FLAVOUR VACUA, SPACE-TIME FOAM & COSMOLOGY

KING'S College LONDON

University of London

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Sarben Sarkar, N. E. Mavromatos and W. Tarantino

King's College London

Physics Department

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OUTLINE

(I) FLAVOUR VACUA: Basic definitions & concepts

(Blasone, Vitiello, Henning)

(II) COSMOLOGICAL IMPLICATIONS OF FLAVOUR VACUA: Non-perturbative contributions to the Cosmological Constant

(Blasone, Vitiello, Capolupo, Capozzielo, Carloni, Barenboim, Mavromatos)

(III) SPACE-TIME FOAM IN STRING/BRANE THEORY (D0-PARTICLE FOAM), FLAVOUR VACUA & NEUTRINO INDUCED COSMOLOGICAL (CONSTANT?) VACUUM ENERGY;

(Mavromatos , SS, Tarantino)

MOTIVATION : Important unresolved issues

- **The flavour Problem**

Flavour mixing,
Neutrino Mass Differences ?
Microscopic origin?

- **How can we quantise a field theory with mixing ?**

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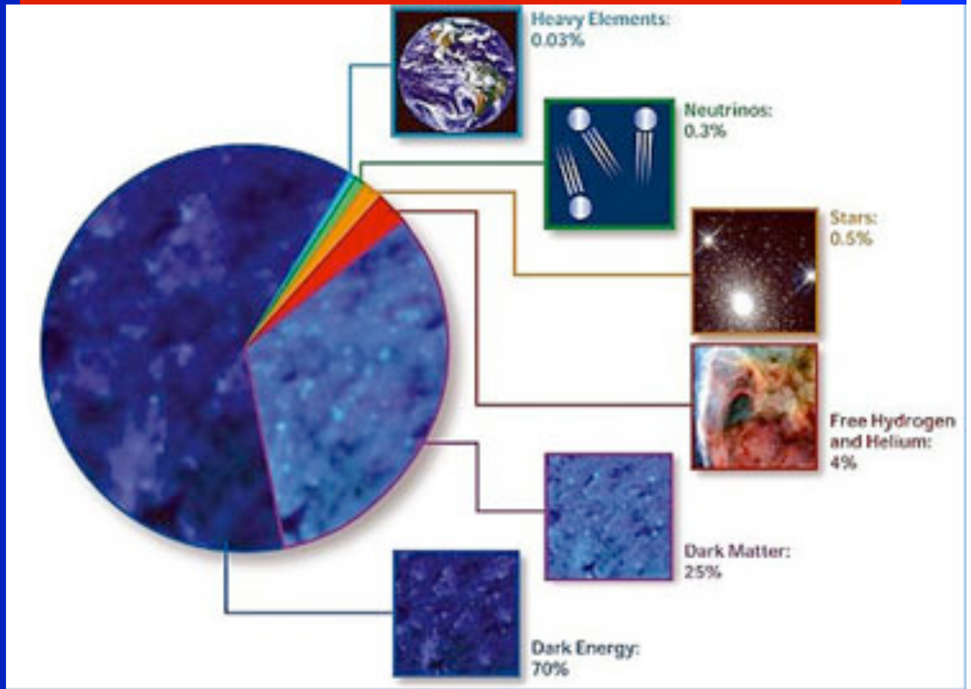

SnIa, CMB
Galaxies, ⟹
Baryon Osc.

Dark Matter(23%)
Dark Energy (73%)
Ordinary matter (4%)

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Is Fock space quantisation Applicable ?
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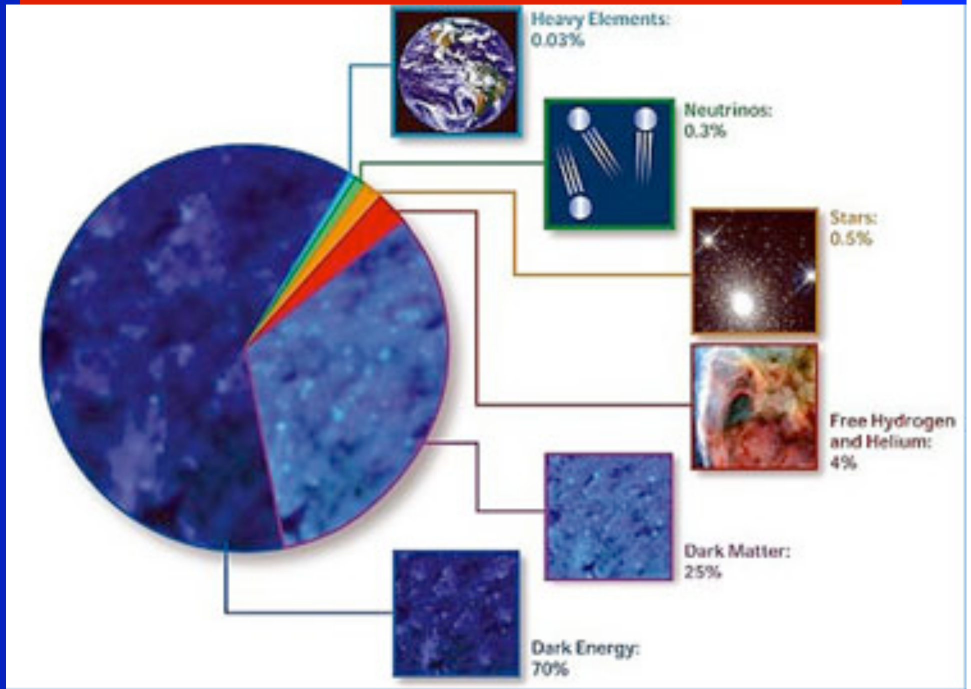

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Are these Problems linked ?

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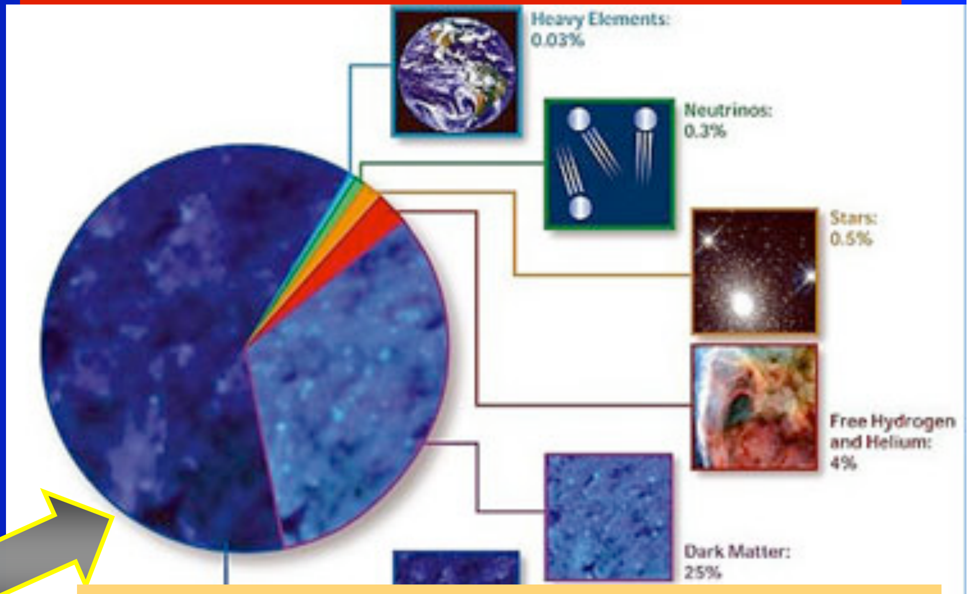

Blasone, Capolupo, Capozzielo, Carloni, Vitiello

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❖ **The flavour Problem**

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I WILL TRY TO ARGUE YES, IN A RIGOROUS WAY,
AT LEAST WITHIN THE CONCEPT OF A PARTICULAR MODEL OF SPACE-TIME FOAM IN STRING THEORY….
ONLY NEUTRINO NON-CLUSTERING CONTRIBUTIONS TO VACUUM ENERGY FOR SPECIFICALLY STRINGY REASONS …

(SS, Mavromatos, Barenboim

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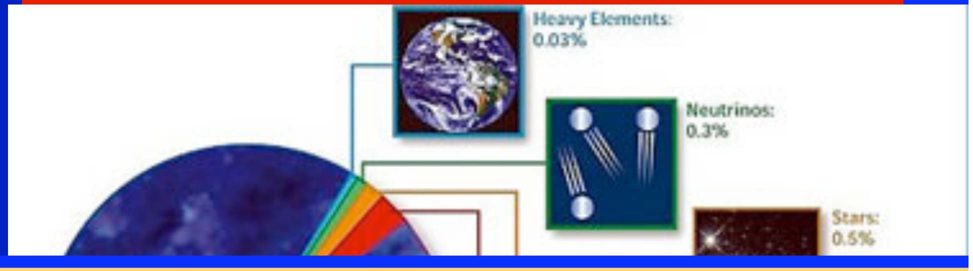

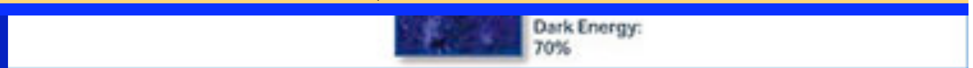

SnIa, CMB
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⇒
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Some properties of flavour vacua in the *conventional* analysis:

- Lorentz symmetry breaking, nonzero expectation value of the stress energy tensor
- Cut-off introduced in a somewhat ad hoc way
- Encompasses all flavoured particles charged or neutral

D-particle Space-Time Foam & Flavour Vacua

Mavromatos, SS (2008)

Motivation:

(i) consider a microscopic model of flavour vacua, with **Dynamical** determination of **cutoff** (if possible)

(ii) Incorporate the Lorentz symmetry breaking of the Flavour condensate by microscopic arguments *in vacuo*

(iii) Select neutrinos among other mixing modes **Dynamically**

D-particle Space-Time Foam & Flavour Vacua

NM, Sarkar (2008)

Motivation:

(i) consider a microscopic model of flavour vacua, with Dynamical determination of cutoff (if possible)

(ii) In

Flavo

(iii) S ... y

At present only TOY effective FIELD TEORY models of flavour states constructed in this framework.

Long way ahead before complete understanding in string theory…

A Model for flavour vacua

- A THEORETICAL MODEL OF SPACE-TIME FOAM INSPIRED FROM NON-CRITICAL STRING THEORY

D-PARTICLE (D0-BRANE) FOAM

(Ellis, Mavromatos, Westmuckett, Nanopoulos, SS, Szabo)

String Theory Basics

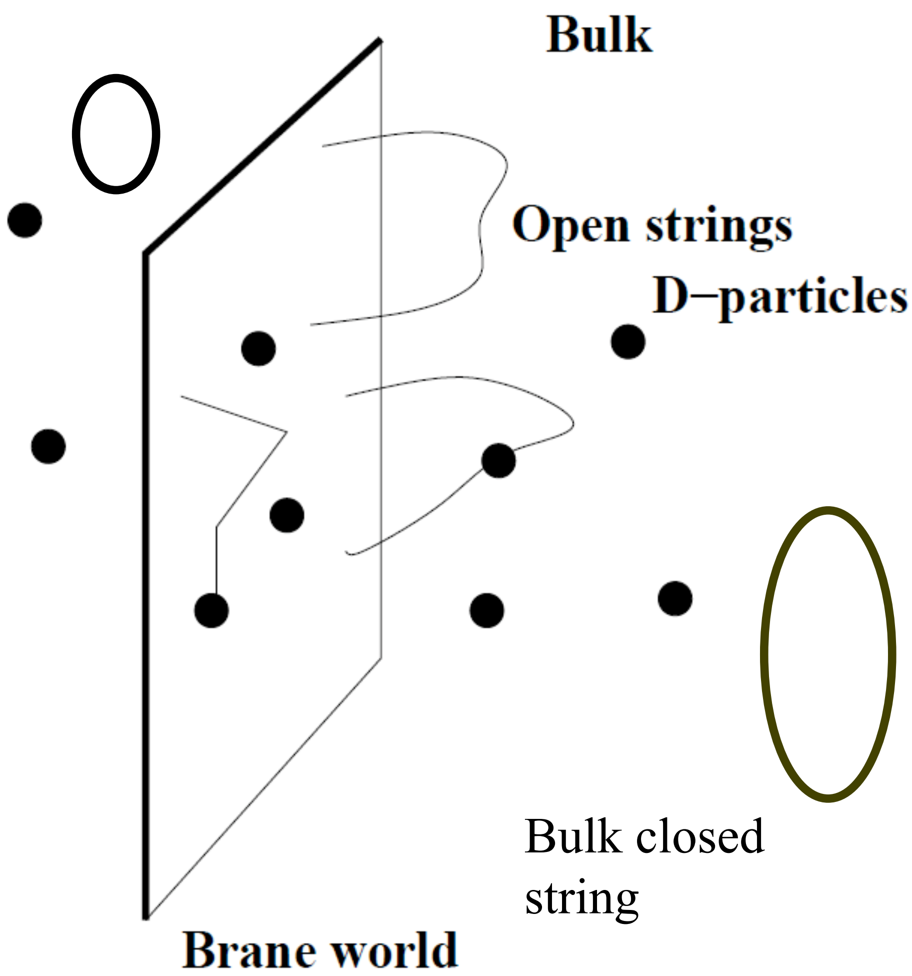
MODERN VERSION OF STRING/BRANE THEORY

Polchinski, Witten, ….

Fundamental (F-) Strings,
ending on Solitonic string states/space-time defects/domain walls called D(irichlet)-branes

D-branes dimensionalities: D(-1) (D-instantons),
D0 (point-like, D-particles)
D1 (string-like, D-strings -> Cosmic Superstrings)
Dp (p > 2)-brane domain walls
(D3-brane Universes, etc.)

D-particle Foam Models

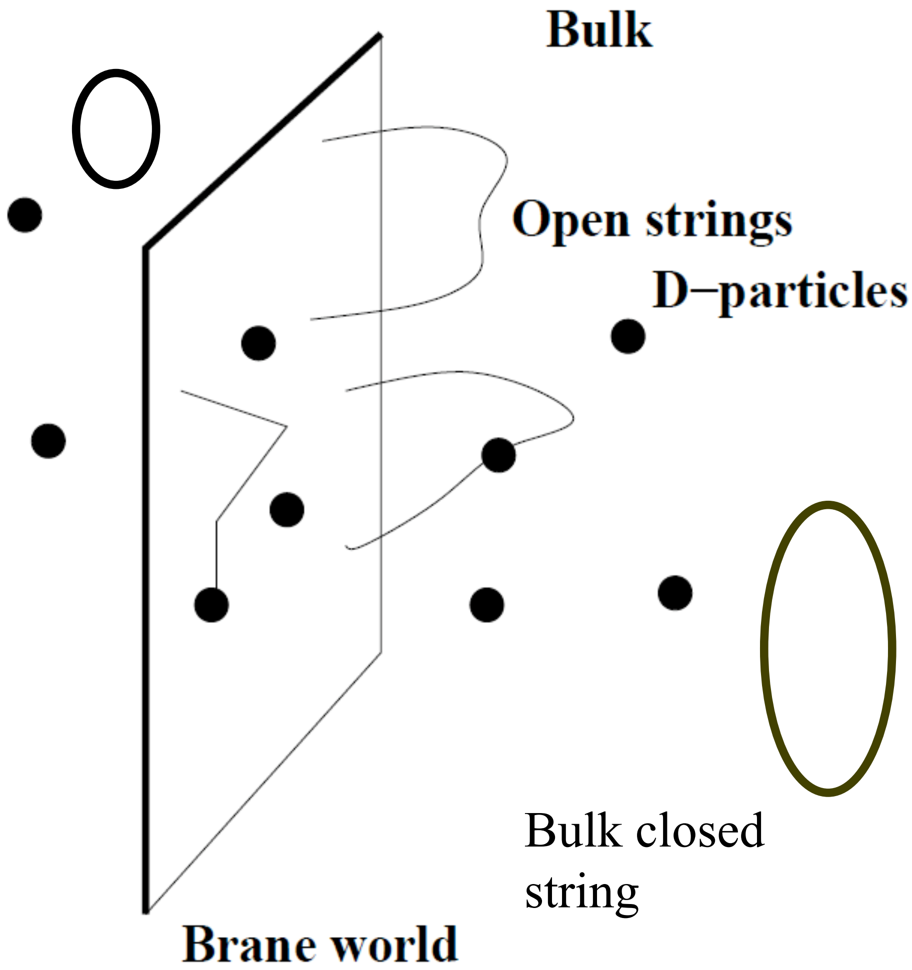


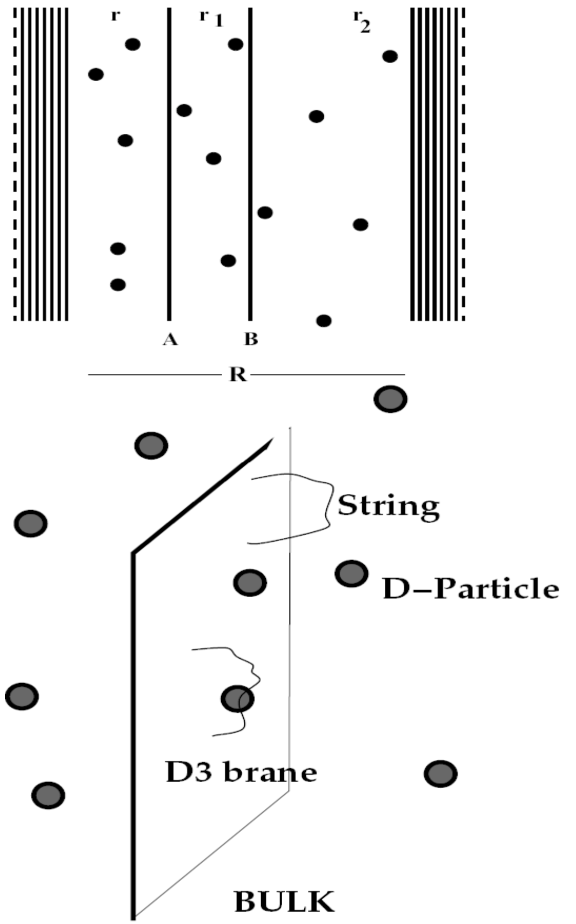
Consistent supersymmetric D−particle foam models can be constructed

No recoil, no brane motion= zero vacuum energy, unbroken SUSY

recoil contributions to vacuum energy
Broken SUSY

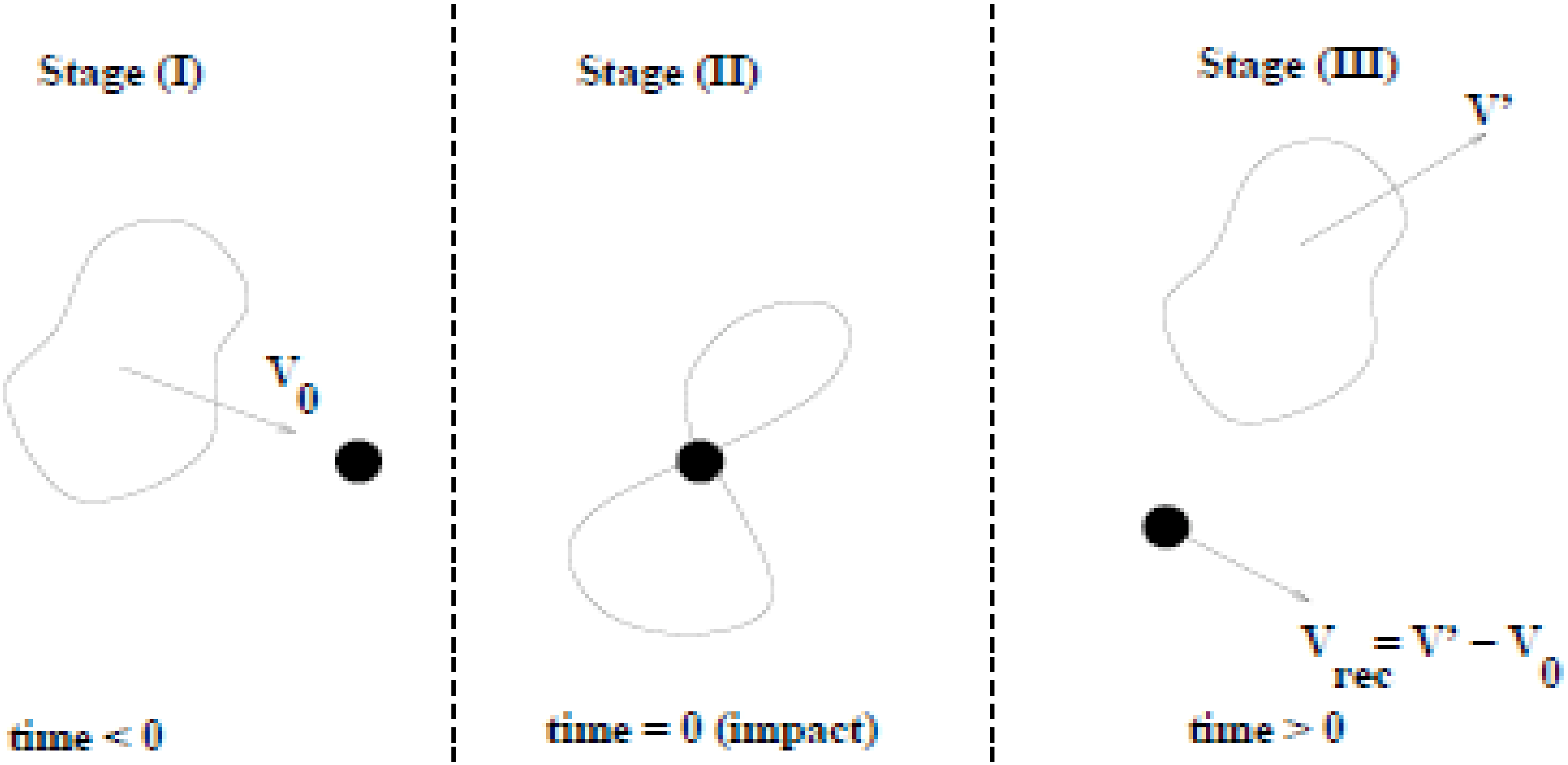
D-particle Foam Models





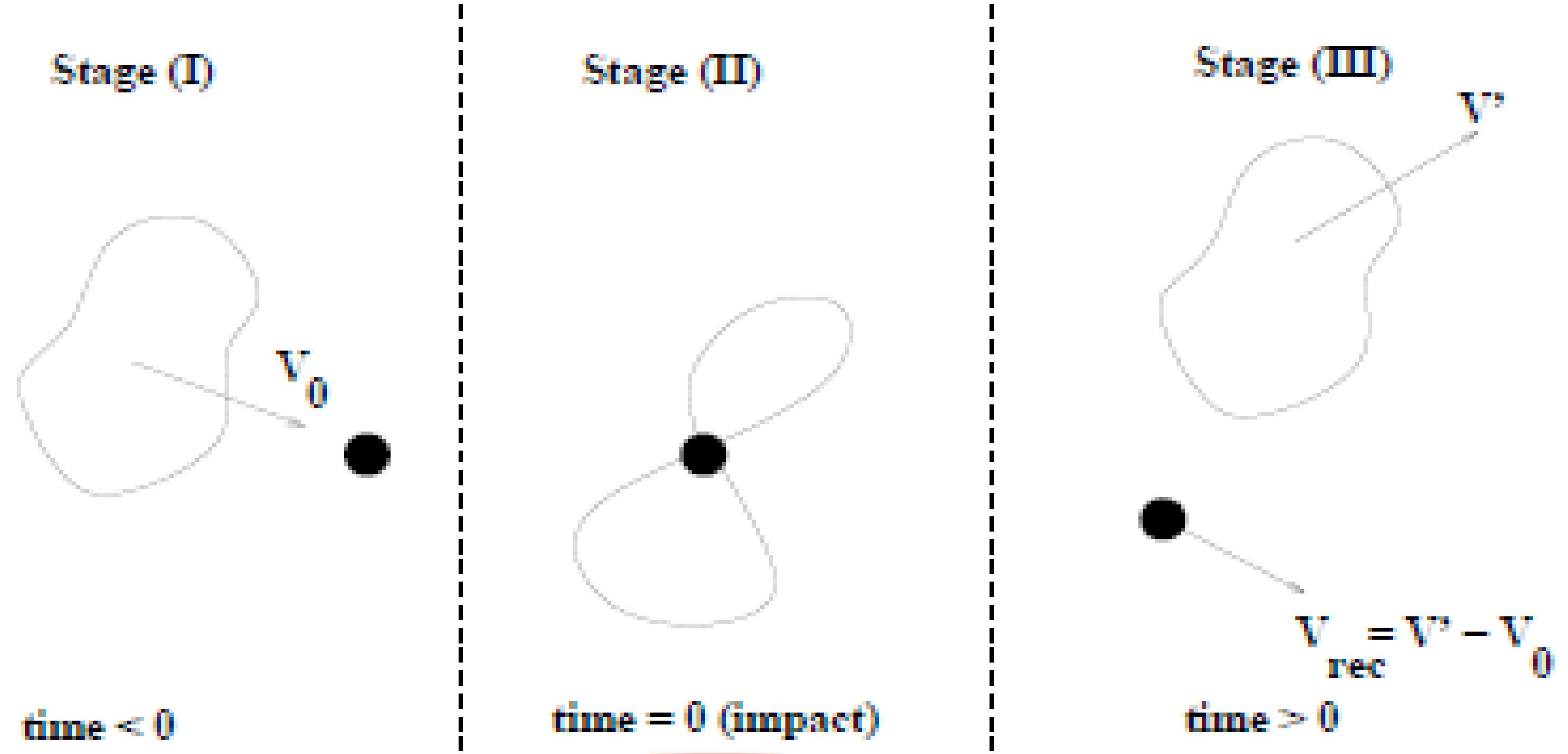
Ellis, Mavromatos, Westmuckett

D-particle Recoil



Logarithmic conformal field theory describes the impulse at stage (II)

D-particle Recoil



Logarithmic conformal field theory describes the impulse at stage (II)

D-particle Recoil

D-particles defects are point-like with masses M_s/g_s

M_s = sting mass scale (free, in general $M_s \neq M_{Plnack}$)

g_s = string scale (assumed weak, $g_s < 1$)

Recoil implies distortion of surrounding space-time, calculated by means of Liouville dressing due to the fact that world-sheet conformal invariance is disturbed. For relatively long times after scattering induced metric has off-diagonal components, Along direction of motion of string, say X :

$$ds^2 = 2\gamma^2(dt + u_x dX)^2 - dt^2 + u_x dX dt - (dX)^2$$

$u_x = g_s \Delta k_x / M_s$ = D-particle Recoil velocity, Δk_x = momentum transfer

For a foam situation we have collection of D-particles. Toy case : Isotropic Gaussian foam

$$\langle\langle u_x \rangle\rangle = 0\ , \qquad \langle\langle u_x u_x \rangle\rangle = \sigma^2$$

For small u_x

$$\boxed{\langle\langle ds^2 \rangle\rangle = (1 + 2\sigma^2)\, dt^2 - (1 - 2\sigma^2) dX^2 + dY^2 + dZ^2}$$

Cosmology from D-foam Recoil

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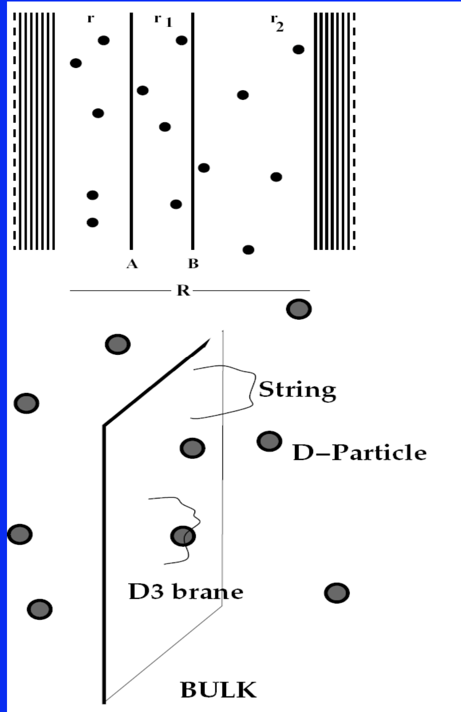


Fluctuations σ could depend on time

$$\sigma = \sigma(t)$$

e.g. non-uniform bulk density of D-particles

$$d\varsigma^2 = (1 + 2\sigma^2(t))dt^2$$

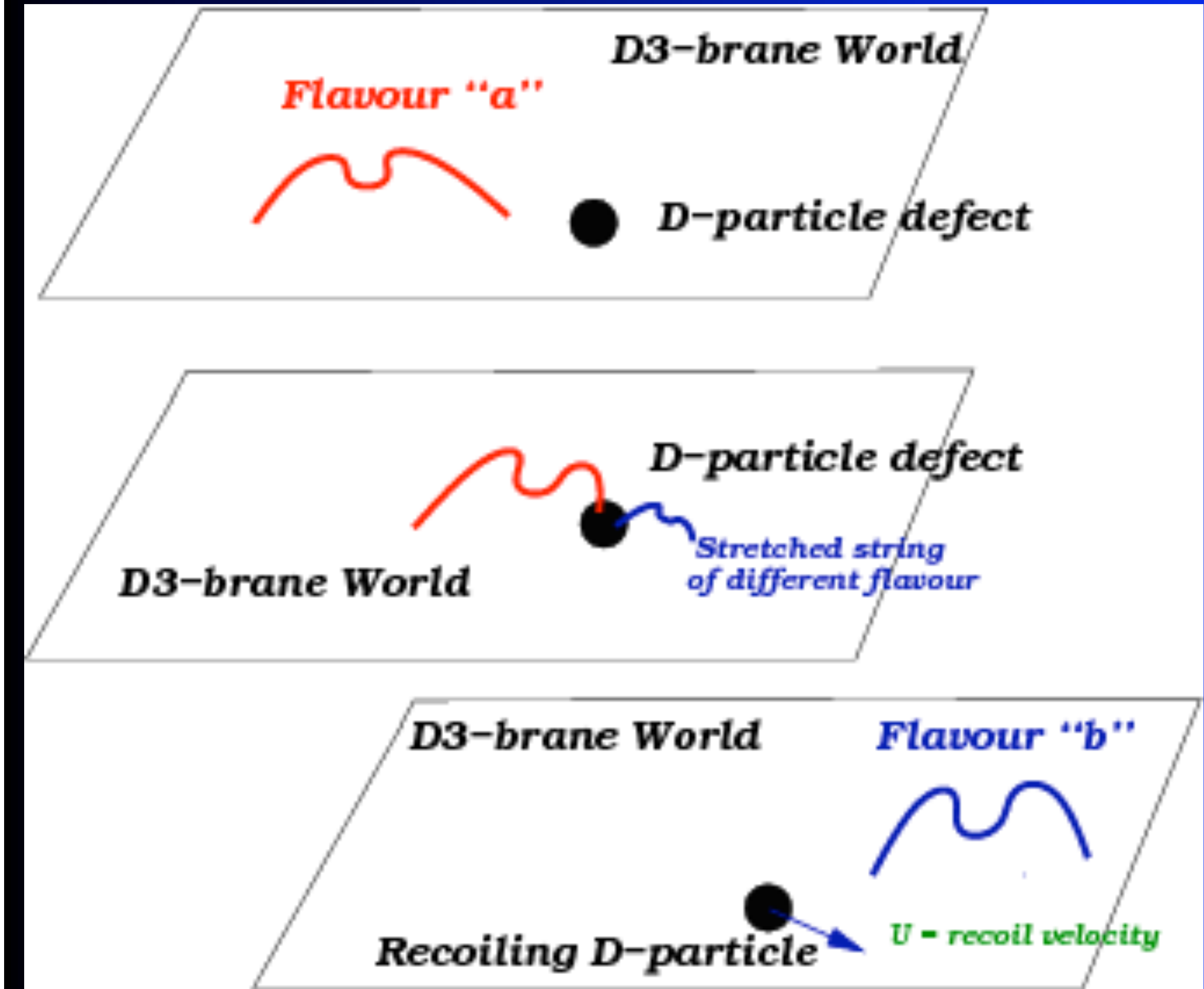
$$\boxed{\langle\langle ds^2 \rangle\rangle = d\varsigma^2 - a^2(\varsigma)(dX)^2 + \dots}$$

If σ(t) decreases as (cosmic) time increases, then we have EXPANDING FRW-type Cosmology on the brane world, as a result of interactions of strings with foam…

Discuss Flavour states in toy effective field theory model of bosons in such a set up…

Mavromatos & SS (2008)

D-particles & Flavour Non-Conservation



Mavromatos & SS 2008

An initially flavoured state ``a'' is **captured** by the D-particle and a string is created, stretching between D-particle and D3-brane. The re-emitted state, after D-particle recoil may be of different flavour ``b''

ONLY electrically neutral open string states (e.g. **Neutrinos**) can be **captured**, due to electric charge conservation.

Toy Model for Bosons/Fermions – effective field theory

Full string theory problem complicated. **Restrict to effective field theory (low-energy limit) of Bosons/Fermions .**
Toy Model , sufficient to qualitatively explain effects of flavour states in this setting.

SEVERAL REASONS FOR CONSIDERING FLAVOUR STATES

(i) Fock space quantization natural in strings.

(ii) Lorentz is violated locally due to D-particle recoil (velocity vectors).

(iii) Flavour is not conserved in the capture process.

D-particle foam induces FRW-cosmological space times, with non-trivial Particle Creation (actually string creation in the full string theory problem).

$$\boxed{ds^2 = dt^2 - a^2(t)dX^2 - dY^2 - dZ^2}$$

Consider effects of Flavour States to vacuum energy self consistently in those curved (anisotropic) space-times. Extension to isotropic situation straightforward.

2-d Cosmic Scalar Fields & Flavour States

NM, Sarkar 2008

Conformal Time, effectively two-dimensional Universes:

$$ds^2 = C(\eta)\left(d\eta^2 - dx^2\right)$$

e.g.

$$C(\eta) = A + B\tanh(\rho\eta)$$

$$C(\eta) = A + B\frac{\eta}{\sqrt{\eta^2 + \frac{1}{\rho^2}}}$$

Incorporates Inflation in the far past, interpolating between de Sitter & Mikowski spacetimes

In cosmic space-times:
In-vacua different from Out-vacua due to Particle Creation (Bogolubov transformation)

Flavour vacuum

- **Blasone, Vitiello, Henning**
  basics

Capolupo, Capozzielo, Carloni,
Barenboim, Mavromatos *cosmological implications*

$$\psi_e(x) = \psi_1(x)\cos\theta + \psi_2(x)\sin\theta$$
$$\psi_\mu(x) = -\psi_1(x)\sin\theta + \psi_2(x)\cos\theta$$

$$\longrightarrow$$

$$\hat{\psi}_e(x) = \hat{G}_\theta^\dagger(t)\hat{\psi}_1(x)\hat{G}_\theta(t)$$
$$\hat{\psi}_\mu(x) = \hat{G}_\theta^\dagger(t)\hat{\psi}_2(x)\hat{G}_\theta(t)$$

$$\hat{G}_\theta(t) = \exp\left[\theta\int d\vec{x}\left(\hat{\psi}_1^\dagger(x)\hat{\psi}_2(x) - \hat{\psi}_2^\dagger\hat{\psi}_1(x)\right)\right]$$

Flavour vacuum $$\left|0\right\rangle_f \equiv \hat{G}_\theta^\dagger(t)\left|0\right\rangle_{mass}$$

Flavour Vacuum Equation of State

Stress-energy tensor of scalar/ fermion field,
L = Lagrangian density

$$T_{\mu\nu}[\psi] = -g_{\mu\nu}L + \frac{1}{2}\left(\bar{\psi}\tilde{\gamma}_{(\mu}D_{\nu)}\psi - D_{(\nu}\bar{\psi}\tilde{\gamma}_{\mu)}\psi\right)$$

$$T^{bos}_{\mu\nu}[\phi] = \frac{1}{2}\left(D_\mu\phi D_\nu\phi + D_\nu\phi D_\mu\phi\right) - g_{\mu\nu}L$$

Flavour vacuum is the physical one. Hence, calculate :

$$\underbrace{{}_{in,\alpha,\beta}\langle 0}_{\text{Flavour Vacuum}} \left| T^{out}_{\mu\nu} \right| \underbrace{0\rangle_{\alpha,\beta,in}}_{\text{Flavour Vacuum}}$$

Flavour Vacuum

Late Times Cosmology, slow expansion (for convenience)

Stochastically fluctuating medium caused by the space-time foam, MSW effect

NM, Sarkar 2006

$$\langle \Delta m^2_{\text{foam}} \rangle \propto G_N \langle n^c_{dp}(r) \rangle k$$

Average number of virtual particles emitted from the foam

In *L*

$$m^2 \rightarrow m_{eff}{}^2 = m^2 \left(1 + g \langle \chi^2 \rangle\right)$$

Normal Ordering Prescription

$$ {}_{in,\alpha,\beta}\langle 0 \left| : T_{11}^{out} : \right| 0 \rangle_{\alpha,\beta,in} = {}_{in,\alpha,\beta}\langle 0 \left| T_{11}^{out} \right| 0 \rangle_{\alpha,\beta,in} - \langle \Psi | T_{11}^{out} | \Psi \rangle $$

$|\Psi\rangle$ is a suitable 'vacuum' state.

In D-particle foam situation: Choose Ψ such that v.e.v. of stress tensor vanishes in the absence of D-particles (ordinary Minkowski vacuum), i.e. subtract the D-particle-Recoil-Velocity-fluctuations independent terms in the above expressions

Result for Late times equation of state p = wρ ,

$$ p = {}_{in,\alpha,\beta}\langle 0 \left| : T_{11}^{out} : \right| 0 \rangle_{\alpha,\beta,in} $$

$$ \rho = {}_{in,\alpha,\beta}\langle 0 \left| : T_{00}^{out} : \right| 0 \rangle_{\alpha,\beta,in} $$

$$ \boxed{w = -1} . $$

Cosmological Constant Type

Two kinds of co-existing vacua in D-particle foam

$$|\mathrm{Flavoured}\rangle \otimes |\mathrm{normal}\rangle$$

Flavoured electrically neutral States (e.g. Neutrinos) with momenta up to a given scale, that can be captured by the D-particles

Rest of the states, not captured by D-particles

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Non-trivial, Non-perturbative Contributions to the Dark Energy of cosmological Constant type ($w \to -1$) at late epochs (slow expansion)…

THANK YOU

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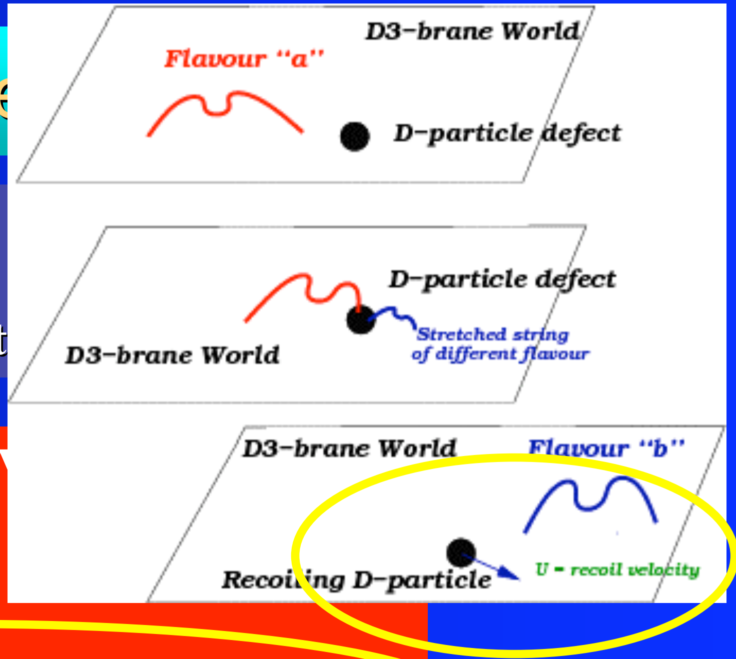

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limit) of Bosons .
Toy Model , sufficient to qualitatively explain effect

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NM, Sarkar 2008

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$$\omega_{in,k} = \sqrt{k^2 + m^2(A-B)},$$

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$$\mathcal{G}(\theta) = \exp(i\theta\mathcal{S})$$

$$\mathcal{S} = \frac{i}{2}\sum_{j=0}^{N}\mathfrak{S}_j$$

$$\mathfrak{S}_j = \gamma_{j-}\left(\tilde{a}_j^{(1)}\tilde{a}_{-j}^{(2)} + \tilde{a}_{-j}^{(1)}\tilde{a}_j^{(2)}\right) - \gamma_{j-}\left(\tilde{a}_j^{(1)\dagger}\tilde{a}_{-j}^{(2)\dagger} + \tilde{a}_{-j,1}^{(1)\dagger}\tilde{a}_j^{(2)\dagger}\right)$$
$$+\gamma_{j+}\left(\tilde{a}_j^{(1)}\tilde{a}_j^{(2)\dagger} + \tilde{a}_{-j}^{(1)}\tilde{a}_{-j}^{(2)\dagger}\right) - \gamma_{j+}\left(\tilde{a}_j^{(1)\dagger}\tilde{a}_j^{(2)} + \tilde{a}_{-j}^{(1)\dagger}\tilde{a}_{-j}^{(2)}\right)$$

$$\gamma_{j\pm} \equiv \sqrt{\frac{\omega_j^{(1)}}{\omega_j^{(2)}}} \pm \sqrt{\frac{\omega_j^{(2)}}{\omega_j^{(1)}}}$$

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$$\gamma_{j\pm} \equiv \sqrt{\frac{\omega_j^{(1)}}{\omega_j^{(2)}}} \pm \sqrt{\frac{\omega_j^{(2)}}{\omega_j^{(1)}}}$$

$$\tilde{a}_j^{(i)} = a_j^{(i)} e^{-i\omega_j^{(i)} t}$$

Can show: Flavour OUT vacua Orthogonal (thermodynamic limit) to Mass eigenstate IN vacua , as in flat space-times.

Enclose system in a Box of size L (periodic boundary conditions) , discrete momenta $k = \frac{2\pi j}{L}$

Mass egeinstate
In & Out Vacua

$$a_j^{(i)in} |0\rangle_{in1,2} = 0 = \ B^\dagger\left(\varphi_j^{(i)out}, v_j^{(i)out}\right) a_j^{(i)out} B\left(\varphi_j^{(i)out}, v_k^{(i)out}\right) |0\rangle_{in,1,2}$$

$$a_j^{(i)out} |0\rangle_{out,1,2} = 0 \quad \forall j.$$

$$|0\rangle_{in,1,2} = \prod_{i=1}^{2}\prod_{j\geq 0} B^\dagger\left(\varphi_j^{(i)out}, v_j^{(i)out}\right) |0\rangle_{out,1,2}$$

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$$B = \mathcal{S}P$$

$$\mathcal{S}\left(v_j^{(i)}\right) = \exp\left[v_j^{(i)} a_{-j}^{(i)\dagger} a_j^{(i)\dagger} - v_j^{(i)*} a_j^{(i)} a_{-j}^{(i)}\right]$$

$$P\left(\phi_j^{(i)}\right) = \exp\left(-i\varphi_j^{(i)}\left[a_j^{\dagger(i)} a_j^{(i)} + a_{-j}^{\dagger(i)} a_{-j}^{(i)}\right]\right)$$

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$$\widetilde{v}_j^{(i)out} \equiv v_j^{(i)out} e^{i\varphi_j^{(i)out}}$$

$$\omega_{\pm,j}^{(i)} = \frac{1}{2}\left(\omega_{out,j}^{(i)} \pm \omega_{in,j}^{(i)}\right)$$

$$\left|\widetilde{v}_j^{(i)in}\right| = \left|\widetilde{v}_j^{(i)out}\right| = \tanh^{-1}\left[\frac{\sinh\left(\frac{\pi\omega_{-,j}^{(i)}}{\rho}\right)}{\sinh\left(\frac{\pi\omega_{+,j}^{(i)}}{\rho}\right)}\right]$$

$$e^{i\varphi_j^{(i)out}} = \sqrt{\frac{\omega_{out,j}^{(i)}}{\omega_{in,j}^{(i)}}}\;\frac{\Gamma\left(1+\frac{i\omega_{in,j}^{(i)}}{\rho}\right)\Gamma\left(\frac{i\omega_{out,j}^{(i)}}{\rho}\right)}{\Gamma\left(\frac{i\omega_{+,j}^{(i)}}{\rho}\right)\Gamma\left(1+\frac{i\omega_{+,j}^{(i)}}{\rho}\right)\cosh\left(\left|\widetilde{v}_j^{(i)in}\right|\right)}$$

Enclose system in a Box of size L (periodic boundary conditions) ,
discrete momenta $k = \frac{2\pi j}{L}$

Mass egeinstate
In & Out Vacua

$$a_j^{(i)in} |0\rangle_{in1,2} = 0 = B^\dagger\left(\varphi_j^{(i)out}, v_j^{(i)out}\right) a_j^{(i)out} B\left(\varphi_j^{(i)out}, v_k^{(i)out}\right)|0\rangle_{in,1,2}$$

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$$\mathcal{G}_*(\theta) = \prod_{j=1}^{N^*} \exp\left(-\tfrac{\theta}{2}\mathfrak{S}_j\right)$$

Flavour vacua

$$|0\rangle_{\alpha,\beta} = \mathcal{G}_*^{-1}(\theta)\,|0\rangle_{1,2}$$

thermodynamic limit $N^* \to \infty$

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$$\mathcal{F}_j\left(\theta\right) = {}_{1,2,out}\left\langle 0\right|\left\{\prod_{i=1}^{2} B\left(\varphi_j^{(i)out}, v_j^{(i)out}\right)\right\}\mathcal{G}_{out,j}^{-1}\left(\theta\right)\left|0\right\rangle_{out,1,2}$$

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Flavour vac.

Mass eigenstate vac.

Flavour Vacuum Equation of State

Stress-energy tensor of scalar field,
L = Lagrangian density

$$T_{\mu\nu} = \frac{1}{2}\left(\phi_{,\mu}\phi_{,\nu} + \phi_{,\nu}\phi_{,\mu}\right) - g_{\mu\nu}\frac{L}{\sqrt{-g}}$$

Flavour vacuum is the physical one. Hence, calculate :

$${}_{in,\alpha,\beta}\langle 0 \,|\, T^{out}_{\mu\nu} \,|0\rangle_{\alpha,\beta,in}$$

Flavour Vacuum

Late Times Cosmology, slow expansion (for convenience)

Flavour Vacuum Equation of State

$$
\begin{aligned}
{}_{in,\alpha,\beta}\left\langle 0 \left| T_{00}^{out} \right| 0 \right\rangle_{\alpha,\beta,in} &= \frac{1}{2}\, {}_{in,\alpha,\beta}\left\langle 0 \left| T_{00}^{(1)out} \right| 0 \right\rangle_{\alpha,\beta,in} + \frac{1}{2}\, {}_{in,\alpha,\beta}\left\langle 0 \left| T_{00}^{(2)out} \right| 0 \right\rangle_{\alpha,\beta,in} \\
&+ \frac{C(\eta)}{2}\, {}_{in,\alpha,\beta}\left\langle 0 \left| T_{00}^{(3)out} \right| 0 \right\rangle_{\alpha,\beta,in} \\
&= -\frac{1}{4L} \sum_{j} \sum_{i=1}^{2} \omega_j^{(i)} \left(\mathfrak{f}_{j,i}^{(+)} + \mathfrak{f}_{j,i}^{(-)} \right) \\
&- 2\pi^2 \sum_{j} \sum_{i=1}^{2} \frac{j^2}{2L^3 \omega_j^{(i)}} \left\{ \mathfrak{f}_{j,i}^{(+)} - \mathfrak{f}_{j,i}^{(-)} \right\} \\
&+ \frac{C(\eta)}{4L} \sum_{j} \sum_{i=1}^{2} \frac{m^{(i)2}}{\omega_j^{(i)}} \left(\mathfrak{f}_{j,i}^{(+)} - \mathfrak{f}_{j,i}^{(-)} \right)
\end{aligned}
$$

Stress-energy tensor of scala

L = Lagrangian density

Flavour vacuum is the physica

$$
\begin{aligned}
{}_{in,\alpha,\beta}\left\langle 0 \left| T_{11}^{out} \right| 0 \right\rangle_{\alpha,\beta,in} &= \frac{1}{2}\, {}_{in,\alpha,\beta}\left\langle 0 \left| T_{00}^{(2)\,out} \right| 0 \right\rangle_{\alpha,\beta,in} + \frac{1}{2}\, {}_{in,\alpha,\beta}\left\langle 0 \left| T_{00}^{(1)\,out} \right| 0 \right\rangle_{\alpha,\beta,in} \\
&- \frac{C(\eta)}{2}\, {}_{in,\alpha,\beta}\left\langle 0 \left| T_{00}^{(3)\,out} \right| 0 \right\rangle_{\alpha,\beta,in} \\
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&- \frac{1}{4L} \left\{ \sum_{j} \sum_{i=1}^{2} \omega_j^{(i)} \left(\mathfrak{f}_{j,i}^{(+)} + \mathfrak{f}_{j,i}^{(-)} \right) \right\} \\
&- \frac{C(\eta)}{4L} \sum_{j} \sum_{i=1}^{2} \frac{m^{(i)2}}{\omega_j^{(i)}} \left(-\mathfrak{f}_{j,i}^{(-)} + \mathfrak{f}_{j,i}^{(+)} \right).
\end{aligned}
$$

RESULT:

$$
{}_{in,\alpha,\beta}\left\langle 0 \left| T_{01}^{out} \right| 0 \right\rangle_{\alpha,\beta,in} = 0
$$

Flavour Vacuum Equation of State

$$
{}_{in,\alpha,\beta}\langle 0 \left| T_{00}^{out} \right| 0\rangle_{\alpha,\beta,in} = \frac{1}{2}\, {}_{in,\alpha,\beta}\langle 0 \left| T_{00}^{(1)out} \right| 0\rangle_{\alpha,\beta,in} + \frac{1}{2}\, {}_{in,\alpha,\beta}\langle 0 \left| T_{00}^{(2)out} \right| 0\rangle_{\alpha,\beta,in}
$$

Stre

L =

$$
:\mathfrak{f}^{(+)}_{k,i}: \; = 2\gamma^{in}_{k,-}\sin^2\theta \sum_{s=1}^{2} (-1)^s\, \delta_{i[s+1]_2} \exp\left(-\frac{\pi}{\rho}\left[\lambda m^{([s+1]_2)} + \frac{k^2}{2\lambda m^{([s+1]_2)}}\right]\right) \times \sin\left(\frac{m^{([s+1]_2)4}\,(\lambda-1)^2}{4\rho\left(k^2 + m^{([s+1]_2)2}\right)^{\frac{3}{2}}}\right)
$$

Flavo

R

$\;{}^{(1)\,out}_{0}\;|0\rangle_{\alpha,\beta,in}$

$in,$

$$
:\mathfrak{f}^{(-)}_{k,i}: \; = -\gamma^{in}_{k,-}\sin^2\theta \sum_{s=1}^{2} (-1)^{s+1}\, \delta_{i[s+1]_2} \left\{ \begin{array}{c} \cos\left(\frac{m^{([s+1]_2)4}(\lambda-1)^2}{2\rho\left(k^2+m^{([s+1]_2)2}\right)^{\frac{3}{2}}}\right) \\ -\exp\left(-\frac{2\pi}{\rho}\left(\lambda m^{([s+1]_2)} + \frac{k^2}{2\lambda m^{([s+1]_2)}}\right)\right) \end{array} \right\}.
$$

$$
-\frac{\quad}{4L} \sum_{j} \sum_{i=1} \frac{\quad}{\omega_j^{(i)}} \left(-\mathfrak{f}_{j,i} + \mathfrak{f}_{j,i} \right).
$$

$$\gamma^{in}_{k,-} \simeq \frac{\delta m}{\sqrt{m^{(1)}m^{(2)}}} - \frac{2\Delta m^2\, m}{4\lambda^2\left(m^{(1)}m^{(2)}\right)^{\frac{5}{2}}}k^2$$

Small momenta

$$A = \frac{1+\lambda^2}{2},\ B = \frac{1-\lambda^2}{2} \text{and } \Delta m^2 \equiv m^{(1)2} - m^{(2)2}$$

Stre

L =

$: \mathfrak{f}^{(+)}_{k,i} :$

$$= 2\gamma^{in}_{k,-}\sin^2\theta \sum_{s=1}^{2}(-1)^s\,\delta_{i[s+1]_2}\exp\left(-\frac{\pi}{\rho}\left[\lambda m^{([s+1]_2)} + \frac{k^2}{2\lambda m^{([s+1]_2)}}\right]\right)$$

$$\times \sin\left(\frac{m^{([s+1]_2)4}(\lambda-1)^2}{4\rho\left(k^2+m^{([s+1]_2)2}\right)^{\frac{3}{2}}}\right)$$

Flavo

R

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$^{(2)out}_{00}\ |0\rangle_{\alpha,\beta,in}$

$^{(1)\,out}_{0}\ |0\rangle_{\alpha,\beta,in}$

$$-\frac{}{4L}\sum_j\sum_{i=1}\frac{}{\omega^{(i)}_j}\left(-\mathfrak{f}_{j,i} + \mathfrak{f}_{j,i}\right).$$

Normal Ordering Prescription

$$ {}_{in,\alpha,\beta}\langle 0 \left| : T_{11}^{out} : \ \right| 0 \rangle_{\alpha,\beta,in} = {}_{in,\alpha,\beta}\langle 0 \left| T_{11}^{out} \right| 0 \rangle_{\alpha,\beta,in} - \langle \Psi | \, T_{11}^{out} \, | \Psi \rangle $$

$|\Psi\rangle$ is a suitable 'vacuum' state.

In D-particle foam situation: Choose Ψ such that v.e.v. of stress tensor vanishes in the absence of D-particles (ordinary Minkowski vacuum), i.e. subtract the D-particle-Recoil-Velocity-fluctuations independent terms in the above expressions

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Subtraction Procedure Independent of choice of C(η) scaling factor

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Result for Late times equation of state p = wρ ,

$$ p = {}_{in,\alpha,\beta}\langle 0 \left| : T_{11}^{out} : \ \right| 0 \rangle_{\alpha,\beta,in} $$

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Cosmological Constant Type

Estimates of Flavoured Vacuum Energy

Dynamical determination of momentum cut-off for ``flavoured''Fock space modes

In D-particle foam models low-momentum modes have a much greater chance of being captured by the D-particle defects

Particle production – particle number at momentum scale k in ``flavoured'' vacuum

$$\boxed{n_\iota\left(k,\eta\right) \equiv {}_{\alpha,\beta}\langle 0 \left| \widetilde{a}^{\dagger}_{\iota,k}\left(\eta,x\right)\widetilde{a}_{\iota,k}\left(\eta,x\right)\right|0\rangle_{\alpha,\beta}}$$

$$\iota = \alpha, \beta$$

$$n_{\alpha,k}\left(t\right) = \gamma^2_{k,-}\left[\frac{\sin^4\theta}{4}\gamma^2_{k,+}\sin^2\omega^{(2)}_{in,k}\eta + \frac{\sin^2 2\theta}{4}\sin^2\left(\omega^{(1)}_{in,k} + \omega^{(2)}_{in,k}\right)\eta\right]$$

$$n_{\beta,k}\left(t\right) = \gamma^2_{k,-}\left[\frac{\sin^4\theta}{4}\gamma^2_{k,+}\sin^2\omega^{(1)}_{in,k}\eta + \frac{\sin^2 2\theta}{4}\sin^2\left(\omega^{(1)}_{in,k} + \omega^{(2)}_{in,k}\right)\eta\right]$$

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Significant fall off for scales higher than:

$$k_0 \sim \frac{1}{\sqrt{2}}\sqrt{\left(m^{(1)2}+m^{(2)2}\right)}$$

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Plausible cutoff

Two kinds of co-existing vacua in D-particle foam

$$|\mathrm{Flavoured}\rangle \otimes |\mathrm{normal}\rangle$$

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No Novel Dark Energy Contributions

Estimates of Flavoured Vacuum Energy

$$
{}_{in,\alpha,\beta}\langle 0 \left| T_{00}^{out} \right| 0\rangle_{\alpha,\beta,in}
= \frac{C(\eta)}{8\pi} \sum_{i=1}^{2} \int_0^{k_0} dk \frac{m^{(i)2}}{\omega_k^{(i)}} \left(\mathfrak{f}_{k,i}^{(+)} - \mathfrak{f}_{k,i}^{(-)} \right)
$$

$$
{}_{in,\alpha,\beta}\langle 0 \left| T_{11}^{out} \right| 0\rangle_{\alpha,\beta,in}
= -\frac{C(\eta)}{8\pi} \sum_{i=1}^{2} \int_0^{k_0} dk \frac{m^{(i)2}}{\omega_k^{(i)}} \left(\mathfrak{f}_{k,i}^{(+)} - \mathfrak{f}_{k,i}^{(-)} \right)
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$$: \mathfrak{f}_{k,i}^{(+)} : $$
$$ = 2\gamma_{k,-}^{in} \sin^2\theta \sum_{s=1}^{2} (-1)^s \delta_{i[s+1]_2} \exp\left(-\frac{\pi}{\rho} \left[\lambda m^{([s+1]_2)} + \frac{k^2}{2\lambda m^{([s+1]_2)}} \right] \right) $$
$$ \times \sin\left(\frac{m^{([s+1]_2)4} (\lambda - 1)^2}{4\rho \left(k^2 + m^{([s+1]_2)2} \right)^{\frac{3}{2}}} \right) $$

$$: \mathfrak{f}_{k,i}^{(-)} : $$
$$ = -\gamma_{k,-}^{in} \sin^2\theta \sum_{s=1}^{2} (-1)^{s+1} \delta_{i[s+1]_2} \left\{ \begin{array}{c} \cos\left(\frac{m^{([s+1]_2)4}(\lambda-1)^2}{2\rho\left(k^2+m^{([s+1]_2)2}\right)^{\frac{3}{2}}} \right) \\ -\exp\left(-\frac{2\pi}{\rho} \left(\lambda m^{([s+1]_2)} + \frac{k^2}{2\lambda m^{([s+1]_2)}} \right) \right) \end{array} \right\}. $$

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$$

$$
\delta m = m^{(1)} - m^{(2)}
$$

Averaging over capture time

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Averaging over capture time

In D-particle foam case, with velocity fluctuations for D-particle $\tilde{\sigma}$

$$ \boxed{C(\eta_0) \sim 1 - 4\tilde{\sigma}^2} $$

Cosmology from D-foam Recoil

$$\langle\langle u_x \rangle\rangle = 0 \;, \qquad \langle\langle u_x u_x \rangle\rangle = \sigma^2$$

$$\langle\langle ds^2 \rangle\rangle = \left(1 + 2\sigma^2\right) dt^2 - (1 - 2\sigma^2) dX^2 + dY^2 + dZ^2$$

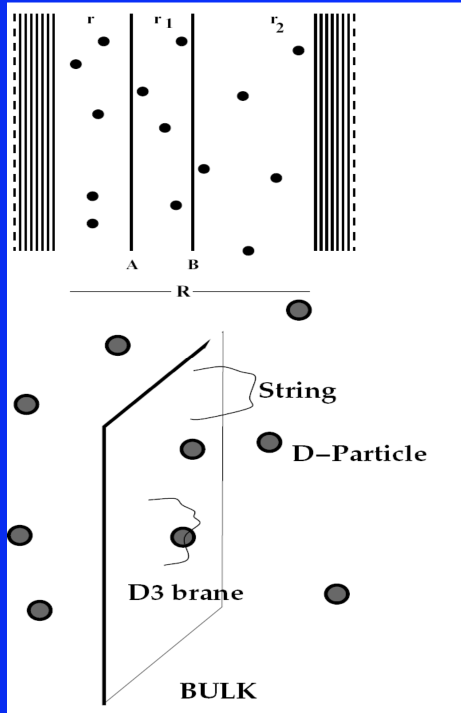
Fluctuations σ could depend on time

σ = σ(t)

e.g. non-uniform bulk density of D-particles

$$d\varsigma^2 = (1 + 2\sigma^2(t))dt^2$$

$$\langle\langle ds^2 \rangle\rangle = d\varsigma^2 - a^2(\varsigma)(dX)^2 + \ldots$$



If σ(t) decreases as (cosmic) time increases, then we have EXPANDING FRW-type Cosmology on the brane world, as a result of interactions of strings with foam…

Discuss Flavour states in toy effective field theory model of bosons in such a set up…

NM & Sarkar (2008)

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$$

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$_{in,\alpha,\beta}\langle 0 \left| T_{00}^{out} \right| 0\rangle$

$= \frac{C(\eta)}{8\pi} \sum_{i=1}^{2} \int_0^{k_0}$ ... $))$

$_{in,\alpha,\beta}\langle 0 \left| T_{00}^{out} \right.$

$\sim -\frac{\sin^2\theta}{24\pi} (\delta m$ ... (2)

Self-consistent back-reaction onto space-time , solution of Einstein equations (consistency of string theory) --- can be de Sitter space-time if σ is constant in cosmic time (approximately true for late eras in ourmodel…)

ime

In D-p rticle foa

$C(\eta_0) \sim 1 -$

$$_{in,\alpha,\beta}\langle 0 \left| : T_{00}^{out} : \right| 0\rangle_{\alpha,\beta,in} \sim \frac{(\delta m)^2}{2\pi} \sin^2\theta\ \tilde{\sigma}^2$$

Comments on Supersymmetry Breaking

In the string effective theory, target space supersymmetry is initially ensured at String scale. Repeat calculations for fermionic partners of flavoured states.

Fermion-Boson contributions of flavour vacuum energy do not cancel out…
Kind of spontaneous breaking of supersymmetry by flavour condensate, but …

Order of breaking very suppressed, of the order of cosmological vacuum energy.

.Phenomenologically realistic mass splittings achieved via SUPERSYMMETRY OBSTRUCTION via compactification to ``magnetised'' manifolds.
Fermions coupled differently than bosons to an external magnetic field (Zeeman).
Induced SUSY obstructing (at excitation level) mass differences, proportional to Magnetic field intensity. String theory have lots of flux fields, playing the role of ``magnetic '' fields in compactified manifolds of extra dimensions.

Gravanis & N.E.M.

Mass diffferences among flavour states insensitive to ``magnetic'' field

SUSY Obstruction does not affect value of Flavour-vacuum contributions to Dark Energy (proportional to flavour-states mass differences…)

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Conclusions & Outlook

- **Flavoured Fock states of (low-momentum) bosons have been considered within a semi-microscopic toy model of D-particle foam in string/brane theory, involving local breaking of Lorentz symmetry (LV), through fluctuations of recoil velocity of D-particles, during capture process.**

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- Long way ahead before drawing definite conclusions in string/brane models, but very encouraging results so far, regarding these novel non-perturbative contributions to Dark Energy of the Cosmos…