Interacting models of dark energy

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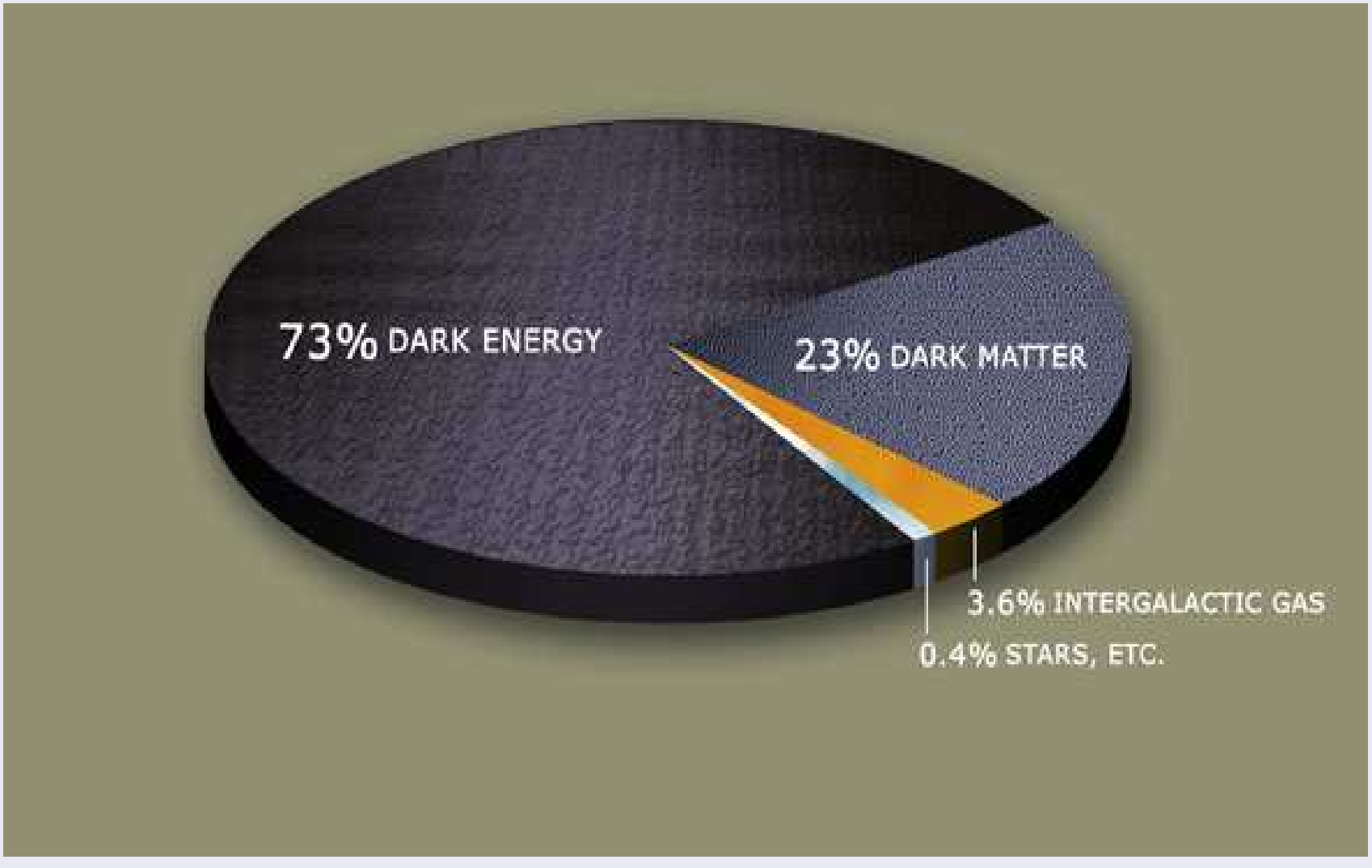
Outlook

Cosmic pie


73% DARK ENERGY
23% DARK MATTER
3.6% INTERGALACTIC GAS
0.4% STARS, ETC.

Main issues

- What is Dark Matter (DM)?
- What is Dark Energy (DM)?
- What are their interactions?
- Can we solve the Cosmic Coincidence problem?
- Are there mutual interactions in the dark sector?

Equations of motion

- Matter components of the model
  - $\rho_b =$ Baryons, $\rho_c =$ DM, $\rho_c =$ DE, $w_x =$ DE EoS
- Equations of energy balance

$$\begin{aligned}
\dot{H} &= -4\pi G\left[\rho_b + \rho_c + (1+w_x)\rho_x\right] , \\
\dot{\rho}_b &= -3H\rho_b , \\
\dot{\rho}_c &= -3H\rho_c + Q , \\
\dot{\rho}_x &= -3(1+w_x)H\rho_x - Q ,
\end{aligned}$$

- Friedmann constraint

$$H^2 = \frac{8\pi G}{3}\left(\rho_b + \rho_c + \rho_x\right) .$$

Interaction term

- Equations of energy balance

$$\begin{aligned} \dot{H} &= -4\pi G\left[\rho_b + \rho_c + (1+w_x)\rho_x\right] , \\ \dot{\rho}_b &= -3H\rho_b , \\ \dot{\rho}_c &= -3H\rho_c + Q , \\ \dot{\rho}_x &= -3(1+w_x)H\rho_x - Q , \end{aligned}$$

General interaction term: Q

- $Q > 0$: Energy transfer DE $\Rightarrow$ DM
- $Q < 0$: Energy transfer DE $\Leftarrow$ DM

Dynamical system with EoS

- Phase space variables

$$x = \frac{8\pi G}{3H^2}\rho_x\,, \quad y = \frac{8\pi G}{3H^2}\rho_c\,, \quad z = \frac{8\pi G}{3H^3}Q\,,$$

- Dynamical equations of motion

$$\begin{aligned} x' &= -3w_x x\,(1-x) - z\,, \\ y' &= 3w_x xy + z\,. \end{aligned}$$

where $' = d/dN$ with $N \equiv \ln(a)$

Dynamical system *without* EoS

- Dynamical equations of motion

$$\begin{aligned} x' &= -3w_x x\,(1-x) - z\,, \\ y' &= 3w_x xy + z\,. \end{aligned}$$

- Dynamical equations of motion *without* EoS

$$\begin{aligned} x' &= 3x\,(1-x) - \tilde{z}\,, \quad \tilde{z} \equiv \frac{z}{(-w_x)} \\ y' &= -3xy + \tilde{z}\,. \end{aligned}$$

where $' = d/d(-w_x N)$. *The EoS is effectively '*hidden*'*.

Interacting models[1, 2, 3, 4, 5]

$$Q = A_c \rho_c + A_x \rho_x$$

Simple realizations

- Model I: $A_I = 3\alpha_I H\,, \quad \alpha_I = \text{const}$
- Model II: $A_I = 3\Gamma_I\,, \quad \Gamma_I = \text{const}$

General requirements

- *Non-negative* DM and DE components: $x, y \geq 0$
- *Early* matter domination: $x \to 0$, $y \to 1$
- *Cosmic coincidence*: $x/y < \infty$ at late times
- *Similar* evolution to ΛCDM model

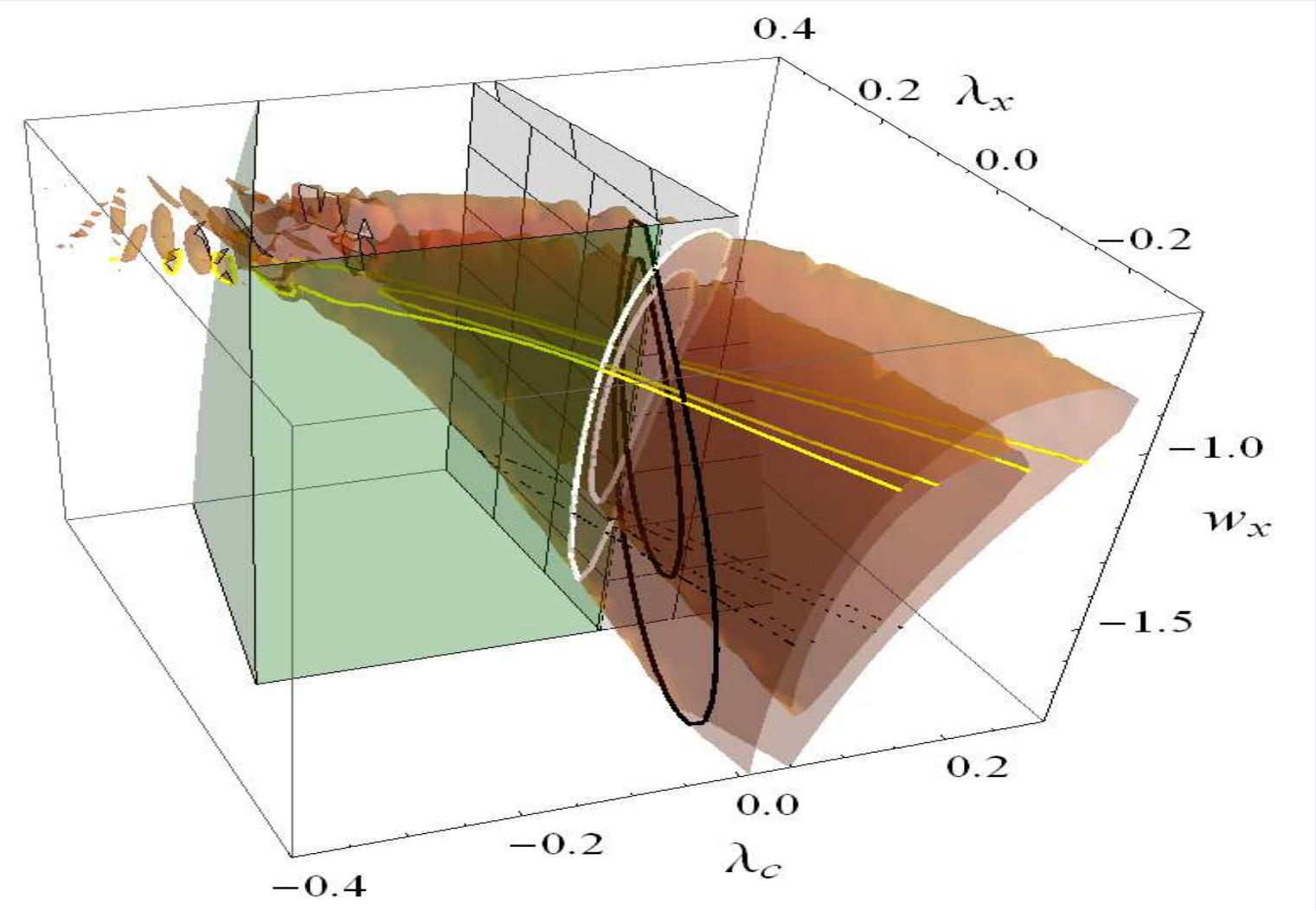
Model I: Equations of motion

$$\begin{aligned} x' &= 3x\left(1-x\right) - 3\left(\tilde{\alpha}_x x + \tilde{\alpha}_c y\right) \\ y' &= -3xy + 3\left(\tilde{\alpha}_x x + \tilde{\alpha}_c y\right) , \\ \tilde{\alpha}_I &\equiv \frac{\alpha_I}{-w_x} \end{aligned}$$

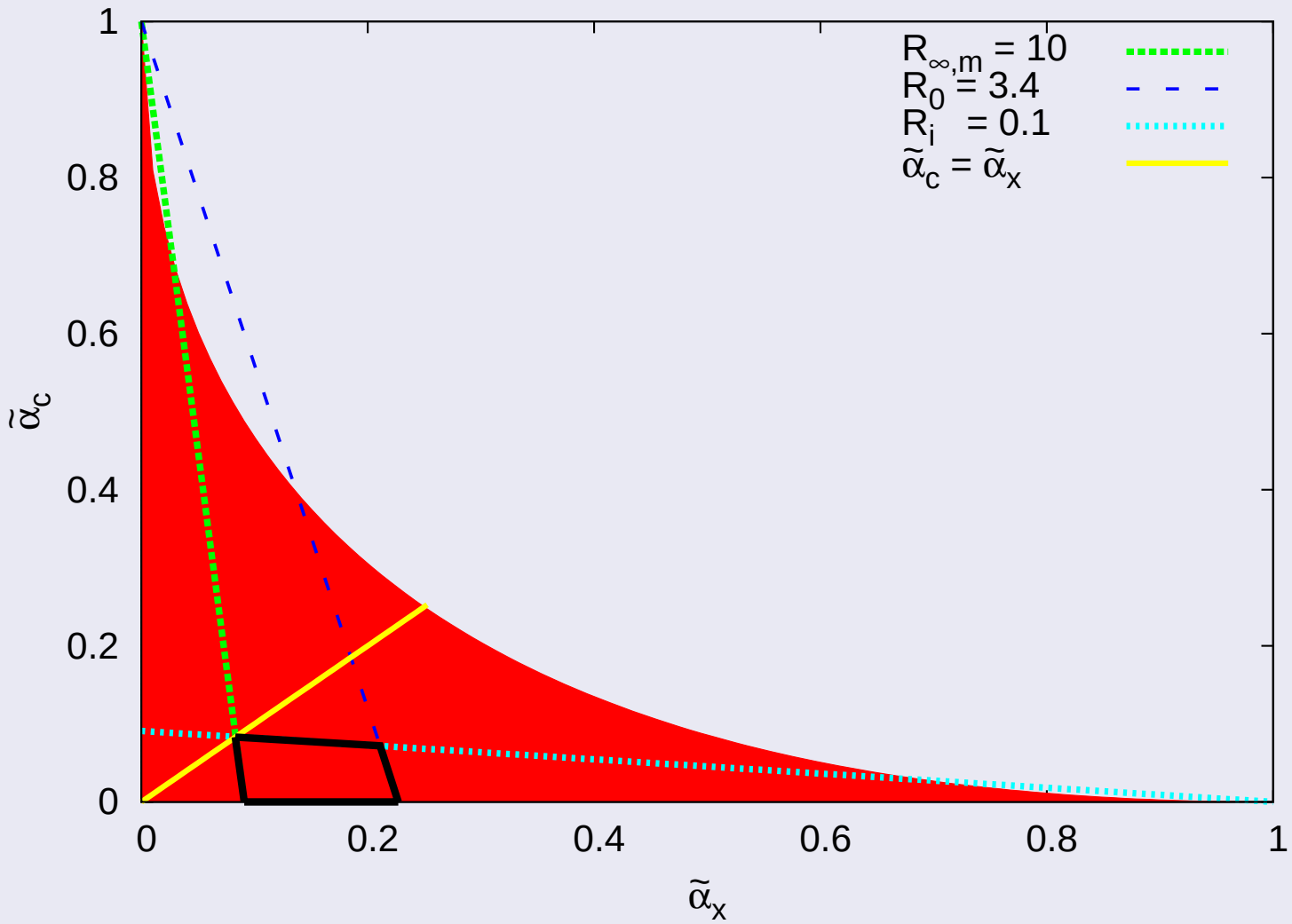
DE-DM ratio: exact solution

$$R' = -3\left[\tilde{\alpha}_x R^2 - \left(1 - \tilde{\alpha}_x - \tilde{\alpha}_c\right) R + \tilde{\alpha}_c\right] .$$

Model I: 3-dim analysis (w_x, α_x, α_c)[4]



Model I: *2-dim* analysis $(\tilde{\alpha}_x, \tilde{\alpha}_c)$[5]



Model II: Equations of motion[6, 7, 8]

$$
\begin{aligned}
x' &= 3x\,(1-x) - 3\left[\tilde{\alpha}_x(u)\,x + \tilde{\alpha}_c(u)\,y\right] , \\
y' &= -3xy + 3\left[\tilde{\alpha}_x(u)\,x + \tilde{\alpha}_c(u)\,y\right] , \\
u' &= \frac{3}{2(-w_x)}(1+w_x x)u(1-u) ,
\end{aligned}
$$

New phase-space variables

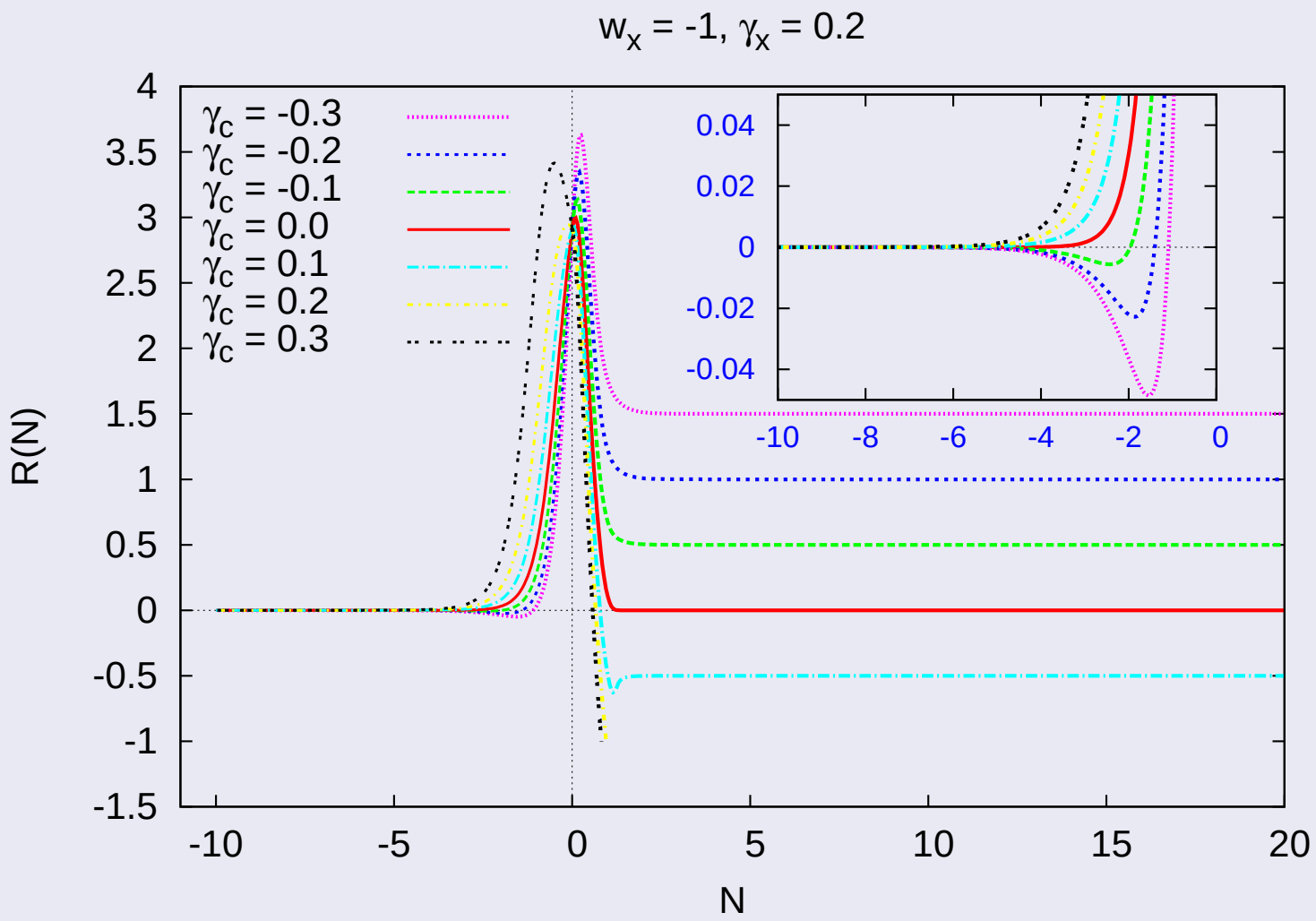
- Evolution variable

$$u := \frac{H_0}{H + H_0} ,$$

- Interaction parameters

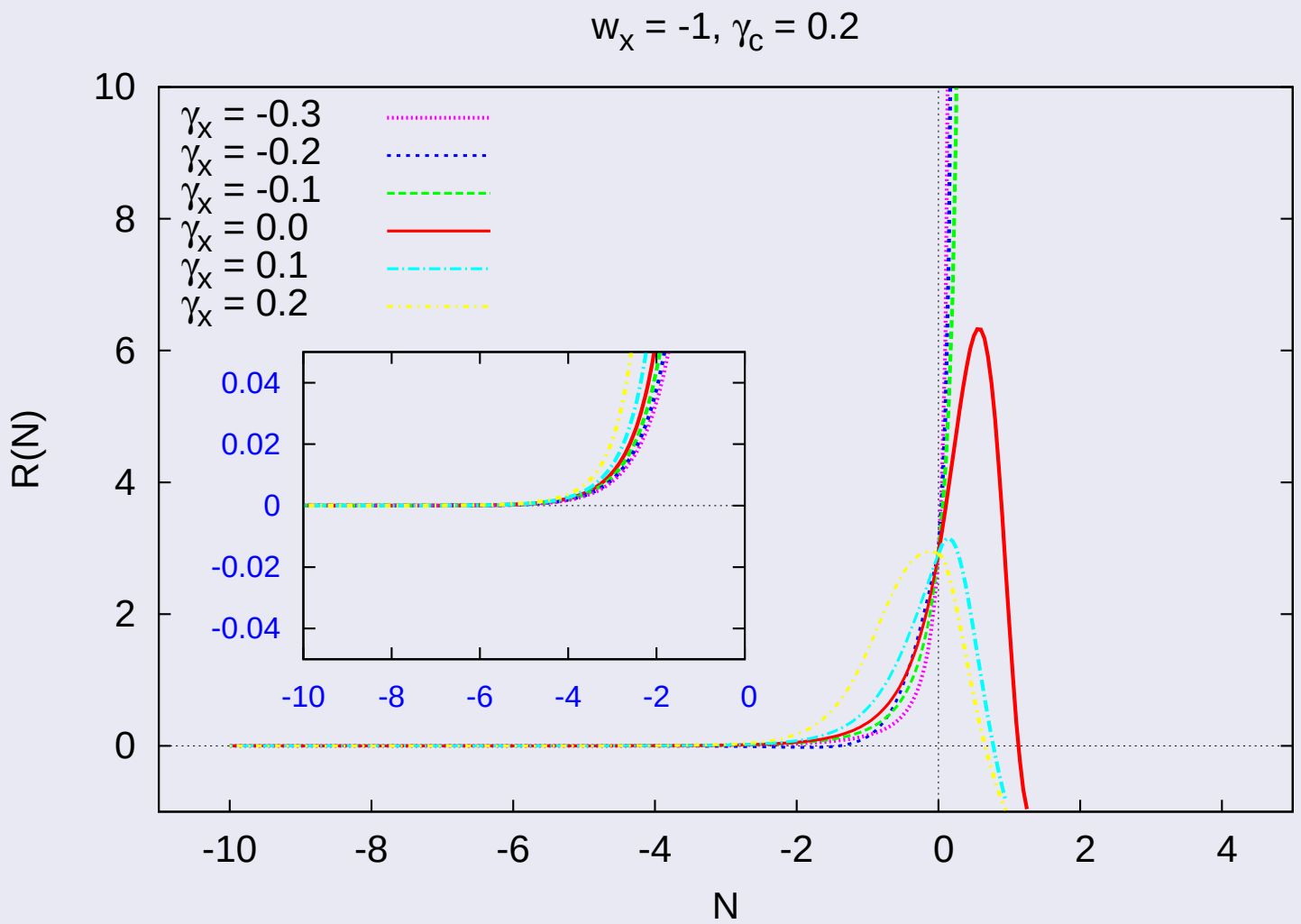
$$\tilde{\alpha}_I(u) := \frac{u}{1-u}\frac{\gamma_I}{(-w_x)} , \quad \gamma_I := \frac{\Gamma_I}{H_0} .$$

Model II: DE-DM ratio $R(N)$[5]



- The $\mathrm{sign}(\gamma_c)$ is the $\mathrm{sign}(\mathrm{DE})$ at early times; thus $\gamma_c > 0$
- A finite late-time attractor appears *only if* $(\gamma_x - \gamma_c) > 0$

Model II: DE-DM ratio $R(N)$[5]



- Positive DE at early times because $\gamma_c > 0$
- However, the only late-time attractor is $\rho_x/\rho_c \to -1$

Final remarks

- EoS are *isomorphic* to the CC case (degeneracy!)
- *Exhaustive* study of the interacting Models I and II
- Model I:
  - *Positive* interacting parameters $\tilde{\alpha}_x, \tilde{\alpha}_c > 0$
  - *Best* option is $\tilde{\alpha}_c = 0$, $\tilde{\alpha}_x > 0$
- Model II:
  - *Time-dependent* version of Model I
  - *Best* option is $\Gamma_c = 0$, $\Gamma_x > 0$
  - DE domination may be *transient*: DM domination *again* at *very* late times

Further reading I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