

# Tools to predict non-adiabatic instabilities in coupled DE-DM models

Laura Lopez Honorez

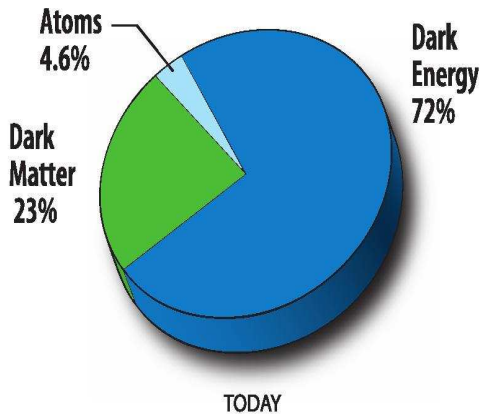
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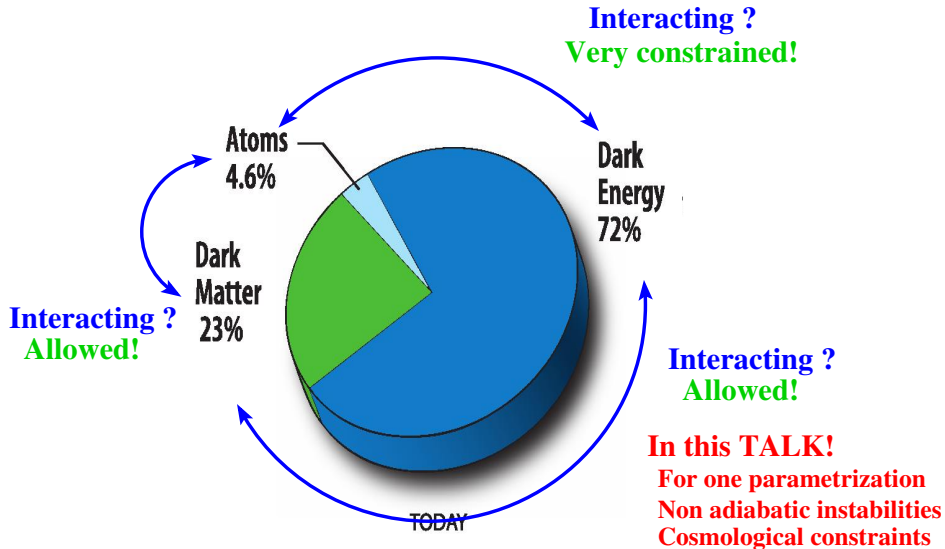
based on *Dark Coupling*

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in collaboration with B. Gavela, D. Hernandez, O. Mena, S. Rigolin

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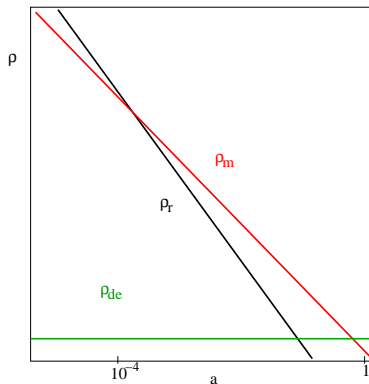
# Background equations - The coupling

- Evolution equation for cosmological fluids :

$$\dot{\rho}_i + 3H(\rho_i + p_i) = 0$$

$$p_i = w_i \rho_i$$

$\Lambda$ CDM model  $w_{de} = -1$



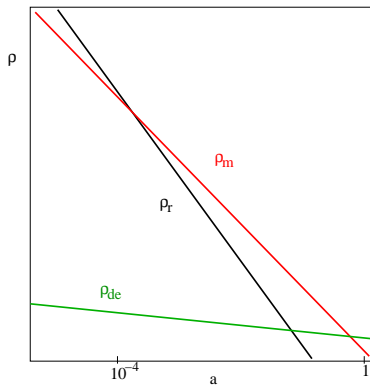
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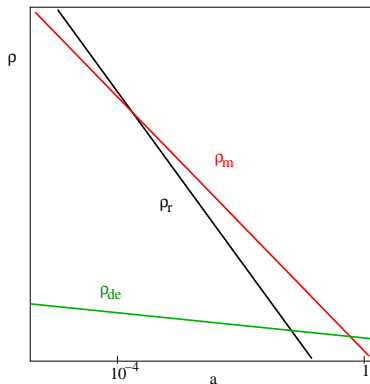
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- Evolution equations for a **Interacting DM-DE System** (see also Ureña talk) :

$$\dot{\rho}_{dm} + 3H\rho_{dm} = 0$$

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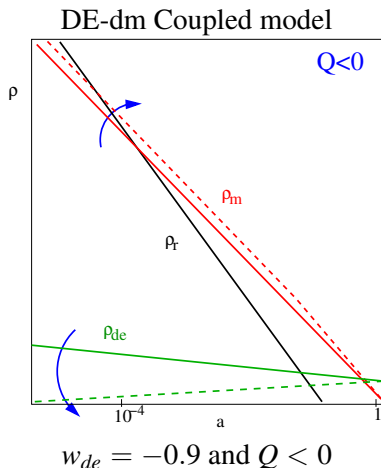
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- Evolution equations for a **Interacting DM-DE System** (see also Ureña talk) :

$$\dot{\rho}_{dm} + 3H\rho_{dm} = Q$$

$$\dot{\rho}_{de} + 3H\rho_{de}(1 + w) = -Q$$

- we took  $w$  constant and  $w < -1/3$
- $Q$  encodes the **interaction**
- $Q$  changes the time of M-Rad equ



# Density perturbation equations

The growth of a perturbation  $\delta_i = \delta\rho_i/\rho_i$  depends on three contributions :

$$\delta_i'' = A_i \frac{\delta_i}{a^2} + B_i \frac{\delta_i'}{a} + \mathcal{F}(\rho_j, \delta_j, \delta_j'; j \neq i)$$



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In the case of DE-DM coupled models :

- $\delta_{de}$  and  $\delta_{dm}$  growth are NOT independent
- $A$  and  $B$  depend on  $Q$  through  $\rho_i(Q)$  and  $\delta P(Q)$  contributions

# Origin of instabilities in coupled models - $\delta P$ sector

- Adiabatic processes :

$$\delta P_{de} \rightarrow c_{ade}^2 \delta \rho_{de}$$
$$c_{ade}^2 = \frac{\dot{P}_{de}}{\dot{\rho}_{de}} \quad \text{which for } w = cst, c_{ade}^2 = w$$

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$\rightsquigarrow$  Instability as  $c_{ade}^2 < 0$ , pressure no more counteract gravity

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In the rest frame of  $I$  :  $\delta P_I = \hat{c}_{sI}^2 \delta \rho_I \neq c_{aI}^2 \delta \rho_I$ ,

In any frame for coupled DE-DM :

$$\delta P_{de} = \delta P_{de}(\hat{c}_{sde}^2, \delta \rho_{de}, c_{ade}^2, w, \mathbf{d})$$

where  $\mathbf{d} \equiv \frac{Q}{3\mathcal{H}\rho_{de}(1+w)}$  is the DOOM FACTOR

# Linear perturbations treatment- Model Parametrisation

To deduce the **evolution of perturbations**, we need a lagrangian or a parametrization at the level of the stress-energy tensor

$$\begin{aligned}\nabla_{\mu} T^{\mu}_{(dm)\nu} &= \mathcal{Q} u_{\nu}^{(dm)} / a , \\ \nabla_{\mu} T^{\mu}_{(de)\nu} &= -\mathcal{Q} u_{\nu}^{(dm)} / a ,\end{aligned}$$

Valiviita, Majorotto & Maartens '08

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- **Conservation** of the total stress-energy tensor  $\nabla_{\mu} T^{\mu}_{(TOT)\nu} = 0$ ,
- $u_{\nu}^{(dm)}$  is the 4-**velocity** of dark matter  $u_{\nu}^{(dm)} = a(-1, v_{dm}^i)$   
 $\rightsquigarrow$  no momentum exchange in the rest frame of dark matter.



# Growth equation - Doom factor - Instability

$$\delta_i'' = A_i \frac{\delta_i}{a^2} + B_i \frac{\delta_i'}{a} + \mathcal{F}(\rho_j, \delta_j, \delta_j'; j \neq i)$$

leads when A,B negligible

Exponential Growth or Oscillations

(Anti)Damping

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In the Strongly Coupled case of  $\nabla_\mu T_{(de)\nu}^\mu = -Q u_\nu^{(dm)}/a$  (i.e. when  $|\mathbf{d}| > 1$ ) at large scale-early time in an unstable scenario :

$$\delta_{de}'' \simeq 3\mathbf{d}(\hat{c}_{sde}^2 + 1) \left( \frac{\delta_{de}'}{a} + 3 \frac{\delta_{de}}{a^2} \frac{(\hat{c}_{sde}^2 - w)}{\hat{c}_{sde}^2 + 1} + \frac{3(1 + w)}{a^2} \delta[\mathbf{d}] \right) + \dots$$

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Assuming  $\hat{c}_{sde}^2 > 0$  :  $\mathbf{d} > 1 \rightsquigarrow$  Non adiabatic source of instability

see also Valiviita - Majerotto - Maartens '08 & He - Wang - Abdalla '08 & Jackson - Taylor - Berera '09

One example :  $Q = \xi \mathcal{H} \rho_{de}$

- Doom factor :  $\mathbf{d} = \frac{\xi}{3(1+w)}$   $\rightsquigarrow$  strong coupling regime  $\equiv |\xi|$  large

STABLE when  $\xi$  and  $1 + w$  have opposite signs

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- Datasets considered for constraints : Run 0
  - WMAP 5-year
  - prior on the Hubble parameter of  $72 \pm 8 \text{ km s}^{-1} \text{ Mpc}^{-1}$  from HST
  - H(z) data
  - Supernovae

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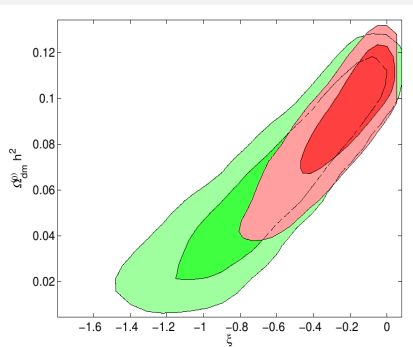
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- Datasets considered for constraints : **Run 1**
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  - prior on the Hubble parameter of  $72 \pm 8 \text{ km s}^{-1} \text{ Mpc}^{-1}$  from HST
  - H(z) data
  - Supernovae
  - Matter Power spectrum (or LSS data from SDSS LRGs)

$\xi - \Omega_{dm}^{(0)}$  degeneracy

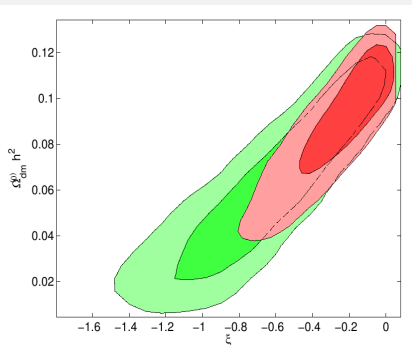
# $\xi - \Omega_{dm}^{(0)}$ degeneracy

For  $\xi < 0$ ,  $|\xi|$  large

$\rightsquigarrow$  more  $\Omega_{dm}(a)$  in the past for the same value of  $\Omega_{dm}^{(0)}$

$\rightsquigarrow$  more clustering

$\rightsquigarrow$  less  $\Omega_{dm}^{(0)}$  needed



LSS data  $\rightsquigarrow$  stringent constraint

due to enormous galaxy clustering for  $\xi < -0.5$

( up to  $\sigma_8 > 2$  compared to WMAP5 analysis  $\sigma_8 = 0.812 \pm 0.026$  )

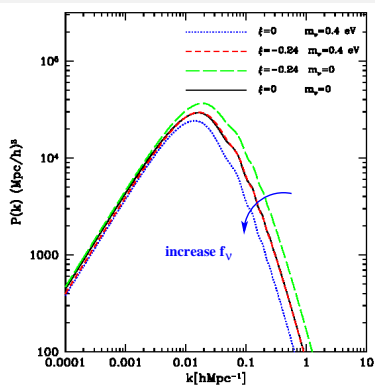


$\xi - f_\nu$  degeneracy

$$f_\nu = \frac{\Omega_\nu^{(0)} h^2}{\Omega_{dm}^{(0)} h^2} = \frac{\sum m_\nu}{93.2 \text{ eV}} \cdot \frac{1}{\Omega_{dm}^{(0)} h^2}$$

Non relativistic neutrinos suppress  
the growth of  $\delta_{dm}$  at small scales

For  $f_\nu \neq 0$  the power spectrum  
is reduced with respect to  $f_\nu = 0$ .

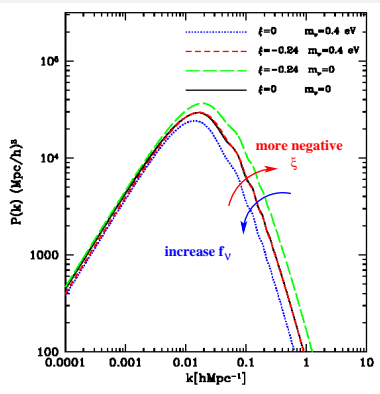


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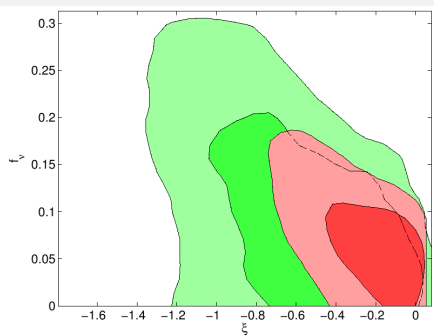
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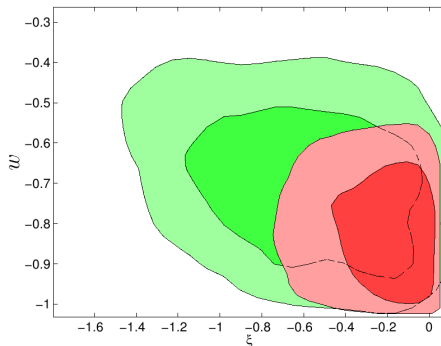


$\rightsquigarrow$  Can be compensated with more negative  $\xi$

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# Viable parameter space in $\xi - w$ plane

In the instability-free region  $\xi < 0$  and  $w > -1$  :



$\leadsto$  Present data are **unable to set strong constraints** on  $\xi - w$ ,  
and large values for both parameters, near -0.5, are easily allowed

# Conclusion for $\nabla_\mu T_\nu^\mu = \pm Q u_\nu^{(dm)}/a$ models

- We have identified the source of non-adiabatic instability as a function of  $Q$  independently of its specific form :

The **doom factor** characterizes the (un)stable regime :

$$\mathbf{d} = \frac{Q}{3\mathcal{H}\rho_{de}(1+w)}$$

- $\mathbf{d} > 1 \Rightarrow$  unstable growth
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- in the other cases, Coupled Models are still viable !
- Confrontation to data for  $Q = \xi H \rho_{de}$ 
  - both  $w$  and  $\xi$  are **not** very **constrained** from data.  
large values for both parameters, near -0.5, are easily allowed !!
  - $\xi$  positively **correlated** with both  $\Omega_{dm} h^2$
  - $m_\nu$  -  $\xi$  **degeneracy** : *i.e.*  $f_\nu$  increases for more negative  $\xi$

This the End  
Thank you for your attention !!

## Backup



# Gauge transformations

- ① There is always some freedom in the way we do the correspondence between the background and the physical perturbed universe  $\equiv$  **Gauge Freedom**
- ② Some quantities are **gauge invariant** like ( $v^j = ik^j v$  and  $c_s^2 = \delta P / \delta \rho$ ) :

$$\begin{aligned} w\Gamma &= (c_s^2 - c_a^2)\delta \\ \Delta &= \delta + \dot{\rho}/\rho(v - B) \end{aligned}$$

For example in synchronous or Newtonian gauge ( $B = 0$ ) :

$$\begin{aligned} w_{de}\Gamma_{de}|_{rf\ de} &= (\hat{c}_s^2 - c_a^2)\hat{\delta}_{de} = (c_s^2 - c_a^2)\delta_{de} = w_{de}\Gamma_{de}|_{any\ frame} \\ \Delta_{de}|_{rf\ de} &= \hat{\delta}_{de} = \delta_{de} + \frac{\dot{\rho}_{de}}{\rho_{de}}v_{de} = \Delta_{de}|_{any\ frame} \end{aligned}$$

$$\rightsquigarrow \delta P_{de} = \hat{c}_{s\ de}^2 \delta \rho_{de} - (\hat{c}_{s\ de}^2 - c_{a\ de}^2)3(1 + w_{de})(1 + \mathbf{d})v_{de}\mathcal{H}\rho_{de}$$

# What would be $\tilde{w}(z)$ reconstructed

...from  $H(z)$  data assuming no coupling and dynamical DE :

$$R_H(z) = \frac{H^2(z)}{H_0^2} = \Omega_{dm}^{(0)}(1+z)^3 + \Omega_{de}^{(0)} \exp \left[ 3 \int_0^z dz' \frac{1 + \tilde{w}(z')}{1 + z'} \right]$$

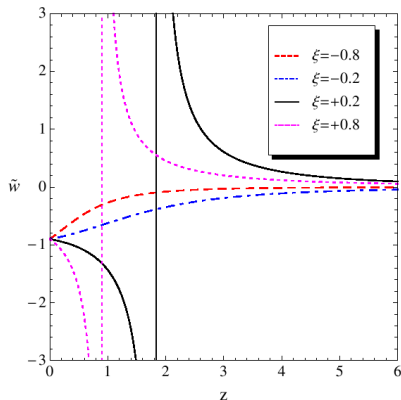
$$\Rightarrow \tilde{w}(z) = \frac{1}{3} \frac{R'_H(1+z) - 3R_H}{R_H - \Omega_{dm}^{(0)}(1+z)^3}.$$

However in presence of dark couplings :

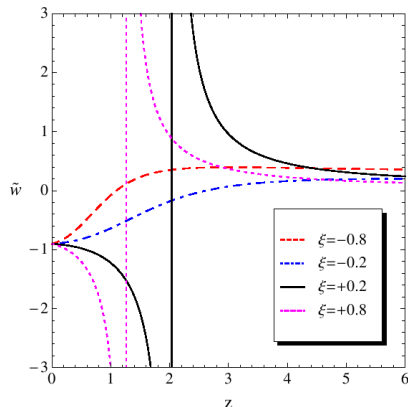
$$R_H(z) = f(w, Q, \Omega_{dm}^{(0)}, \Omega_{de}^{(0)})$$

# Reconstructing $\tilde{w}(z)$ as a function of $w$ and $\xi$

For  $Q = \xi H \rho_{de}$



For  $Q = \xi H \rho_{dm}$



$\leadsto$  divergent  $\tilde{w}(z)$  for  $\xi > 0$

Similar behaviour in  $f(R)$  cosmologies see e.g. Amendola & Tsujikawa '07

# Cosmo constraints

Parameter	Prior
$\omega_b$	0.005-0.1
$\omega_{dm}$	0.01-0.99
$\theta_{CMB}$	0.5-10
$\tau$	0.01-0.8
$\Omega_k$	-0.1-0.1
$f_\nu$	0-0.3
$w$	-1-0
$\xi$	-2-0
$n_s$	0.5-1.5
$\ln(10^{10} A_s)$	2.7-4.0

**TAB.:** *Priors for the cosmological fit parameters considered in this work. All priors are uniform in the given intervals.*

- $\omega_b = \Omega_b h^2$  and  $\omega_{dm} = \Omega_{dm} h^2$
- $\theta_{CMB}$  is proportional to the ratio of the sound horizon to the angular diameter distance,
- $\tau$  is the reionisation optical depth,
- $\Omega_k$  is the spatial curvature,
- $f_\nu = \Omega_\nu / \Omega_{dm}$  refers to the neutrino fraction,
- $n_s$  is the scalar spectral index
- $A_s$  the scalar amplitude.

# Cosmo ref

J. Dunkley et al. Five-Year Wilkinson Microwave Anisotropy Probe (WMAP) Observations: Likelihoods and Parameters from the WMAP data. 2008, 0803.0586.

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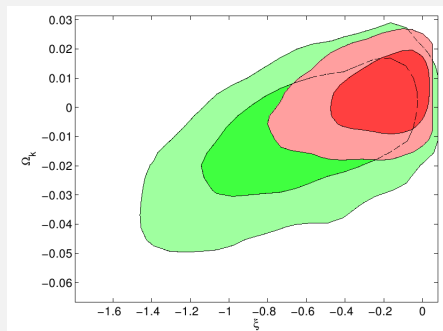
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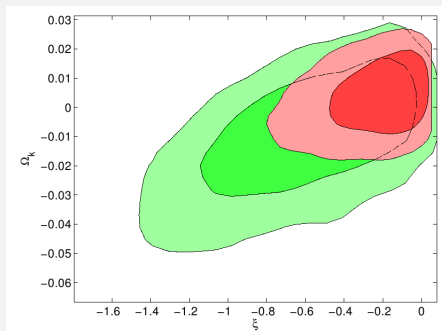
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# $\xi - \Omega_k^{(0)}$ degeneracy



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For  $\xi < 0$ ,  $|\xi|$  large

$\rightsquigarrow$  more  $\Omega_{dm}$  in the past

small negative  $\Omega_K$  can  
compensate this effect

to describe well CMB data.

The degeneracy between  $\xi$  and  $\Omega_k$  gets alleviated  
if one adds LSS data to the analysis.

# The Very Coupled Case : $Q = \xi \mathcal{H} \rho_{de}$

$$\mathbf{d} = \frac{\xi}{3(1 + w)} .$$

$\rightsquigarrow$  strong coupling regime  $\equiv |\xi|$  large

Model: $Q \propto \rho_{de}$	$1 + w$	$\xi$	$\rho_{dm}$	$\rho_{de}$	$\mathbf{d}$	Early time instability?
	+	+	$\mp$	+	+	Yes
	+	-	+	+	-	No
	-	-	+	+	+	Yes
	-	+	$\mp$	+	-	No

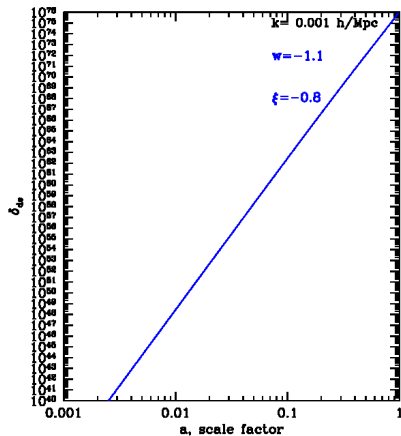


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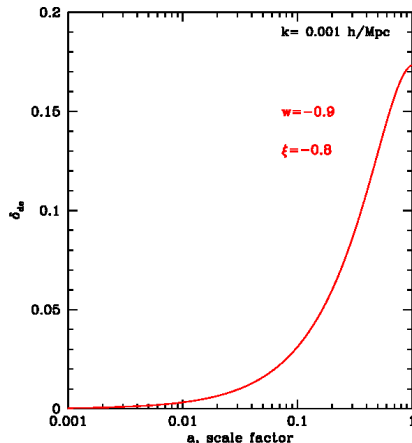
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	-	-	+	$\mp$	-	No

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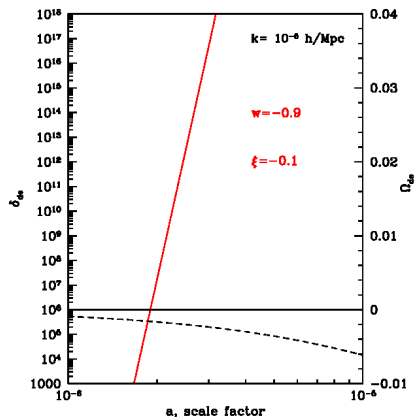
$$\mathbf{d} = \frac{\xi}{3(1+w)} \frac{\rho_{dm}}{\rho_{de}}.$$

Model: $Q \propto \rho_{dm}$	$1+w$	$\xi$	$\rho_{dm}$	$\rho_{de}$	$d$	Early time instability?
	+	-	+	$\mp$	+	Yes
	+	+	+	+	+	Yes
	-	+	+	+	-	No
	-	-	+	$\mp$	-	No

$\frac{\rho_{dm}}{\rho_{de}}$  enhance  $\mathbf{d}$  for small  $\xi$

$\rightsquigarrow |\xi| < 1$  always strong coupling regime

$\rightsquigarrow$  Ok with Valiviita *et al* analysis



# The Very Coupled Case : $Q = \xi \mathcal{H} \rho_{dm}$

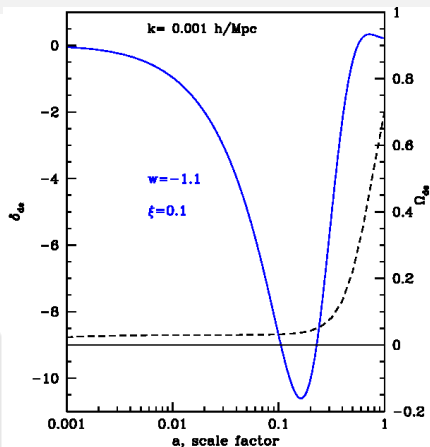
$$\mathbf{d} = \frac{\xi}{3(1+w)} \frac{\rho_{dm}}{\rho_{de}}.$$

Model: $Q \propto \rho_{dm}$	$1+w$	$\xi$	$\rho_{dm}$	$\rho_{de}$	$d$	Early time instability?
	+	-	+	$\mp$	+	Yes
	+	+	+	+	+	Yes
	-	+	+	+	-	No
	-	-	+	$\mp$	-	No

$\frac{\rho_{dm}}{\rho_{de}}$  enhance  $\mathbf{d}$  for small  $\xi$

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New :  $Q = \xi \mathcal{H} \rho_{dm}$  model is free from early time non-adiabatic instabilities when  $1 + w < 0$

# Some coupling from conformal transformation

From a Brans-Dicke action (with  $\omega = 0$ ) in the Jordan (string) frame :

$$S_J = \frac{M_{Pl}^2}{2} \int d^4x \sqrt{-g_J} \Phi R_J + S_M(\psi, g_{\mu\nu}^J)$$

we get in the Einstein frame ( $\Phi = \Omega^{-1}$ ) :

$$S_E = \int d^4x \sqrt{-g_E} \left\{ \frac{M_{Pl}^2}{2} R_E - \frac{1}{2} \partial_\mu \varphi \partial^\mu \varphi \right\} + S_M(\psi, \Omega^2 g_{\mu\nu}^E)$$

Using conformal transformation with

$$\begin{aligned} g_{\mu\nu}^E &= \Omega^{-2} g_{\mu\nu}^J \\ \varphi / M_{Pl} &= -\sqrt{6} \ln \Omega. \end{aligned}$$

In that framework, assuming that in the Jordan Frame :  $\nabla_\mu T_M^{\mu\nu} = 0$   
we get in the Einstein frame coupled DE-DM system :

$$\nabla_\mu T_M^{\mu\nu} = T_M^\mu{}_\mu g_E^{\mu\nu} \partial_\nu \ln \Omega = -\nabla_\mu T_\varphi^{\mu\nu}$$

# Mass varying DM and Couplings

Non minimal couplings can appear for :

$$S_E = \int d^4x \sqrt{-g_E} \left\{ \frac{M_{Pl}^2}{2} R_E - \frac{1}{2} \partial_\mu \varphi \partial^\mu \varphi - V(\varphi) \right\} + S_M(\psi, \Omega(\varphi)^2 g_{\mu\nu}^E)$$

with :

$$\nabla_\mu T_M^{\mu\nu} = T_{M\mu}^\mu g_E^{\mu\nu} \partial_\nu \ln \Omega(\varphi) = - \nabla_\mu T_\varphi^{\mu\nu}$$

this implies for pressureless DM :  $d \ln \rho_{dm} = d \ln \Omega(\varphi) - 3d \ln a$  ; such that :

$$m_{dm} \propto \Omega(\varphi)$$

In *e.g.* [Amendola, Camargo & Rosenfeld '07](#) they parametrize the time variation of the  $m_{dm}$  as a function of the scale factor :

$$\begin{aligned} m(a) &= m_0 e^{\int_1^a \zeta(a') d \ln(a')} \\ \rightsquigarrow \dot{\rho}_{dm} + 3H\rho_{dm} &= \zeta \mathcal{H} \rho_{dm} \\ \dot{\rho}_{de} + 3H\rho_{de}(1+w) &= -\zeta \mathcal{H} \rho_{dm} \end{aligned}$$