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PS5 – DE as new energy component

QCD, q –theory, and the cosmological constant

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0. Introduction

“Dark Energy” (“DE”):

- effect is more or less established (accelerating Universe);
- nature and origin remain unclear.

At this moment, there is a need for new physical mechanisms. One has been proposed by Volovik and the speaker under the name of ‘ q –theory.’

This talk consists of two parts:

1. basic idea of q –theory applied to QCD [1];
2. modified-gravity model of the present Universe [2].

[1] FRK & G.E. Volovik, PRD 79, 063527 (2009), arXiv:0811.4347.

[2] FRK, arXiv:0904.3276.

1. Gravitating gluon condensate

QCD gluon condensate [3]:

$$q = \langle 0 | \frac{1}{4\pi^2} G^{a\mu\nu} G^a_{\mu\nu} | 0 \rangle = \langle 0 | \frac{1}{4\pi^2} G_{a\kappa\lambda} g^{\kappa\mu} g^{\lambda\nu} G^a_{\mu\nu} | 0 \rangle, \quad (1)$$

with Yang–Mills field strength $G^a_{\mu\nu} = \partial_\mu A^a_\nu - \partial_\nu A^a_\mu + f^{abc} A^b_\mu A^c_\nu$ for $su(3)$ structure constants f^{abc} .

particle physics experiments: $q \sim (100 \text{ MeV})^4$

observational cosmology: $\rho_{\text{vac}} \sim (1 \text{ meV})^4$

⇒ how to reconcile the typical QCD vacuum energy density $\epsilon_{\text{QCD}} \sim 10^{32} \text{ eV}^4$ with the observed value $\rho_{\text{vac}} \sim 10^{-12} \text{ eV}^4$?

[3] M.A. Shifman, A.I. Vainshtein, & V.I. Zakharov, NPB 147, 385 (1979).

1. Gravitating gluon condensate

General q -theory argument [1]:

1. there exists a conserved microscopic variable q whose macroscopic behavior can be studied;
2. the microscopic vacuum energy density (ϵ_{vac}) differs from the one (ρ_{vac}) that enters the gravitational equations;
3. in equilibrium, q has self-adjusted to the value q_0 with $\rho_{\text{vac}}(q_0) = 0$.

Here, q explicitly given by (1).

1. Gravitating gluon condensate

Effective action:

$$S_{\text{eff}} = S_{\text{grav}} + S_{\text{vac}} = \int d^4x \sqrt{-g} \left(\frac{1}{16\pi G_N} R[g] + \epsilon(q) \right) \quad (2)$$

Energy-momentum tensor for the gravitational field equations:

$$\begin{aligned} T_{\mu\nu}^{\text{vac}} &= -\frac{2}{\sqrt{-g}} \frac{\delta S_{\text{vac}}}{\delta g^{\mu\nu}} \\ &= \epsilon(q) g_{\mu\nu} - 2 \frac{d\epsilon(q)}{dq} \frac{\delta q}{\delta g^{\mu\nu}} \\ &= \left(\epsilon(q) - q \frac{d\epsilon(q)}{dq} \right) g_{\mu\nu} \\ &\equiv \rho_{\text{vac}}(q) g_{\mu\nu} \end{aligned} \quad (3)$$

In equilibrium: $q = q_0$, $\rho_{\text{vac}}(q_0) = 0$, and $g_{\mu\nu}(x) = \eta_{\mu\nu}^{\text{Minkowski}}$.

1. Gravitating gluon condensate

In a nonequilibrium state such as the expanding Universe (Hubble parameter $H \neq 0$), there is a perturbation of the vacuum:

$$q = q_0 + \delta q(H) \neq q_0 \Rightarrow \rho_{\text{vac}}(q) \sim \frac{d\rho_{\text{vac}}}{dq} \delta q(H) \neq 0. \quad (4)$$

For QCD, this is a difficult IR problem (cf. [4]). *A priori*, we can have

$$\begin{aligned} \rho_{\text{vac}}(H) &\sim 0 + H^2 \Lambda_{\text{QCD}}^2 + H^4 + \dots \\ &+ |H| \Lambda_{\text{QCD}}^3 + |H|^3 \Lambda_{\text{QCD}} + \dots \end{aligned} \quad (5)$$

Linear term in H gives the correct order of magnitude for ρ_{vac} , a.k.a. the cosmological “constant.”

[4] F.R. Urban & A.R. Zhitnitsky, arXiv:0906.2162.

2. Modified-gravity cosmology

Flat FRW universe has Ricci curvature scalar $R = -6(2H^2 + \dot{H})$.

So, previous $|H| \Lambda_{\text{QCD}}^3$ term suggests modified-gravity action [2]:

$$S_{\text{eff}} = \int_{\mathbb{R}^4} d^4x \sqrt{-g} \left(K R + \eta |R|^{1/2} |q|^{3/4} + \epsilon[q] + \mathcal{L}^M[\psi, g] \right), \quad (6)$$

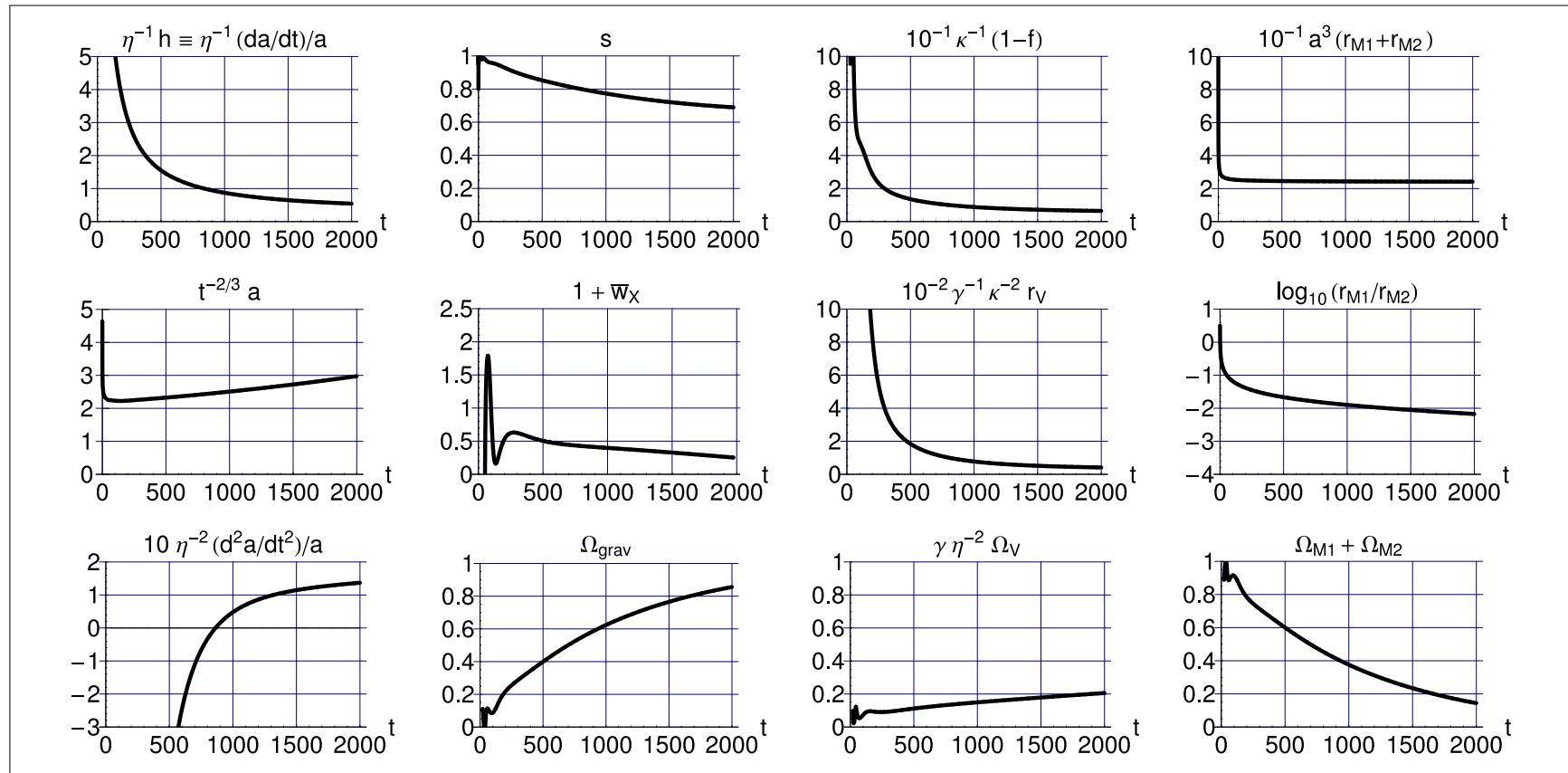
with q the QCD gluon condensate (1) and coupling constants $K \equiv (16\pi G)^{-1} > 0$ and $\eta > 0$ [standard GR has $\eta = 0$ and $G = G_N$].

Theory (6) has basically one unknown coupling constant, η .

Resulting cosmological equations solved in the scalar-tensor formalism [Brans–Dicke (BD) scalar $s(t) < 1$].

Exploratory numerical results for $\eta \sim 10^{-3}$ in the Figure.

2. Modified-gravity cosmology



Numerical solution for model parameters $(\gamma, \eta^2) = (10^2, 0.9 \times 10^{-6})$, with $\kappa \equiv (3/32) \eta^2 / \gamma = 8.4375 \times 10^{-10}$. Dimensionless energy densities: r_{M1} [relativistic matter], r_{M2} [nonrelativistic matter], and $r_V = \gamma(1 - f)^2$ [vacuum].

2. Modified-gravity cosmology

Taking $\eta^2 = 0.9 \times 10^{-6}$ and defining the “present universe” to be at $\Omega_{\text{M,tot}}(t_{\text{p}}) = 0.25$, there are these dimensionless results:

$$t_{\text{p}} h(t_{\text{p}}) \equiv t \dot{a}(t)/a(t) \Big|_{t=t_{\text{p}}} \approx 0.91 , \quad (7a)$$

$$\overline{w}_{\text{X}}(t_{\text{p}}) \equiv -\frac{2}{3} \left(\frac{\ddot{a} a}{(\dot{a})^2} + \frac{1}{2} \right) \frac{1}{1 - \Omega_{\text{M}}} \Big|_{t=t_{\text{p}}} \approx -0.66 , \quad (7b)$$

$$z_{\text{inflect}} \equiv a(t_{\text{p}})/a(t_{\text{inflect}}) - 1 \approx 0.5 , \quad (7c)$$

and, with $q_0 = (210 \text{ MeV})^4$ and $G \sim s G_N$, these dimensionful results:

$$\tau_{\text{p}} = t_{\text{p}} K q_0^{-3/4} \sim 13.1 \text{ Gyr} , \quad (8a)$$

$$H_{\text{p}} = h(t_{\text{p}}) K^{-1} q_0^{3/4} \sim 68 \text{ km s}^{-1} \text{ Mpc}^{-1} , \quad (8b)$$

$$\rho_{V,\text{p}}^{(\text{BD})} = \frac{1}{4} \eta^2 / (1 - s(t_{\text{p}})) K^{-1} q_0^{3/2} \sim (2 \times 10^{-3} \text{ eV})^4 . \quad (8c)$$

3. Conclusions

- self-adjustment of a conserved microscopic variable q can give $\rho_{\text{vac}} = 0$ for an equilibrium state and $\rho_{\text{vac}} \neq 0$ for a perturbation of the equilibrium state (e.g., by the Hubble expansion);
- gravitational effects of the QCD gluon condensate can be described with q -theory;
- a possible QCD-induced modified-gravity model can give a satisfactory description of the present Universe, both qualitatively and quantitatively.