Tachyon cosmology, supernovae data and the Big Brake singularity

Phys. Rev. D **79**, 083504-1-8 (2009)

László Árpád Gergely
University of Szeged, Hungary

with ***Zoltán Keresztes*** *(Szeged),*
Alexander Yu. Kamenshchik *(Bologna, Moscow),*
Vittorio Gorini *&* ***Ugo Moschella*** *(Como, Milan)*

2009

Outline:

Why do we need a new model?

Modified dynamics? Modified matter?

Friedmann universe with tachyons,
a dark energy candidate

Dynamics (negative parameter)

Dynamics (positive parameter)

Singularities

Let the supernovae select: The past evolution

Future evolution: de Sitter or Big Brake?

How, where and when?

Why do we need a new model?

- Type Ia supernovae: standard(izable) candles.

  Calibration methods from time-dependence of emitted luminosity and spectrum

1. Multi-Color Light Curve analysis (shape of the light curve and spectral distribution of the emission converted into the reference system of the host galaxy).
2. For distant supernovae: time dilation and K-correction

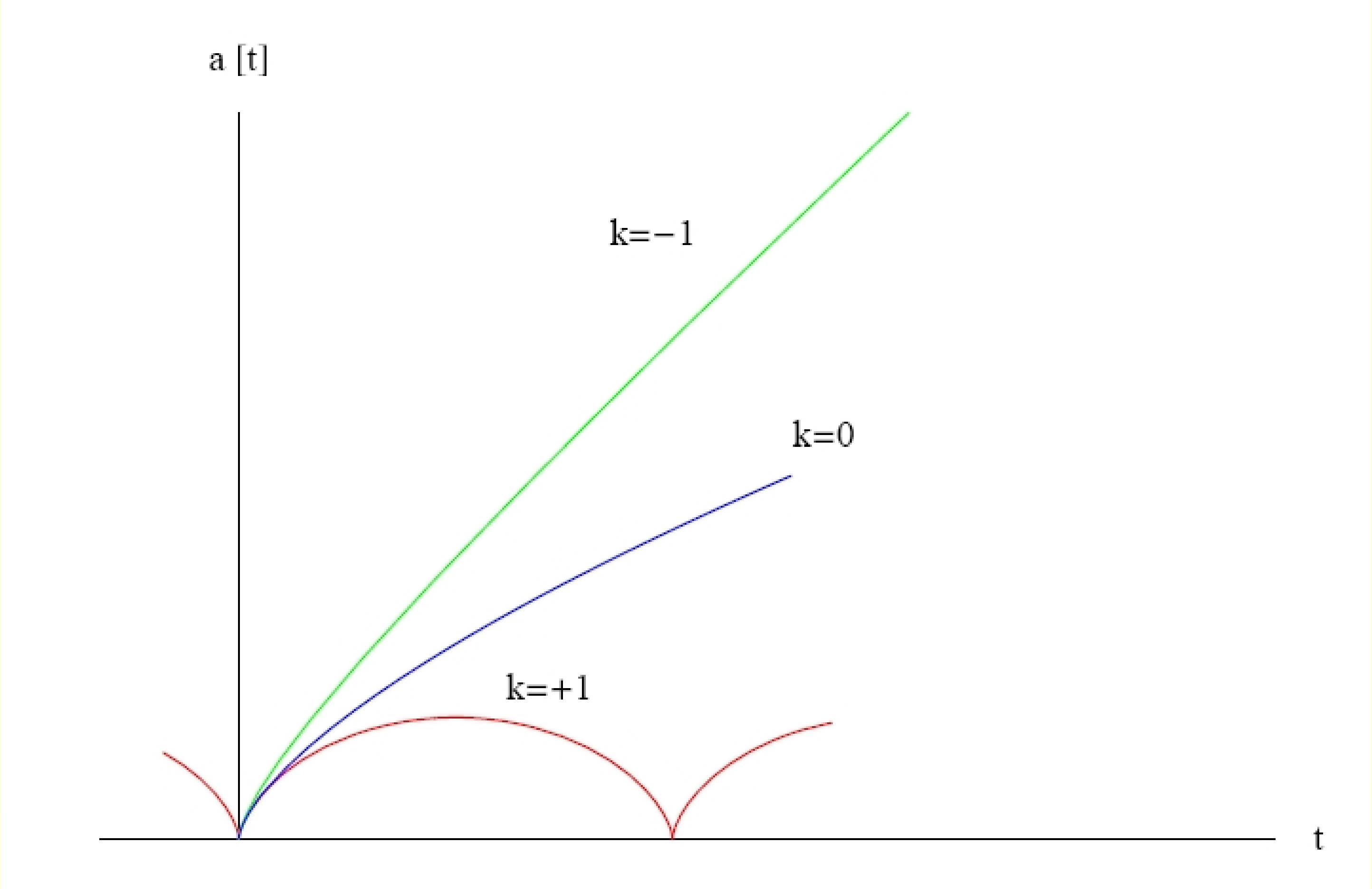
- luminosity distance = f (redshift, cosmological model)

$$d_L(z) = (1+z)\int_0^z \frac{dz'}{H(z')}.$$

What does not fit?

- Standard cosmological model

  without cosmological constant

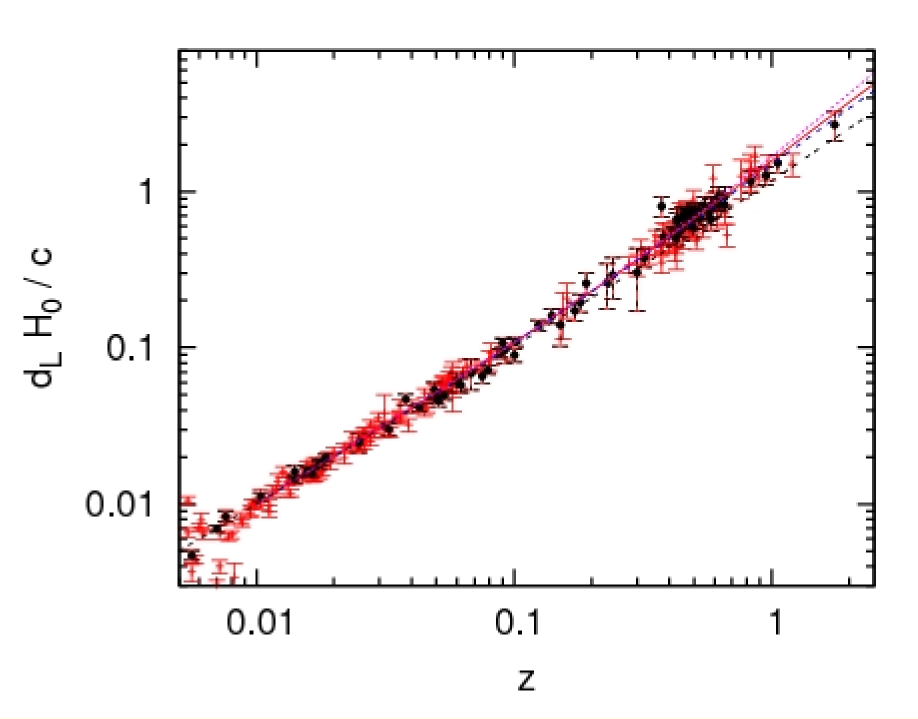


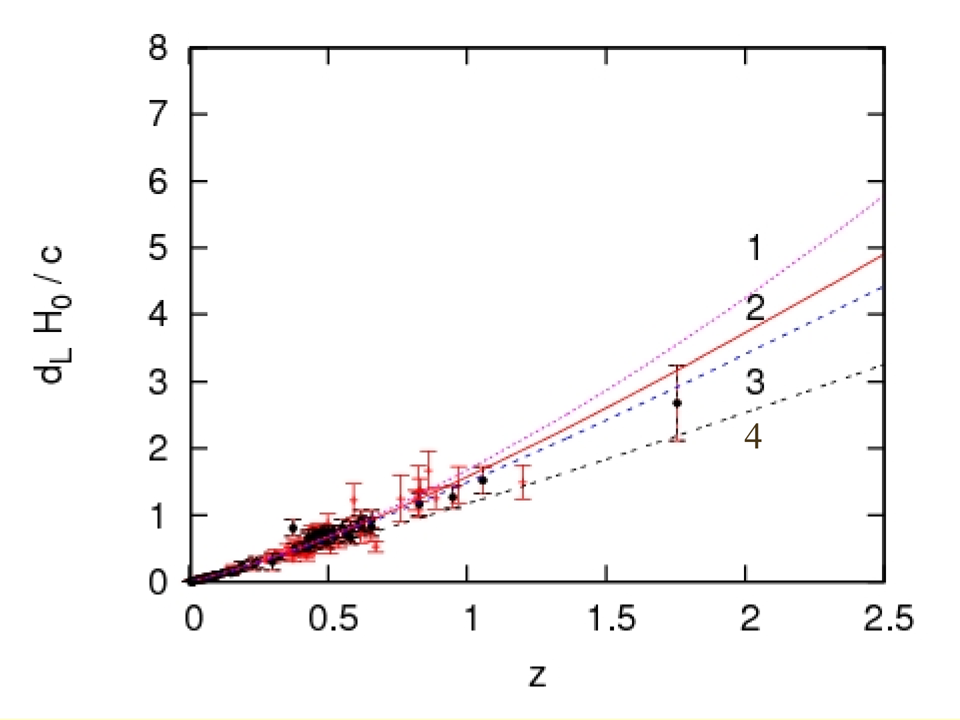
What does fit?

- LCDM
- and many other models ...

(1,3 brane-world models, 2=LCDM;

- 4=CDM does not fit!





GM Szabó, **LÁ Gergely**, Z Keresztes
PMC Physics A **1** : 8-1-18 (2007)

Different routes to reconcile with observations

- Einstein-equation:

$$G_{ab} = 8\pi G\, T_{ab}$$

- modified with dark matter, dark energy terms:

$$G_{ab} = 8\pi G\, (T + \text{unusual matter})_{ab}$$

- modified gravitational dynamics

$$[G + f\,(\text{metric, extrinsic curvature})]_{ab} = 8\pi G\, T_{ab}$$

Tachyonic cosmological model

- Friedmann universe $$ds^2 = g_{ab}dx^a dx^b = dt^2 - a^2(t)dl^2,$$
- Tachyonic Lagrangian (Sen) $$L = -V(T)\sqrt{1 - g_{00}\dot{T}^2}$$
- Metric variation

energy density: $$\varepsilon = \frac{V(T)}{\sqrt{1-\dot{T}^2}}$$

pressure: $$p = -V(T)\sqrt{1-\dot{T}^2}.$$

Tachyonic field variation

$$\frac{\ddot{T}}{1-\dot{T}^2} + 3\frac{\dot{a}\dot{T}}{a} + \frac{V_{,T}}{V} = 0$$

How does it relate to a minimally coupled scalar field?

- Minimally coupled scalar field

$$L = \frac{1}{2}\dot{\varphi}^2 - U(\varphi)$$

$$\varepsilon = \frac{1}{2}\dot{\varphi}^2 + U(\varphi),$$

$$p = \frac{1}{2}\dot{\varphi}^2 - U(\varphi).$$

- equivalent with the tachyonic field iff

$$U = \frac{1}{2}(\varepsilon - p) = \frac{(\varepsilon a^6)'}{6a^5}. \qquad \varphi = F(a) = \int^a \frac{dx}{x}\sqrt{\frac{-\varepsilon'(x)\,x}{3\varepsilon(x)}}$$

- comparison in terms of the Hubble parameter:

$$\dot{T}^2 = \frac{\varepsilon + p}{\varepsilon} = -\frac{2\dot{h}}{3h^2}, \qquad V(T) = \sqrt{h^2\left(\frac{2}{3}\dot{h} + h^2\right)},$$

$$\Rightarrow \quad \dot{h} \geq -\frac{3}{2}h^2$$

V. Gorini, A.Yu. Kamenshchik, U. Moschella and V. Pasquier, Phys. Rev. D **69**, 123512 (2004).

$$\dot{\varphi}^2 = \varepsilon + p = -\frac{2}{3}\dot{h}, \qquad U(\varphi) = h^2 + \frac{1}{3}\dot{h}.$$

$$\Rightarrow \quad \dot{h} \leq 0$$

Tachyonic field more restrictive!!!

A toy tachyonic model with unexpected features

- Potential:

$$V(T) = \frac{\Lambda}{\sin^2\left(\frac{3}{2}\sqrt{\Lambda(1+k)}T\right)} \times \sqrt{1-(1+k)\cos^2\left(\frac{3}{2}\sqrt{\Lambda(1+k)}T\right)}$$

Λ is a positive constant and $-1 < k < 1$.

(mixture of a cosmological constant and a perfect fluid)

- Tachyonic dynamics:

$$\dot{T} = s,$$

$$\dot{s} = -3\sqrt{V}(1-s^2)^{3/4}s - (1-s^2)\frac{V_{,T}}{V}$$

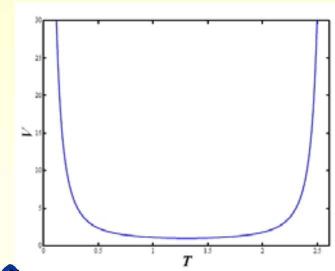
Gravitational dynamics:

$$H^2 = \varepsilon,$$

+ continuity eq.

Dynamics for $-1<k\leq 0$

- Potential:

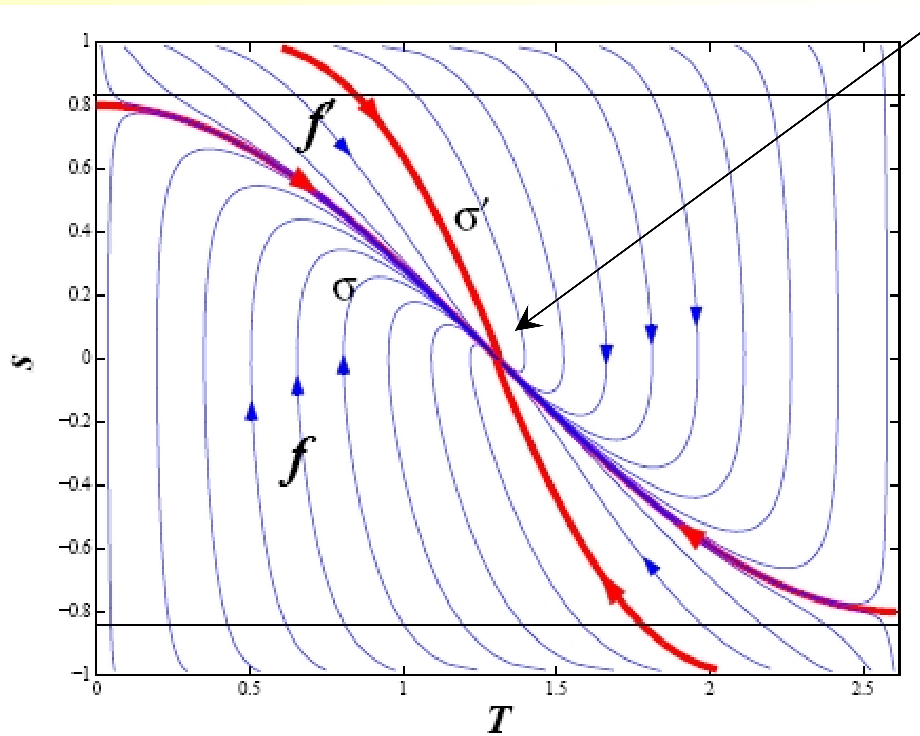


- Phase space:

$-1<k\leq -1/3$

$-1/3<k\leq 0$

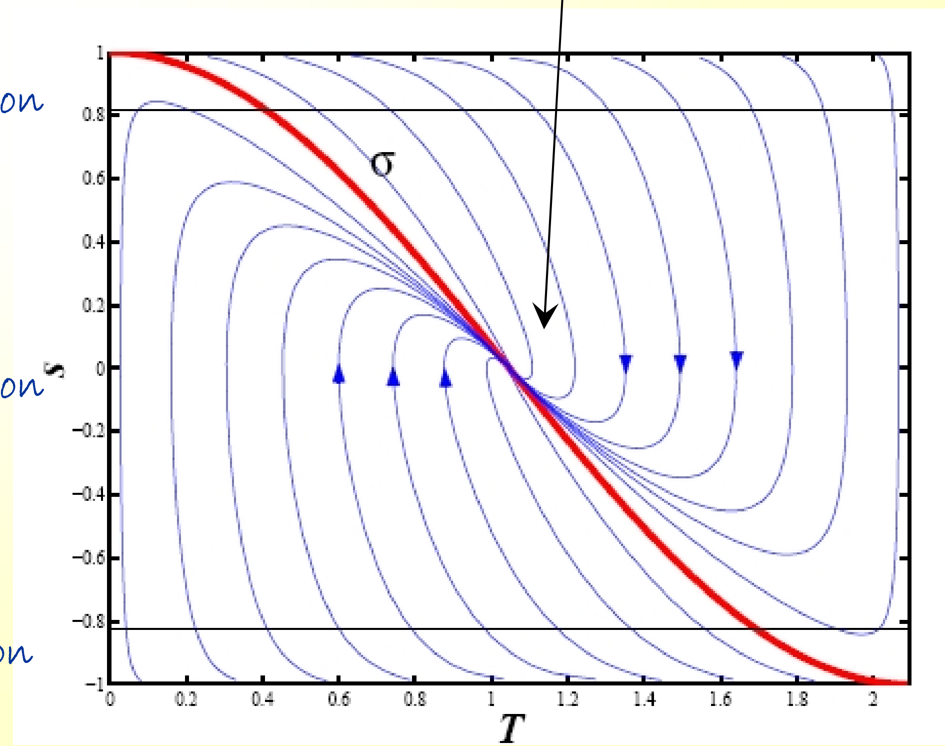
de Sitter node



deceleration

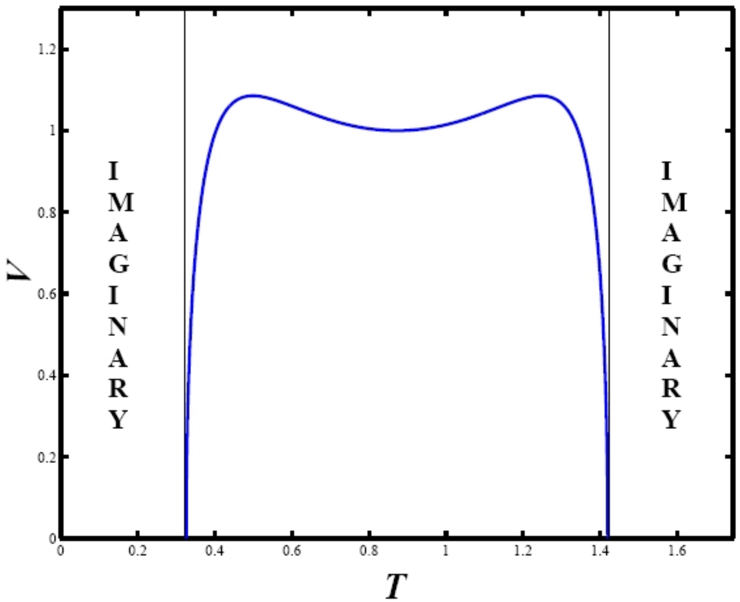
acceleration

deceleration

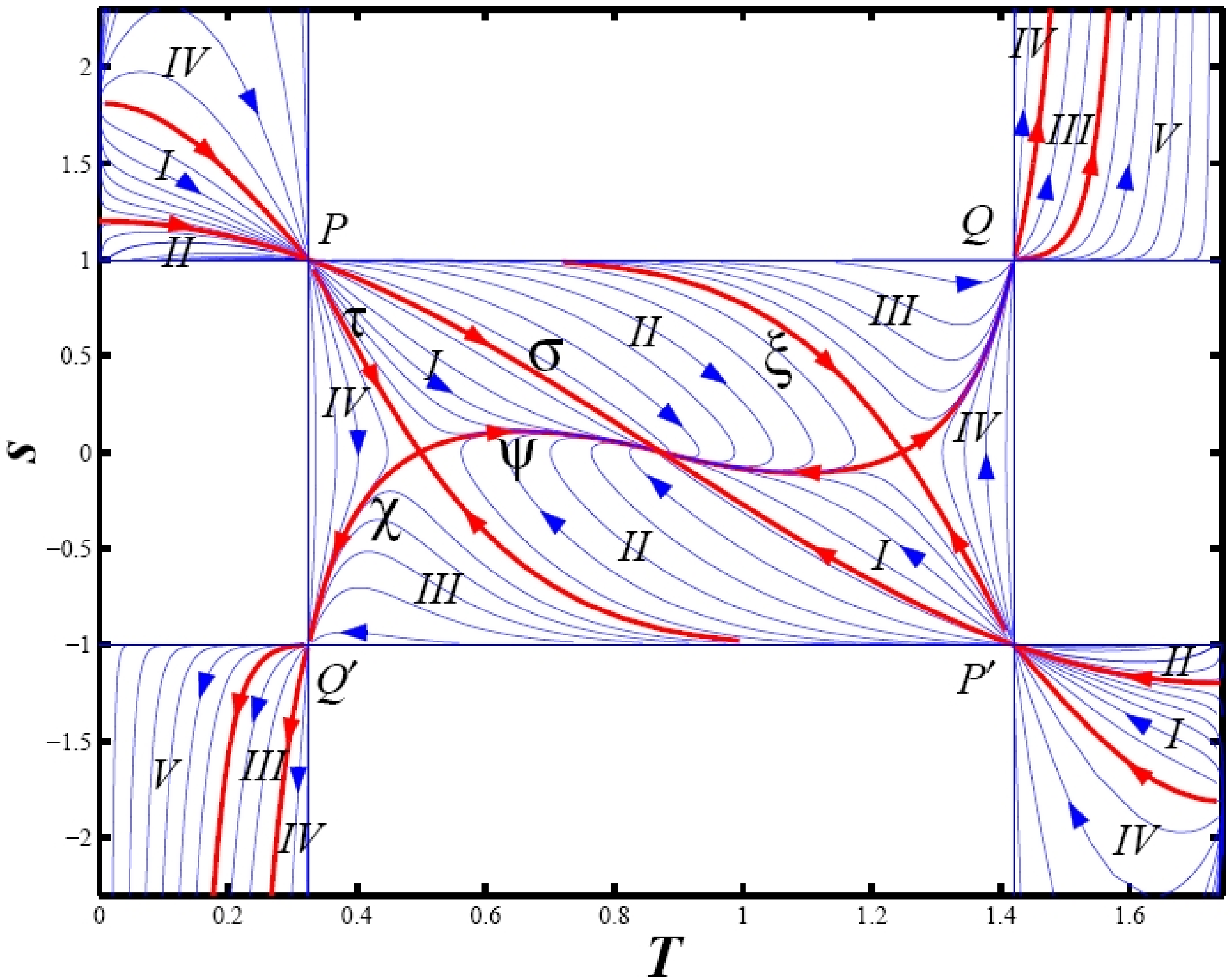


Dynamics for 0<k<1

- Potential:



Phase space:



Singularities for 0<k<1

- curvature scalar $R = 3h^2(4-3s^2) = \dfrac{3V(T)(4-3s^2)}{\sqrt{1-s^2}}$

singular at 1. $s = \pm 1$, except when V(T)=0

dust-like behaviour $\varepsilon = \dfrac{4}{9t^2}$, $H = \dfrac{2}{3t}$

and 2. $s = \pm\infty$

- V(T) singular at $s = \sqrt{\frac{1+k}{k}}, T = 0$

$\varepsilon = \dfrac{4k^2}{9(1+k)^2 t^2}$, $H = \dfrac{2k}{3(1+k)t}$

perfect barotropic fluid with $w \equiv \dfrac{p}{\varepsilon} = \dfrac{1}{k}$

Through the corners

- R is not singular → must continue the evolution

- present epoch | transition | tachyonic

present epoch	transition	tachyonic
$s^2<1$	$s^2=1$	$s^2>1$
$p<0$	$p=0$	$p>0$

- in the tachyonic regime:

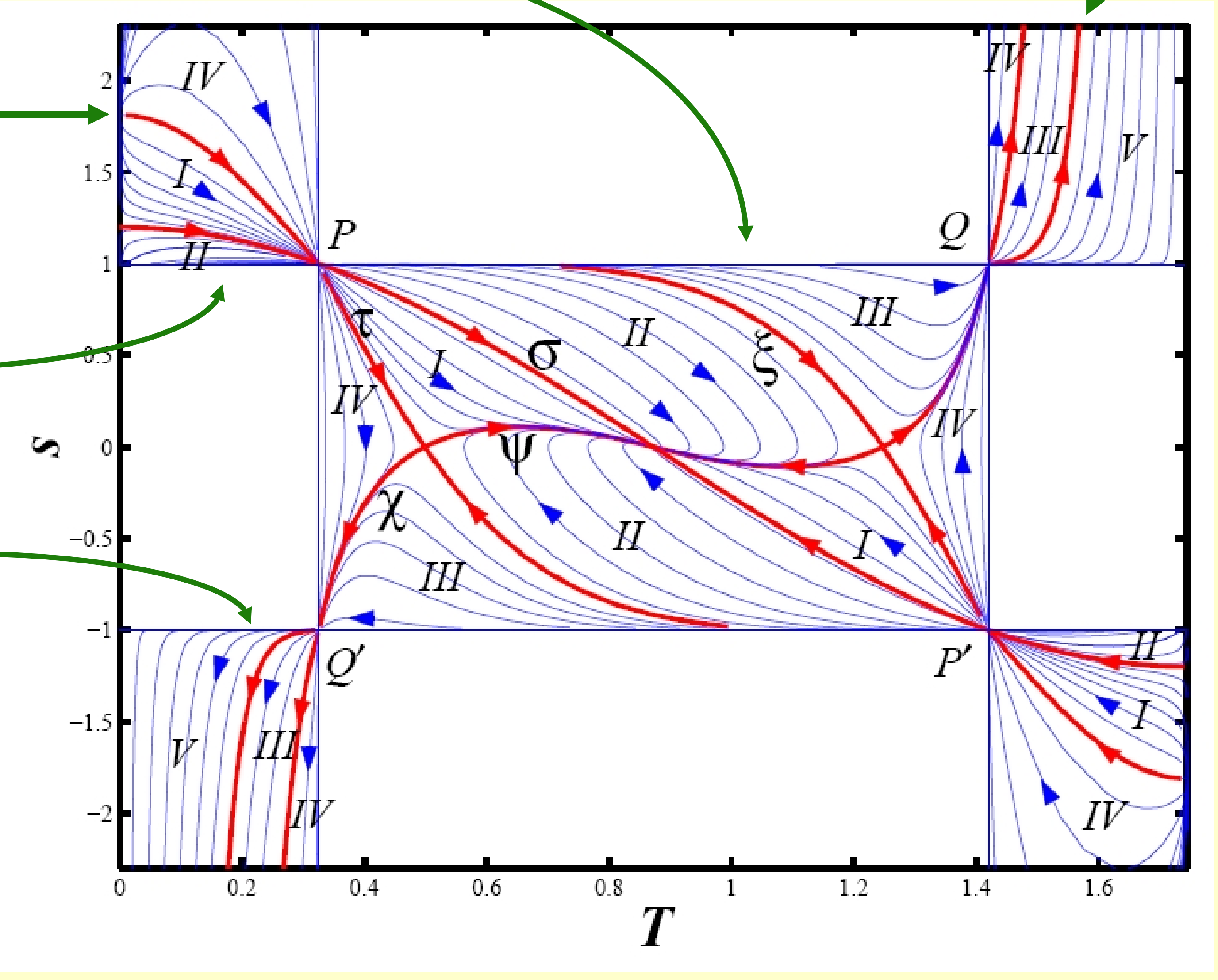
$$V(T)=i\,W(T), \quad (1-s^2)^{1/2}=i\,(s^2-1)^{1/2}$$

- but the energy density and pressure stay real!

Type I-V evolutions for 0<k<1

- Big Bang for III
- Big Bang for I and IV
- Big Bang for II
- Big Bang for V

Big Brake



Type I-V evolutions for 0<k<1

- **Type I:** tachyonic Big Bang → tachyonic evolution → tachyonic crossing → accelerated expansion → de Sitter (infinite exponential expansion)
- **Type II:** $s^2=1$ Big Bang → tachyonic evolution → tachyonic crossing → accelerated expansion → de Sitter
- **Type III:** $s^2=1$ Big Bang → accelerated expansion → tachyonic crossing → Big Brake
- **Type IV:** tachyonic Big Bang → tachyonic evolution → tachyonic crossing → accelerated expansion → tachyonic crossing → Big Brake
- **Type V:** $s^2=1$ Big Bang → Big Brake

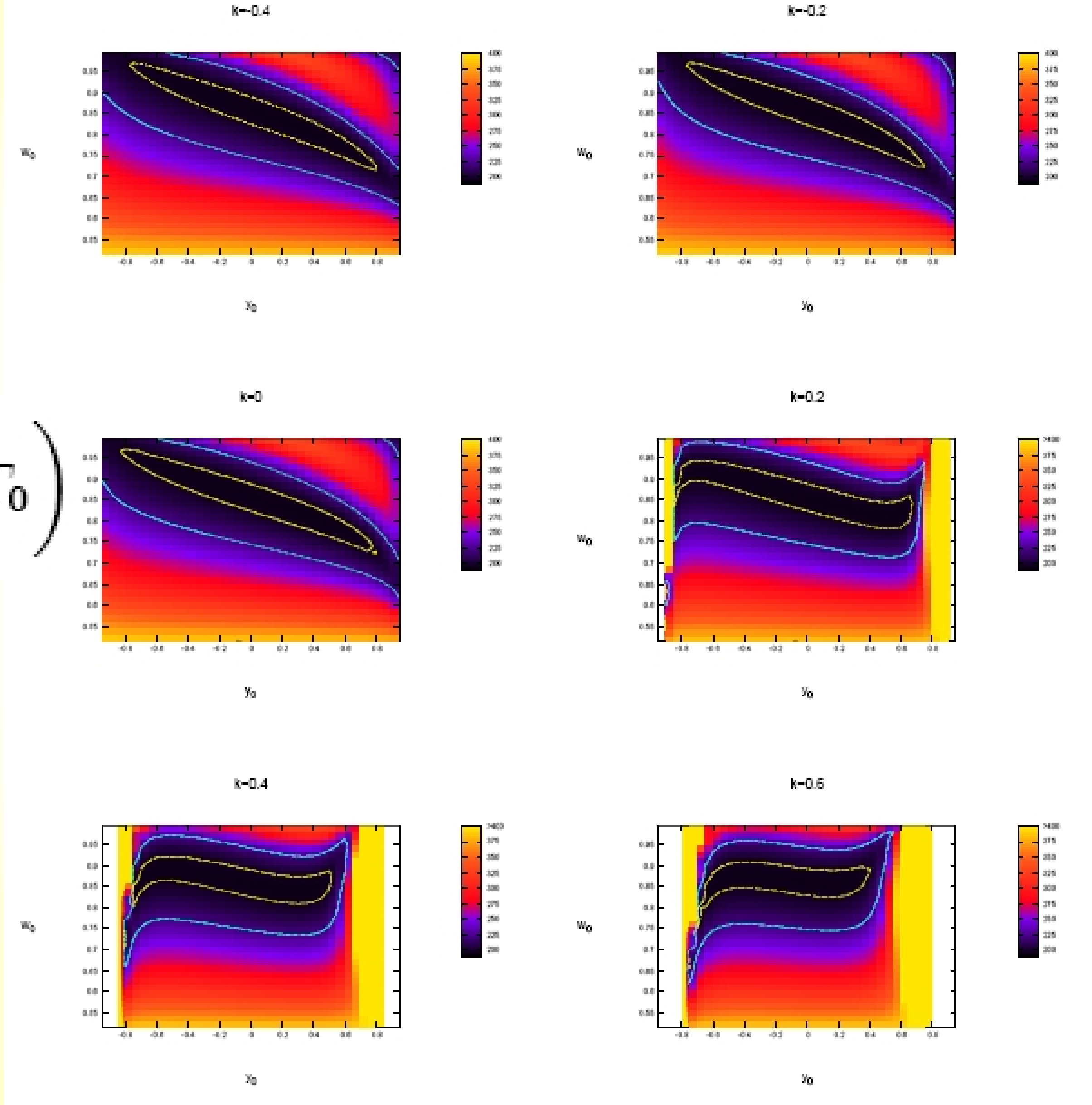
Past evolution: Let the supernovae decide …

- 1σ and 2σ contours in the phase space

$$y_0 = \cos\left(\frac{3}{2}\sqrt{\Omega_\Lambda(1+k)}H_0 T_0\right)$$

$$w_0 = 1/\left(1+s_0^2\right)$$

for various k:

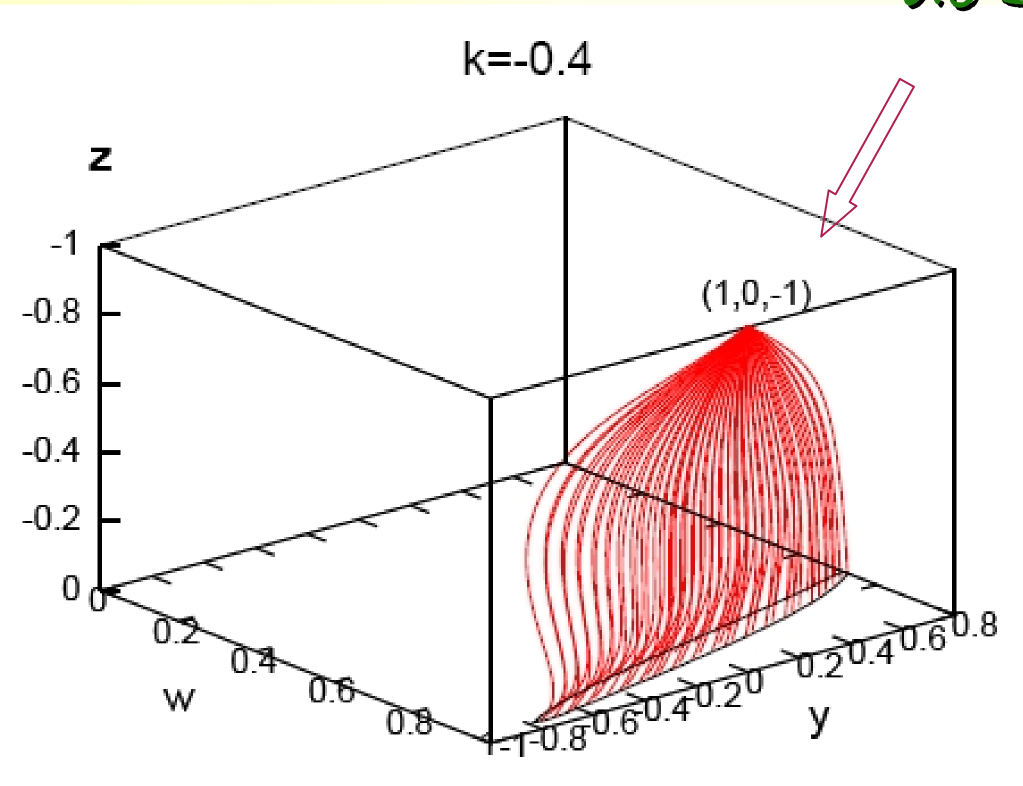


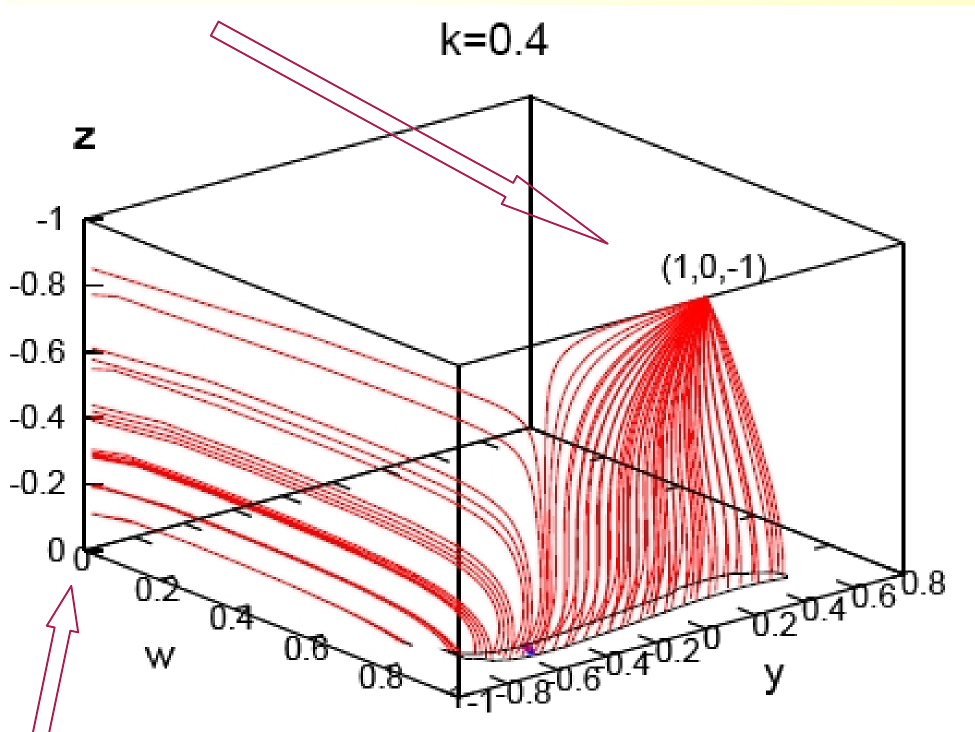
Future evolution from the 1σ contours

negative k:

positive k:

de Sitter





Big Brake (w=0)

Big Brake: How?

- scale factor regular (finite)
- Hubble parameter and energy density → 0
- acceleration → - ∞
- velocity of tachyonic field s → ± ∞
- pressure → ∞

- Kinematical analysis (Barrow 2003) predicted such singularities, named sudden future singularities
- From a combined kinematical and observational viewpoint, Big Brake could be even earlier (Dabrowski, Denkiewitz, Hendry 2007)
- Quantum study of the Big Brake showed singularity avoidance (Kamenshchik, Kiefer, Sandhoefer 2007)
- The Big Brake singularity is stable (Barrow 2009, private communication)

BUT: the speed of sound becomes imaginary and its module diverges

→ need to reconcile this more carefully with the stability argument

Big Brake: Where?

Everywhere !!!

(homogeneous, isotropic model)

Big Brake: When?

TABLE II: Properties of the tachyonic universes with $k = 0.2$ which (a) are within 1σ confidence level fit with the type Ia supernova data and (b) evolve into a Big Brake singularity. Columns (1) and (2) represent a grid of values of the allowed model parameters. Columns (3) and (4): the redshift z_* and time t_* at the future tachyonic crossing (when $s = 1$ and the pressure becomes positive). Columns (5) and (6): the redshift z_{BB} and time t_{BB} necessary to reach the Big Brake. The former indicates the relative size of the universe when it encounters the Big Brake. (The values of t_* and t_{BB} were computed with the Hubble parameter $H_0 = 73$ km/s/Mpc.)

y_0	w_0	z_*	t_* $(10^9 yrs)$	z_{BB}	t_{BB} $(10^9 yrs)$
−0.90	0.635	−0.024	0.3	−0.068	1.0
−0.85	0.845	−0.158	2.4	−0.194	3.1
−0.85	0.860	−0.162	2.4	−0.198	3.1
−0.85	0.875	−0.166	2.5	−0.201	3.2
−0.80	0.890	−0.363	6.2	−0.390	6.9
−0.80	0.905	−0.384	6.7	−0.409	7.3
−0.80	0.920	−0.408	7.2	−0.432	7.9

Future tasks

- What happens with the observers at the tachyonic crossing?
- … and at the Big Brake?
- A meticulous stability analysis
- Be patient and wait to see …

  … whether the Big Brake happens