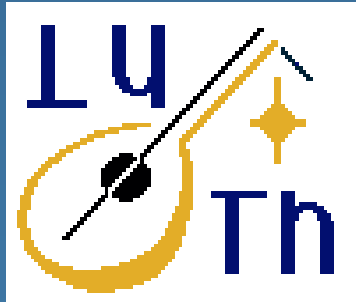


Slow-roll suppression of adiabatic instabilities in coupled scalar field-dark matter models

Phys. Rev. D (2008) 78, 083538



Pier Stefano Corasaniti

CNRS

LUTH, Observatoire de Paris

Non-Minimally Coupled Scalar Fields

where are they from?

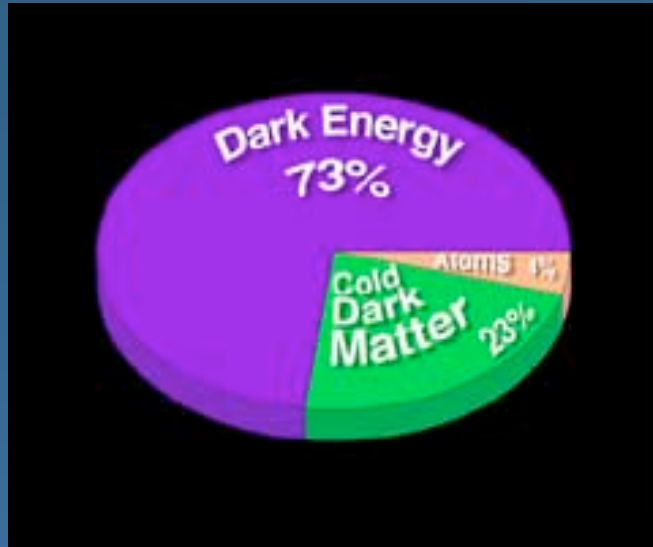
Particle Physics

- Coupled scalar and hierarchy problem (Wetterich '87; Ellis et al. '89)

String Theory

- Coupled scalars in the low energy from compactification of extra-dimensions
- Dilaton couples to all matter fields \Rightarrow EP violation
- Several mechanisms can reconcile EP violation with local tests of GR: **e.g. non-universality of the couplings** (Damour et al. '90, Casas et al. '92); **string-loops corrections to dilaton coupling and GR attractor** (Damour et al. '93, '94)

The Dark Energy Problem



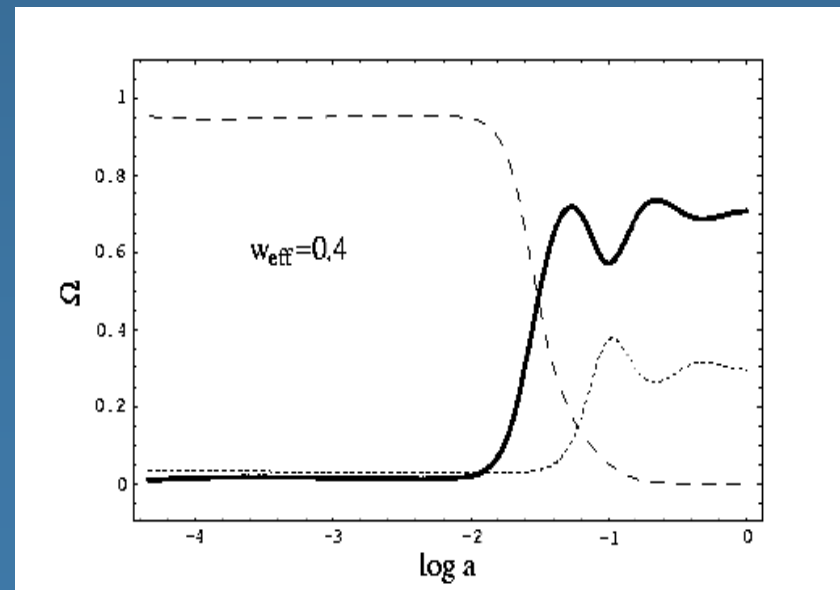
Dark Energy:

$$\Omega_{DE} = 0.73 \pm 0.03$$

- DE = Cosmological Constant Λ
 - Λ = vacuum energy \Rightarrow Fine Tuning
 - Λ and Structure Formation \Rightarrow Cosmic Coincidence (?)

- DE = Non-Minimally Coupled Scalar Field with Exponential Potential
 - Late time accelerating attractor solution \Rightarrow IC dependence erased
 - Along the attractor Matter to Dark Energy density ratio is constant

(Amendola '99)



Dark Sector Interaction Model Building

ϕ -DM:

- Non-minimally coupled scalar + exponential potential + DM-coupling
 - **Runaway Dilaton** (Gasperini et al. '02)
 - **Moduli Fields and Brane World Scenario** (Brax et al. '03)

Mass Varying Neutrinos:

- Non-minimally coupled scalar to Neutrinos (Fardon et al. 2004)

Chameleon Model:

- Non-minimally coupled scalar + inverse power law + universal-coupling
 - **local density dependent $m_\phi \Rightarrow$ EP test satisfied** (Khoury & Weltman '04)
 - **gravity constraints $\Rightarrow M < 10^{-3}$ eV \Rightarrow dark energy = chameleon + adiabatic evolution** (Brax et al. '04)

What the action looks like?

$$S = \int d^4x \sqrt{-g} \left\{ \frac{M_{\text{Pl}}^2}{2} \mathcal{R} - \frac{1}{2} (\partial\phi)^2 - V(\phi) \right\} - \int d^4x \mathcal{L}_{\text{m}}(\psi_{\text{m}}^{(i)}, g_{\mu\nu}^{(i)})$$

and if you do not like working with the action,
assume two interacting fluids in FRW with given eos, such that
 $T_{\text{Tot}}^{\mu\nu}$ is conserved, eq. of motion follows:

$$\begin{aligned}\rho'_X &= -3H\rho_X(1 + w_X) - aQ \\ \rho'_Y &= -3H\rho_Y(1 + w_Y) + aQ\end{aligned}$$

...and Q is some function parameterizing the type of interaction

Are These Still Viable Models?

Linear Perturbations:

instabilities found

- in some realizations of coupled scalar field with exponential potential (Koivisto '05; Kaplinghat & Rajamaran '06) and in specific MaVaN models (Afshordi et al. '05)
- in scenarios with adiabatic background evolution (Bean et al. '08)

$$\frac{1}{c_a^2} = -1 - \frac{\alpha + 1}{\beta} \frac{M_{Pl}}{\phi} \quad \text{negative!!}$$

- in more generic fluid-description instabilities found for couple dark energy models with constant $w \sim -1$ (Valiviita et al. '08)

$$\psi = A_\psi (k\tau)^{n_\psi}$$

$$n_\psi = n_\pm = \frac{-(1 + 2w_x) \pm \sqrt{3w_x^2 - 2}}{1 + w_x}$$

see also Wang et al. '08

What is the relation between the background scalar field evolution and the onset of the instabilities? Are instabilities really generic and dangerous?

A non-minimally coupled Model

- Ingredients: $f(\phi/M_{Pl})\bar{\psi}\psi$ with $f(\phi) = e^{\beta\phi/M_{Pl}}$ and $V(\phi)$:

$$\Gamma \equiv \frac{V_{,\phi\phi}V}{V_{,\phi}^2} > 1$$

- $T^{\text{tot}}_{\mu\nu}$ conservations

$$\begin{aligned} T^{\mu(DM)}_{\nu;\mu} &= \beta\phi_{;\nu}T^{\gamma(DM)}_{\gamma}, \\ T^{\mu(\phi)}_{\nu;\mu} &= -\beta\phi_{;\nu}T^{\gamma(DM)}_{\gamma} \end{aligned}$$

\Rightarrow

- Equation of motions

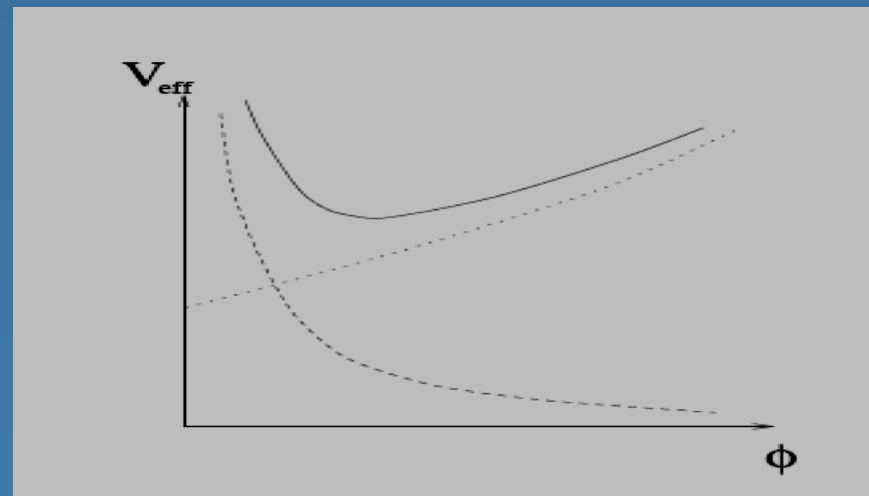
$$\begin{aligned} \dot{\rho}_{DM} + 3H\rho_{DM} &= \beta\dot{\phi}\rho_{DM} \\ \ddot{\phi} + 3H\dot{\phi} + V_{,\phi} &= -\beta\rho_{DM} \end{aligned}$$

- DM density evolution

$$\rho_{DM} = \frac{\rho_{DM}^{(0)}}{a^3} e^{\beta(\phi-\phi_0)}$$

- Scalar Effective Potential

$$V_{\text{eff}}(\phi) = V(\phi) + \frac{\rho_{DM}^{(0)}}{a^3} e^{\beta(\phi-\phi_0)}$$



Adiabatic Regime

- Minimum of the potential:

$$V_{,\phi} = -\beta \frac{\rho_{DM}^{(0)}}{a^3} e^{\beta(\phi - \phi_0)} \quad \text{which gives} \quad \left(\frac{\phi_0}{\phi_{\min}} \right)^{\alpha+1} = \frac{1}{a^3} e^{\beta(\phi_{\min} - \phi_0)}$$

- 'Adiabatic' Attractor Solution: $m_\phi > H$

$$m^2 = \frac{\rho_{DM}^{(0)}}{a^3} \left(\frac{\beta}{M_{Pl}} \right)^2 \frac{f}{f_0} \left[1 + \frac{\Gamma}{V} \frac{\rho_{DM}^{(0)}}{a^3} \frac{1}{f_0} \right] \Rightarrow \text{e.g. today} \quad \frac{m_0^2}{H_0^2} > 3\Omega_{DM}^{(0)} \left(\frac{\beta}{M_{Pl}} \right)^2 \left[1 + \frac{\Gamma}{f_0} \frac{\Omega_\phi^{(0)}}{\Omega_{DM}^{(0)}} \right]$$

It can be shown to be the case at all times

- Slow-Roll Evolution: $(d\phi_{\min}/dt)^2/2 \ll V_{\text{eff}}(\phi_{\min})$

$$\dot{V}_{\text{eff}}^\phi = 0 \Rightarrow \dot{\phi} = \frac{3H}{m^2} \frac{\rho_{DM}^{(0)}}{a^3} \frac{f_{,\phi}}{f_0} = -\frac{3H}{m^2} V_{,\phi} \Rightarrow \frac{\dot{\phi}^2}{2V} = \frac{9H^2}{2m^4} \frac{V_{,\phi}^2}{V} < \frac{9H^2}{2m^2} \frac{1}{\Gamma}$$

(Brax et al. '04; Das et al. '06)

A Specific Example

- Inverse Power-Law Potential:

$$V(\phi) = \frac{M^{4+\alpha}}{\phi^\alpha},$$

- Attractor and Slow-Roll Condition:

- $\alpha \leq O(1)$

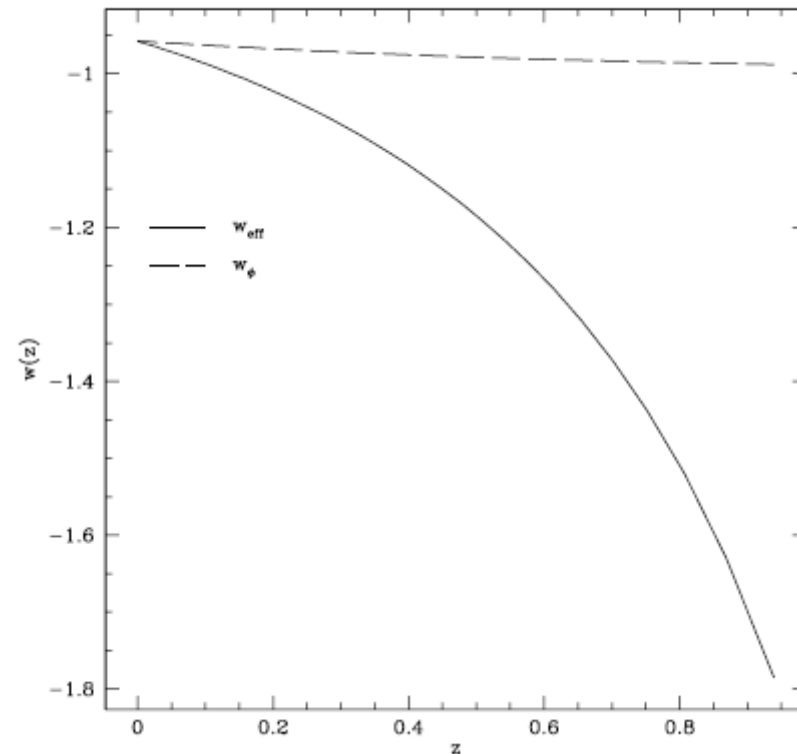
- $\beta \geq O(1)$

- Mimicking Phantom:

$$w_{\text{eff}} = \frac{w_\phi}{1-x}$$

$$x \equiv -\frac{\rho_{\text{DM}}^{(0)}}{a^3 \rho_\phi} \left[\frac{f(\phi/M_{\text{Pl}})}{f(\phi_0/M_{\text{Pl}})} - 1 \right] \geq 0$$

- $\alpha = 0.2, \beta = 1 \Rightarrow \bar{w}_{\text{eff}} \approx -1.1$



Linear Perturbations

- Einstein Equation about linearly perturbed FRW:

$$\dot{\delta}_{DM} = - \left(\frac{\theta_{DM}}{a} + \frac{\dot{h}}{2} \right) + \beta \delta \dot{\phi},$$

$$\dot{\theta}_{DM} = -H\theta_{DM} + \beta \left(\frac{k^2}{a} \delta\phi - \dot{\phi}\theta_{DM} \right),$$

$$\begin{aligned} \delta\ddot{\phi} + 3H\delta\dot{\phi} + \left(\frac{k^2}{a^2} + V_{,\phi\phi} \right) \delta\phi + \frac{1}{2}\dot{h}\dot{\phi} &= \\ &= -\beta\rho_{DM}\delta_{DM}, \end{aligned}$$

$$\dot{h} = \frac{2k^2\eta}{a^2H} - \frac{8\pi G}{H} [\delta\rho_\phi + \rho_{DM}\delta_{DM}]$$

$$\dot{\eta} = \frac{4\pi G}{k^2} a \left[\rho_{DM}\theta_{DM} + ak^2\dot{\phi}\delta\phi \right]$$

Unified Fluid Description

- Total Energy momentum is conserved

$$\dot{\rho}_T = -3H(1 + w_T)\rho_T,$$

- The unified fluid is non-barotropic:

$$\begin{aligned} \dot{\delta}_T = & -3H(c_{sT}^2 - w_T)\delta_T + \\ & - (1 + w_T) \left\{ \left[\frac{k^2}{a^2 H^2} + 9(c_{sT}^2 - c_{aT}^2) \right] \frac{aH^2}{k^2} \theta_T + \frac{\dot{h}}{2} \right\}, \end{aligned} \quad (16)$$

$$\dot{\theta}_T = -H(1 - 3c_{sT}^2)\theta_T + \frac{c_{sT}^2 k^2}{a(1 + w_T)}\delta_T, \quad (17)$$

- Adiabatic Sound Speed

$$c_{aT}^2 = \dot{p}_T / \dot{\rho}_T$$

- Non-Adiabatic Sound Speed

$$c_{sT}^2 = \delta p_T / \delta \rho_T$$

Slow-Roll Suppression of Adiabatic Instabilities

- Sound speeds in terms of individual components:

$$c_{aT}^2 = \frac{3H\dot{\phi}^2 + \dot{\phi}[2V_{,\phi} + \beta\rho_{DM}]}{3H\dot{\phi}^2 + 3H\rho_{DM}}, \quad (18)$$

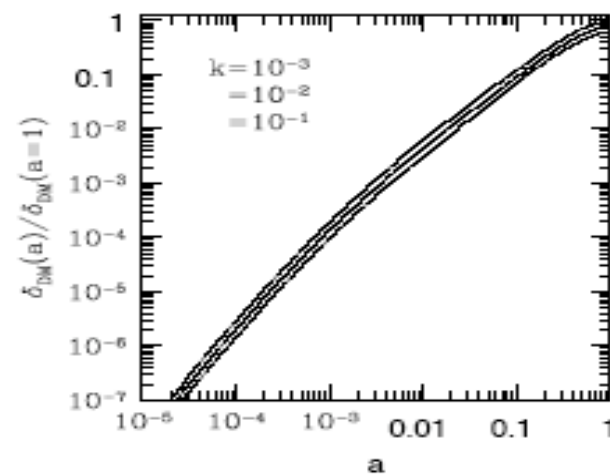
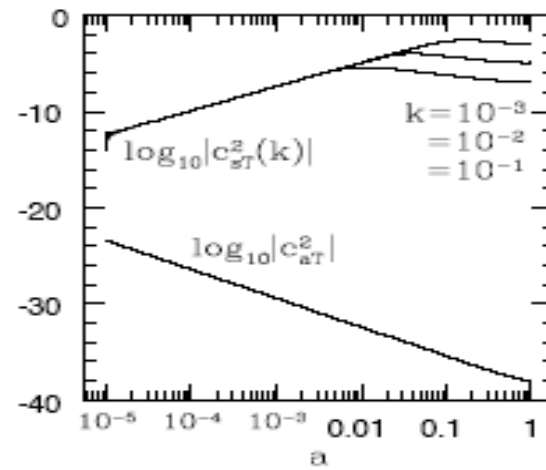
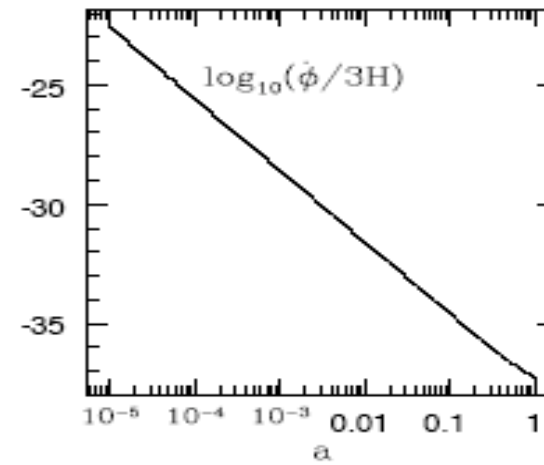
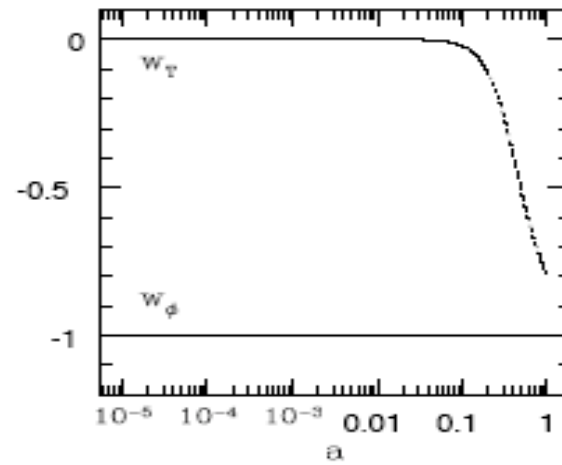
$$c_{sT}^2 = \frac{\dot{\phi}\delta\dot{\phi} - V_{,\phi}\delta\phi}{\dot{\phi}\delta\dot{\phi} + V_{,\phi}\delta\phi + \rho_{DM}\delta_{DM}}. \quad (19)$$

- Along the adiabatic solution:

$$c_{aT}^2 = -\beta\frac{\dot{\phi}}{3H}, < 0 \quad \text{but} \quad 3H\dot{\phi} \approx 0 \quad \Rightarrow \quad c_{aT}^2 \approx 0^-$$

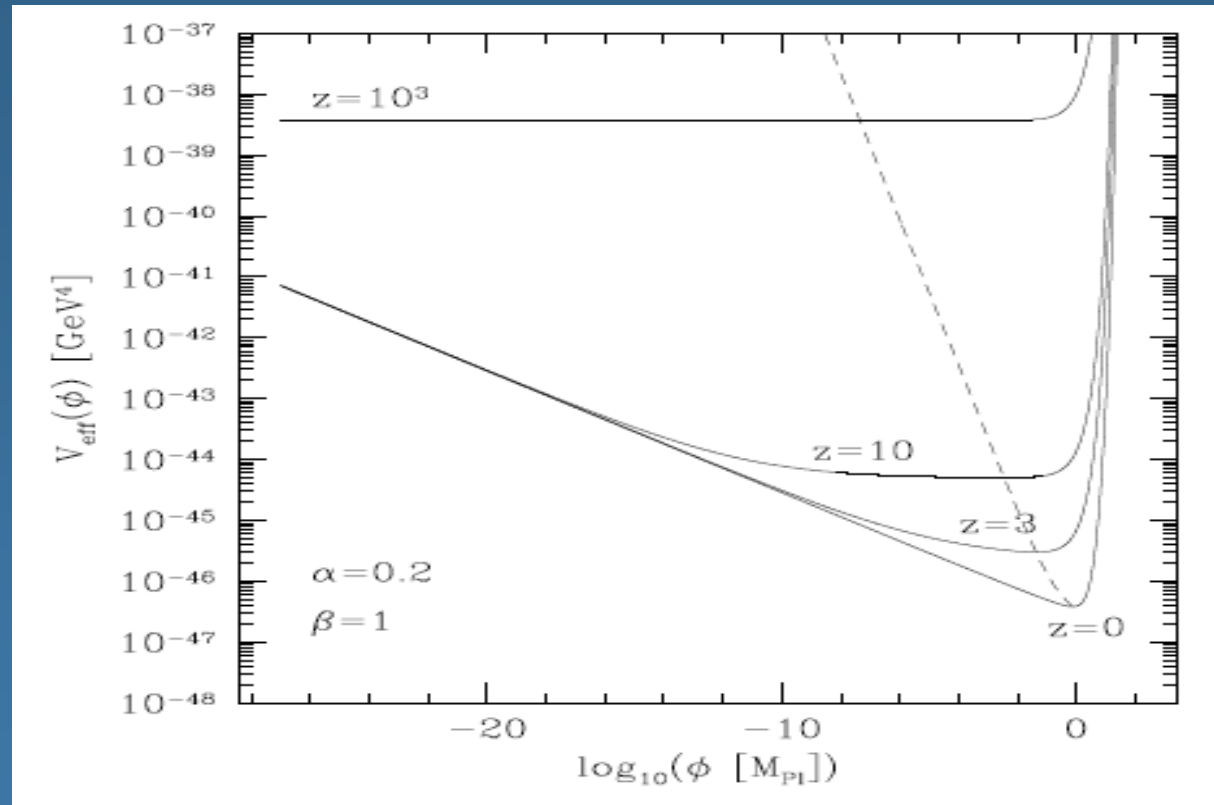
$$c_{sT}^2 = -\frac{1}{1 - \frac{1}{\beta}\frac{\delta_{DM}}{\delta\phi}} \quad \text{if } \delta\phi < \delta_{DM} \quad \Rightarrow \quad c_{sT}^2 \approx \beta\frac{\delta\phi}{\delta_{DM}} \cong 0$$

Numerical Results



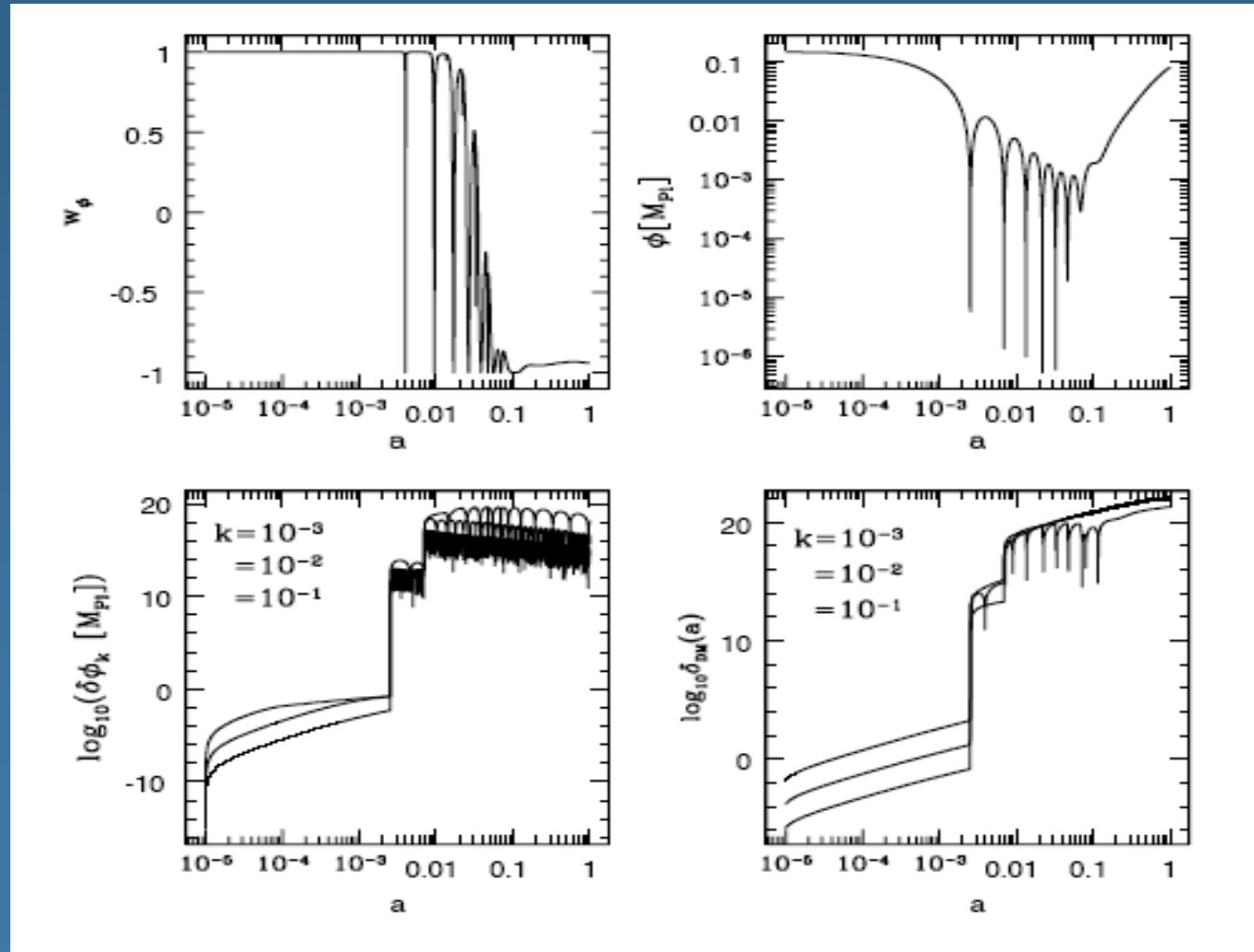
Non-Adiabatic Regime

- Initial Large Field Values: $\phi_i > \phi_i^{\text{MIN}}$



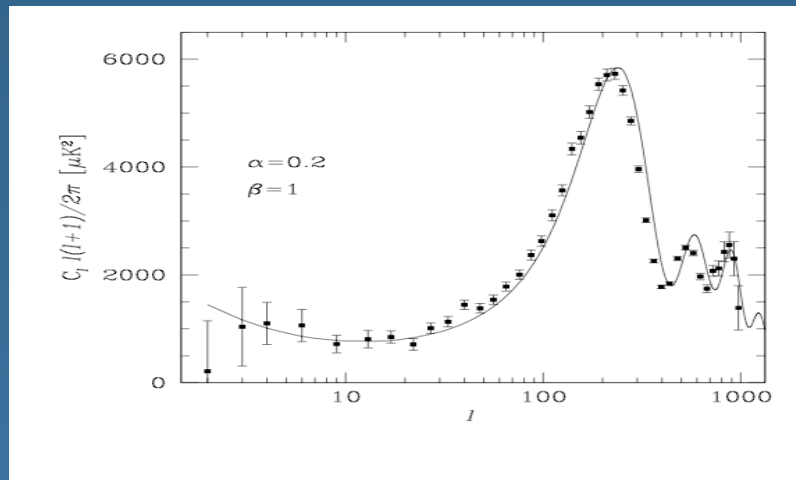
- Oscillating Scalar Field:
 - $\delta\phi$ instability like in inflationary reheating (TsujiKawa '99)

Large Field Oscillations and Resonance



Phenomenology

- Full CMB calculation and data model comparison

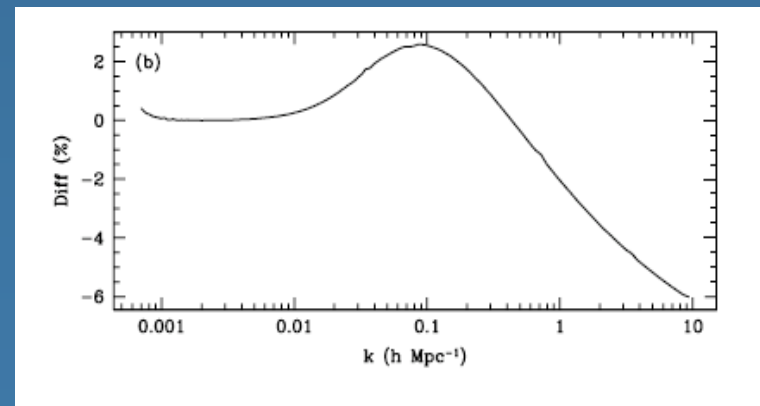


- Non-linear regime
 - Voids phenomenon (Farrar & Peebles '04)
 - Missing satellite problem

- Structure Formation at $k > 0.1$

$$\delta'' + aH\delta' = \frac{3}{2}a^2H^2 \left[1 + \frac{2\beta^2}{1 + a^2V_{,\phi\phi}/k^2} \right] \delta$$

$$\lambda \equiv V_{,\phi\phi}^{-1/2} = \sqrt{\frac{\phi^{\alpha+2}}{\alpha(\alpha+1)M^4}} \Rightarrow \lambda_0 \approx 0.7H_0^{-1}$$



Cosmological Constraints

MCMC Likelihood Analysis:

$\beta = 1$ (gravitational coupling)

- $(\Omega_{\text{DM}}, \Omega_b h^2, H_0, n_s, \tau, \log[\alpha], A_s, b)$
- WMAP5+UNION SNIa+SDSS

$$\Omega_{\text{DM}} = 0.22 \pm 0.07$$

$$\Omega_b h^2 = 0.022 \pm 0.002$$

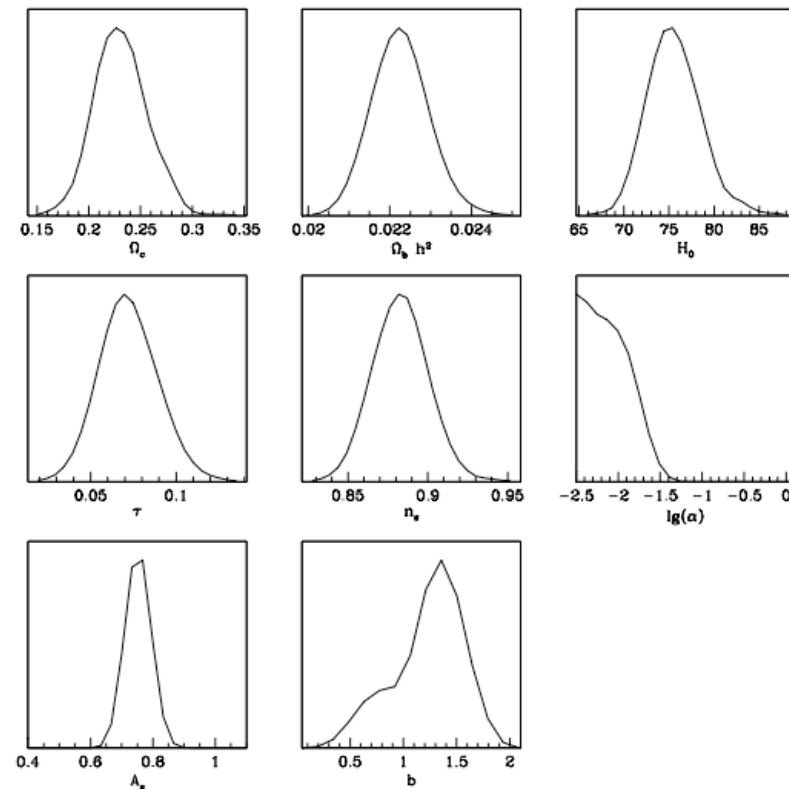
$$H_0 = 75 \pm 8 \text{ Km/s/ Mpc}^{-1}$$

$$n_s = 0.89 \pm 0.03$$

$$\log[\alpha] < -1.5 \quad (\alpha < 0.032)$$

$$A_s(k_p=0.02) = 0.74 \pm 0.07$$

$$b = 1.45^{+2.35}_{-0.98}$$



Conclusions

- Non-Minimally coupled Scalar Field-Matter Scenario can still provide viable cosmological models
- In models with coupling $O(1)$ the slow-roll dynamics of the field along the adiabatic solution suppress the rise of instabilities of the perturbations
- Instabilities can arise only in extremely strongly coupled cases or non-adiabatic background field solutions
- Compatible fits to homogeneous and linear cosmological data
- Yukawa-correction to the DM newtonian potential on cosmological scales, imprint on the DM clustering and halo formation. N-Body simulations needed
- LOTS OF WORK STILL TO BE DONE!