{\omega}{2}\frac{\dot{\psi}^2}{\psi^2} - 3\frac{\dot{a}}{a}\frac{\dot{\psi}}{\psi} + \frac{V}{\psi} \\
2\frac{\ddot{a}}{a} + \frac{\dot{a}^2}{a^2} &= -\frac{\omega}{2}\frac{\dot{\psi}^2}{\psi^2} - \frac{\ddot{\psi}}{\psi} - 2\frac{\dot{a}}{a}\frac{\dot{\psi}}{\psi} + \frac{V}{\psi} \\
\ddot{\psi} + 3H\dot{\psi} &= \frac{\rho_m}{2\omega+3} + \frac{1}{2\omega+3}\left[4V - 2\psi\frac{dV}{d\psi}\right]
\end{aligned}\right\} \Rightarrow \dot{\rho_m} + 3\frac{\dot{a}}{a}\rho_m = 0$$

# Field equations in Einstein's frame

- We now make a conformal transformation

$$\bar{g}_{\mu\nu} = e^{\frac{\phi}{\sqrt{\zeta}}} g_{\mu\nu} \qquad \left(\zeta = \frac{2\omega+3}{2} \text{ and } \ln\psi = \frac{\phi}{\sqrt{\zeta}}\right)$$

such that $dt = e^{-\frac{\phi}{2\sqrt{\zeta}}} d\bar{t}$ and $a = e^{-\frac{\phi}{2\sqrt{\zeta}}} \bar{a}$ .

- The corresponding equations in the new frame look like

$$\left.\begin{aligned}
3\bar{H}^2 &= \bar{\rho}_m + \tfrac{1}{2}\dot{\phi}^2 + \bar{V} \\
2\dot{\bar{H}} + 3\bar{H}^2 &= -\tfrac{1}{2}\dot{\phi}^2 + \bar{V} \\
\ddot{\phi} + 3\bar{H}\dot{\phi} &= \frac{\bar{\rho}_m}{2\sqrt{\zeta}} - \frac{d\bar{V}}{d\phi} \\
\dot{\bar{\rho}}_m + 3\bar{H}\bar{\rho}_m &= -\sqrt{\tfrac{2}{3}} W \dot{\phi}\bar{\rho}_m
\end{aligned}\right\} \quad \left(\begin{aligned}
W &= \sqrt{\frac{3}{2}}\frac{1}{2\sqrt{\zeta}} \\
\bar{\rho}_m &= e^{-\frac{2\phi}{\sqrt{\zeta}}}\rho_m \\
\bar{V} &= e^{-\frac{2\phi}{\sqrt{\zeta}}} V
\end{aligned}\right)$$

- These equations can be written in the form of a plane - autonomous system by introducing variables ::
  - $x = \frac{\dot{\phi}}{\sqrt{6}\bar{H}}$
  - $y = \frac{\sqrt{\bar{V}}}{\sqrt{3}\bar{H}}$
  - $\lambda = -\frac{1}{\bar{V}}\frac{d\bar{V}}{d\phi}$
- These imply ::
  - $\bar{\Omega}_\phi = x^2 + y^2$
  - $\gamma \equiv 1 + w_\phi = \frac{2x^2}{x^2+y^2}$

- In terms of the new variables, the field equations become ::

$$\left.\begin{aligned} x' &= -3x + \lambda\sqrt{\tfrac{3}{2}}y^2 + \tfrac{3}{2}x(1+x^2-y^2) \\ &\qquad + W(1-x^2-y^2) \\ y' &= -\lambda\sqrt{\tfrac{3}{2}}xy + \tfrac{3}{2}y(1+x^2-y^2) \\ \lambda' &= -\sqrt{6}\lambda^2(\Gamma-1)x \end{aligned}\right\} \quad \left( \begin{aligned} \Gamma &\equiv \frac{\bar{V}\,\frac{d^2\bar{V}}{d\phi^2}}{\left(\frac{d\bar{V}}{d\phi}\right)^2} \\ x' &\equiv \frac{dx}{d\ln a} \end{aligned} \right)$$

- Changing the dependent variables from $x$ and $y$ to the observable quantities $\Omega_\phi$ and $\gamma$ ::

$$\Omega_\phi{}' = 3(1-\Omega_\phi)\left[(1-\gamma)\Omega_\phi + \frac{W}{3}\sqrt{2\gamma\Omega_\phi}\right]$$

$$\begin{aligned}\gamma' = &-3\gamma(2-\gamma) + \lambda(2-\gamma)\sqrt{3\gamma\Omega_\phi} \\ &+ \sqrt{\frac{2\gamma}{\Omega_\phi}}W(1-\Omega_\phi)(2-\gamma)\end{aligned}$$

$$\lambda' = -\sqrt{3}\lambda^2(\Gamma-1)\sqrt{\gamma\Omega_\phi}$$

- From these one arrives at :

$$\begin{aligned}\frac{d\gamma}{d\Omega_\phi} \quad &= \quad \frac{-3\gamma(2-\gamma)+\lambda(2-\gamma)\sqrt{3\gamma\Omega_\phi}}{3(1-\gamma)(1-\Omega_\phi)\left[\Omega_\phi + \frac{W}{3}\sqrt{2\gamma\Omega_\phi}(1+\gamma)\right]} \\ &+ \quad \frac{\sqrt{\frac{2\gamma}{\Omega_\phi}}W(1-\Omega_\phi)(2-\gamma)}{3(1-\gamma)(1-\Omega_\phi)\left[\Omega_\phi + \frac{W}{3}\sqrt{2\gamma\Omega_\phi}(1+\gamma)\right]}\end{aligned}$$

- At this point, we make two assumptions ::
  - $\gamma << 1 \implies w_\phi$ is very close to $-1$
  - The potential is nearly flat, i.e, slow-roll conditions

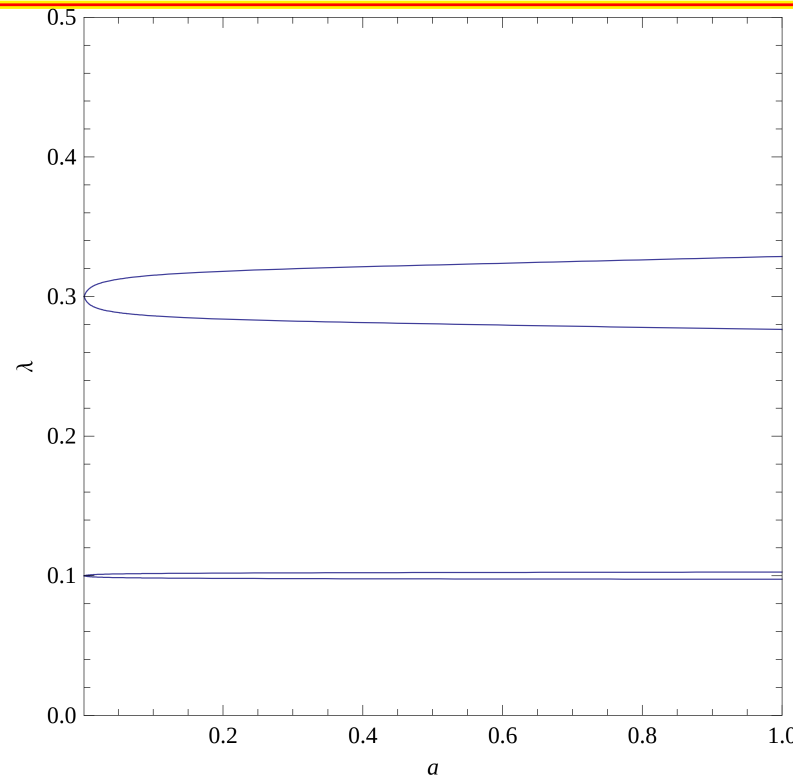
$$\left(\frac{1}{V}\frac{dV}{d\phi}\right)^2 << 1$$

$$\frac{1}{V}\frac{d^2V}{d\phi^2} << 1$$

    are satisfied.

- This implies, $\lambda$ is approximately constant ($\lambda = \lambda_0$).

# Plot of $\lambda$ vs. $a$ for $W = 0.05$ :




- The lower set is for $\lambda_0 = 0.1$ and the upper set is for $\lambda_0 = 0.3$
- In each case upper line is for $V(\phi) = \phi^2$ and lower line is for $V(\phi) = \phi^{-2}$

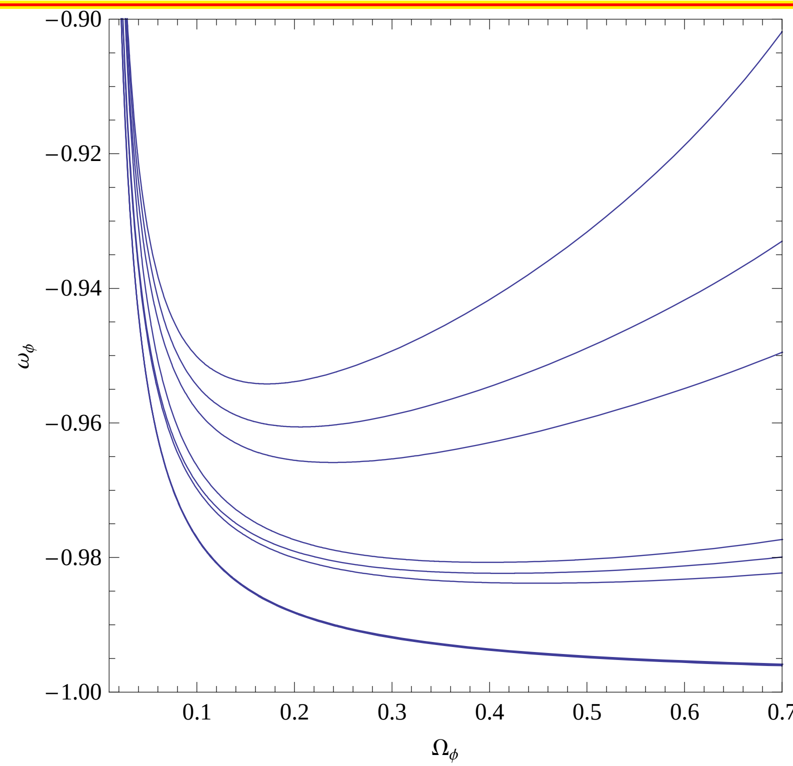
- Taking $\lambda = \lambda_0$ and retaining terms only upto lowest order in $\gamma$, we get ::

$$\frac{d\gamma}{d\Omega_\phi} = \frac{-2\gamma}{\Omega_\phi(1-\Omega_\phi)} + \frac{2}{\sqrt{3}}\frac{\lambda_0\sqrt{\gamma}}{\sqrt{\Omega_\phi}(1-\Omega_\phi)} + \frac{2\sqrt{2}}{3}\frac{W\sqrt{\gamma}}{\Omega_\phi^{3/2}}$$

- Solving this equation, we get the expression for the equation of state parameter as ::

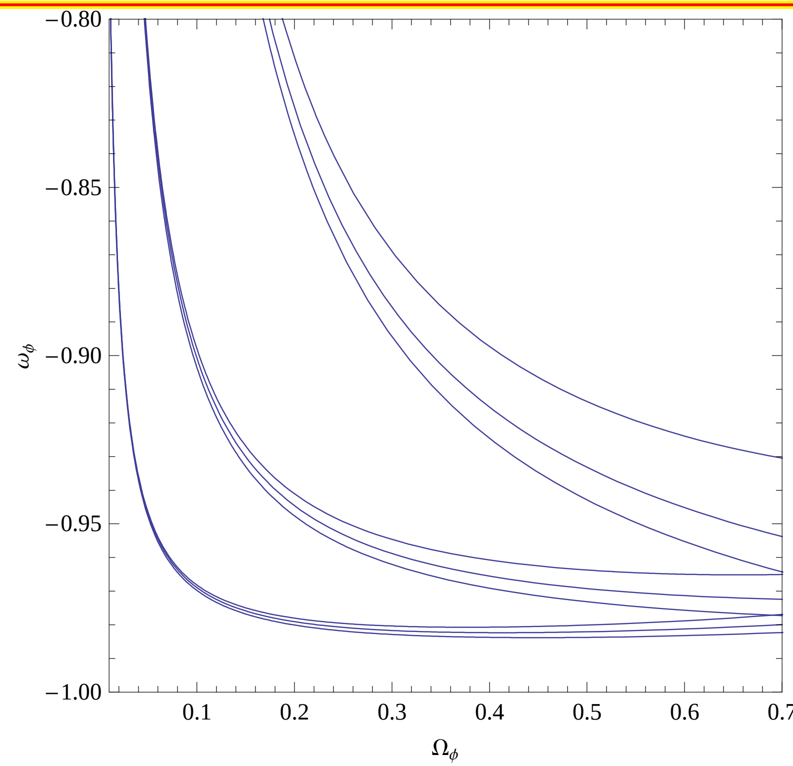
$$1+\omega_\phi = [\frac{\lambda_0}{\sqrt{3\Omega_\phi}} - \left(\frac{1}{\Omega_\phi}-1\right)[(\frac{\lambda_0}{2\sqrt{3}} - \frac{\sqrt{2}W}{3}) \times \log\left(\frac{1+\sqrt{\Omega_\phi}}{1-\sqrt{\Omega_\phi}}\right) - \alpha]]^2$$

# Plot of $w_\phi$ vs. $\Omega_\phi$ for different potentials :




- The three sets are for $\lambda_0 = 0.1,\ 0.2,\ 0.3$ from bottom to top ($W = 0.05$)
- For each set, top line is for $V(\phi) = \phi^2$, middle line is for analytical expression and bottom line is for $V(\phi) = \phi^{-2}$

# Plot of $w_\phi$ vs. $\Omega_\phi$ for different potentials :

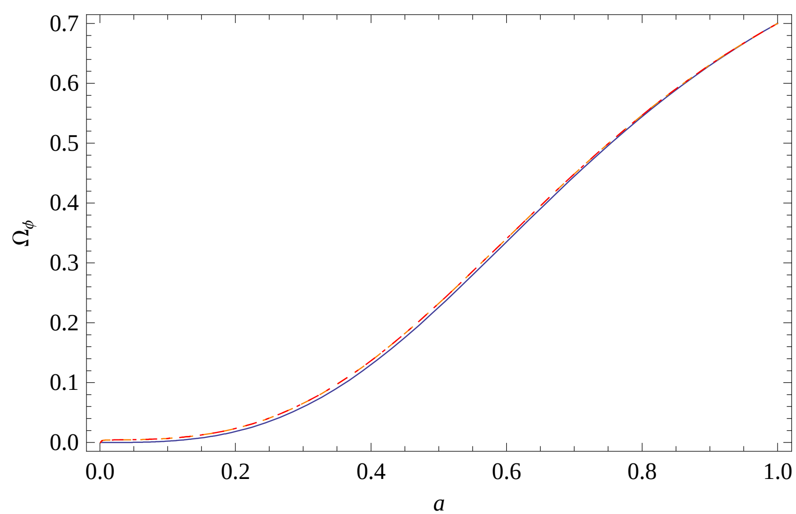



- The three sets are for $W = 0.05,\ 0.1,\ 0.2$ from bottom to top ($\lambda_0 = 0.3$)
- For each set, top line is for $V(\phi) = \phi^2$, middle line is for analytical expression and bottom line is for $V(\phi) = \phi^{-2}$

# Plot of $\Omega_\phi$ vs. $a$ for different potentials :

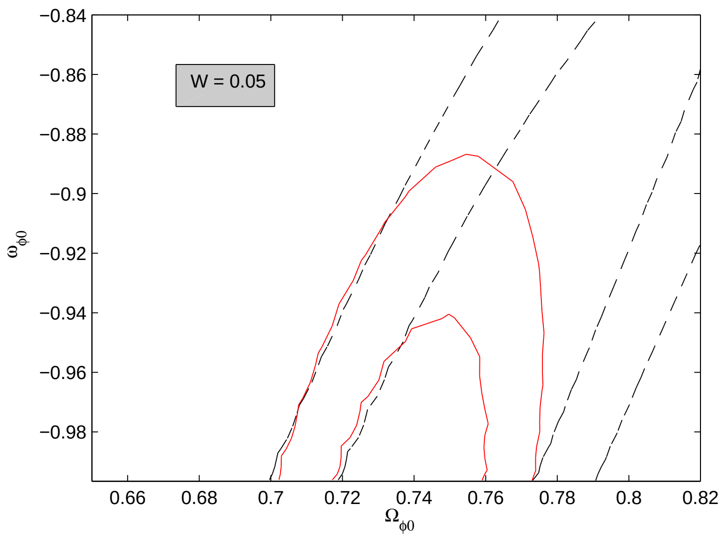
- We solve for $\Omega_\phi(a)$ which in turn gives $w_\phi(a)$.
- This is essential for confronting the model with any observational result.

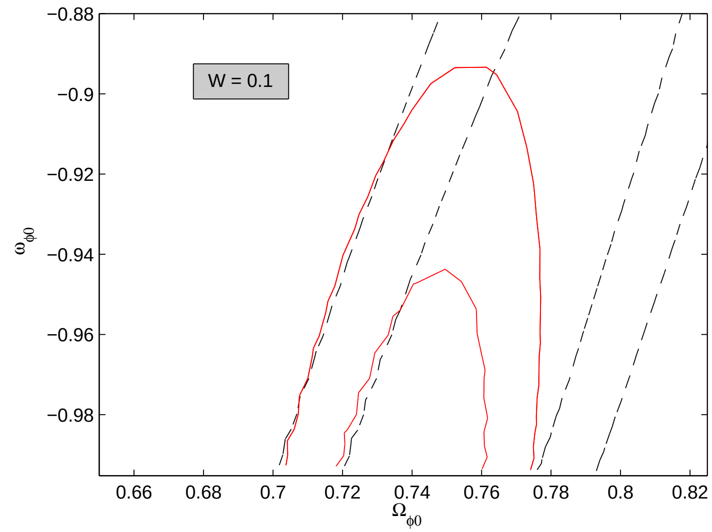
$$\Omega_\phi = \left[1 + \left(\frac{1}{\Omega_{\phi 0}} - 1\right) a^{-3}\right]^{-1}$$

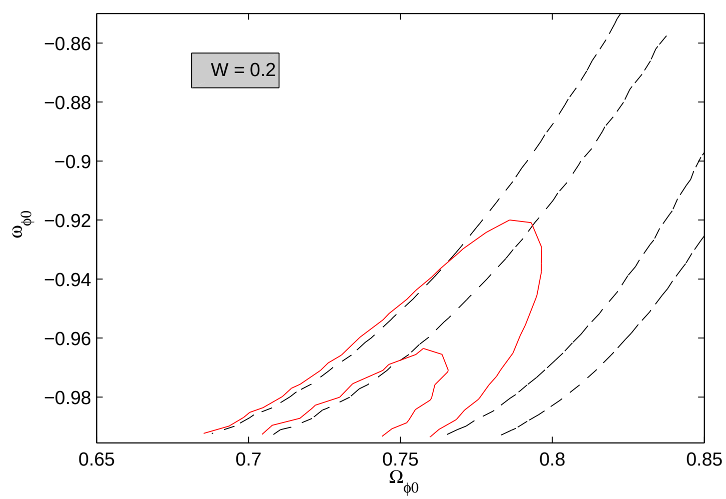



- Plot of $\Omega_\phi$ vs $a$ for $\lambda_0 = 0.3$, $\Omega_{\phi 0} = 0.7$ and $W = 0.1$.

# Observational Constraints :




- Dashed lines are for SNIa data and red lines are for SNIa + BAO data.

# Conclusions :

- We have studied the dark energy models in Brans-Dicke theory.
- We show that various scalar field potentials satisfying the slow-roll conditions actually converge to a universal behaviour.
- We have obtained a general form of $w_\phi$ in the BD theory.
- The evolution of the universe is insensitive to the form of the potential.
- SnIa data indicates that to get strong bound on $w_\phi$, one needs to have strong bound on $\Omega_\phi$.
- BAO data put a strong upper bound on $w_\phi$ irrespective of the value of $W$.

# Thank You