The w < -1 side of Dark Energy

Filippo Vernizzi - IPhT (CEA Saclay)

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with P. Creminelli, G. D'Amico and J. Noreña

Invisible Universe, 2 July 2009

The Universe accelerates

Since 1998, the Universe has been accelerating…

Friedman equations require negative pressure component

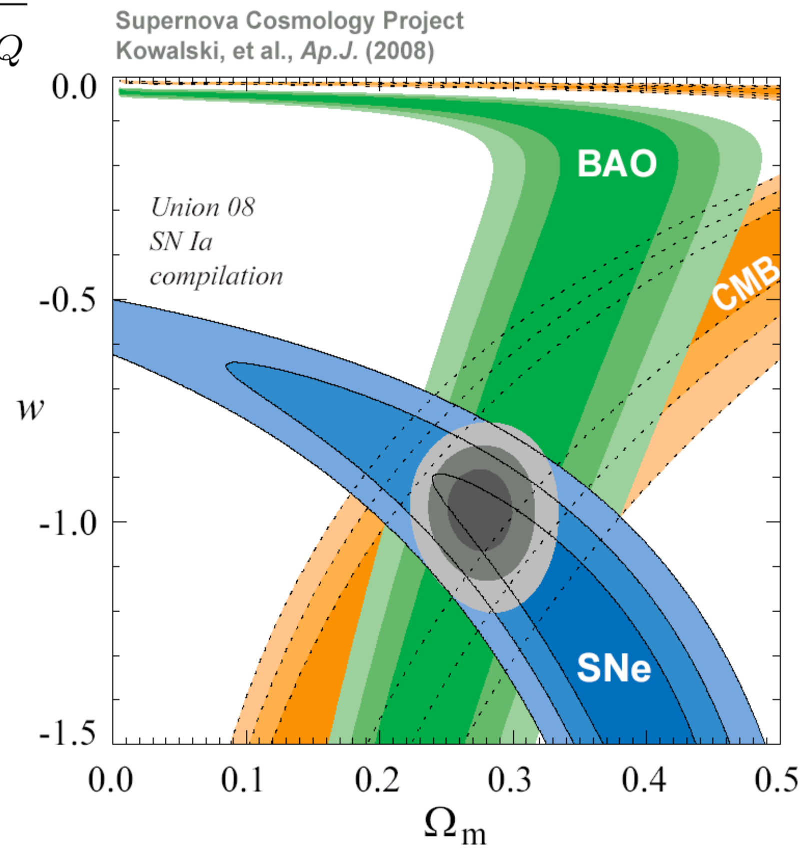
$$\frac{\ddot{a}}{a} = -\frac{4\pi G}{3}(\rho + 3p)$$

$$\rho + 3p = \rho_m + (1 + 3w_Q)\rho_Q, \qquad w_Q = \frac{p_Q}{\rho_Q}$$

- Data are converging towards w ≈ -1
- Λ is the simplest explanation: w = -1
- **Quintessence** (here a general single-field Dark Energy):

  1) Time-varying: $w_Q(t) \neq -1$

  2) Space-varying: not spatially homogeneous → speed of sound c_s^2



Is there a life below -1?

• Energy density of "standard" matter decreases with expansion: $\rho_Q \propto a^{-3(1+w_Q)}$

Ex: $\rho_m \propto a^{-3}, \quad \rho_r \propto a^{-4}, \quad \rho_\Lambda = \text{const}$

• Can we consistently have a scalar field with w < -1 (**phantom**)?

$$\mathcal{L} = -\frac{1}{2}(\partial\phi)^2 - V(\phi) \quad \longrightarrow \quad w > -1$$

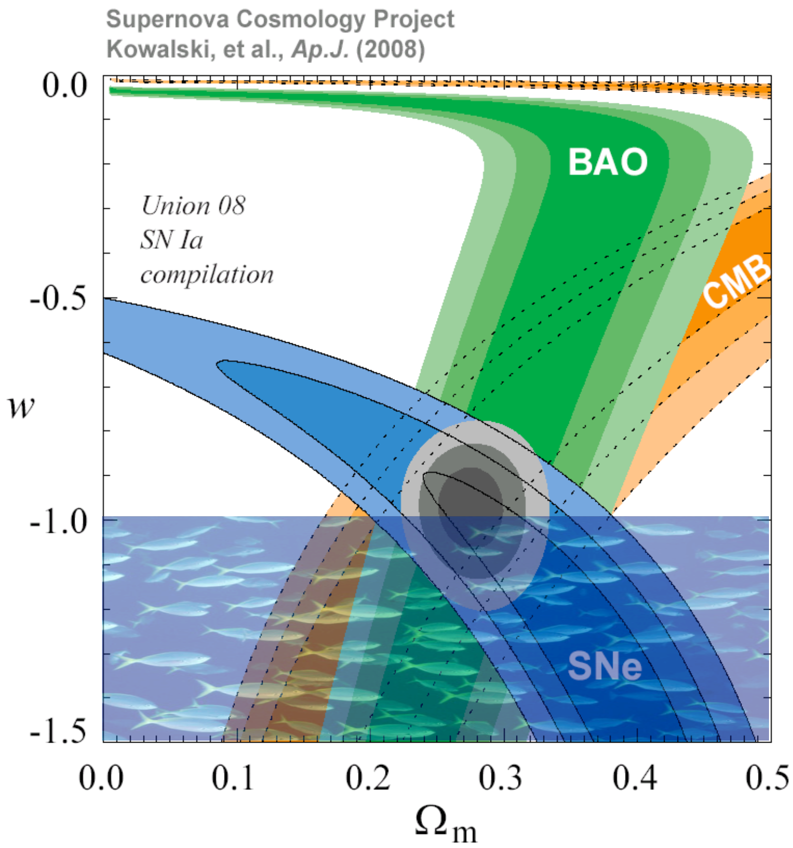
• Can we cross w = -1 (**phantom divide**)?

• What are the implication of w < -1 on the speed of sound of fluctuations?

$$c_s^2 = \left.\frac{d\rho}{dp}\right|_{T^0{}_i=0}$$

Ex: $\mathcal{L} = -\frac{1}{2}(\partial\phi)^2 - V(\phi) \quad \longrightarrow \quad c_s^2 = 1$

Phenomenologically important: **clustering of dark energy**



Canonical scalar field

Scalar field filling the Universe: $\mathcal{L} = -\frac{1}{2}(\partial\phi)^2 - V(\phi)$

Homogeneous Friedman metric and field: $ds^2 = -dt^2 + a(t)^2 d\vec{x}^2$ $\qquad \phi = \phi_0(t)$

$$\rho_Q = \frac{\dot{\phi}_0^2}{2} + V(\phi_0), \quad p_Q = \frac{\dot{\phi}_0^2}{2} - V(\phi_0)$$

$$w_Q = \frac{\dot{\phi}_0^2/2 - V(\phi_0)}{\dot{\phi}_0^2/2 + V(\phi_0)}$$

Phantom from canonical field?

Can we get **$w_Q < -1$ (phantom)**?

Scalar field filling the Universe: $\mathcal{L} = +\frac{1}{2}(\partial\phi)^2 - V(\phi)$ Caldwell '02

Homogeneous Friedman metric and field: $ds^2 = -dt^2 + a(t)^2 d\vec{x}^2$ $\phi = \phi_0(t)$

$$\rho_Q = -\frac{\dot\phi_0^2}{2} + V(\phi_0), \quad p_Q = -\frac{\dot\phi_0^2}{2} - V(\phi_0)$$

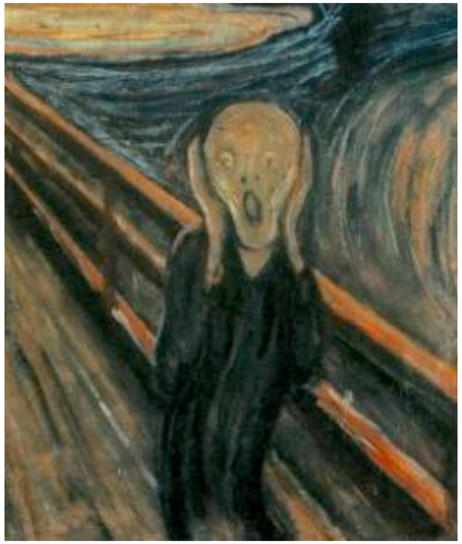
$$w_Q = \frac{-\dot\phi_0^2/2 - V(\phi_0)}{-\dot\phi_0^2/2 + V(\phi_0)}$$

Using the "strange" sign for $w_Q < -1$:

$$\mathcal{L} = +\frac{1}{2}(\partial\phi)^2 - V(\phi) = -\frac{1}{2}\dot\phi^2 + \frac{1}{2}(\vec\nabla\phi)^2 - V(\phi)$$

$w_Q < -1 \longrightarrow$ wrong sign of the kinetic term: **Ghost!**

Can we live with a ghost?

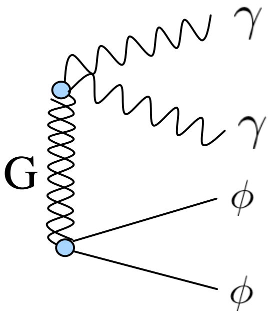


• **Classically**: Hamiltonian not bounded. Possibility of exchanging energy between positive and negative energy sectors.

Vikman '05

No pathology until linear theory remains valid.

• **Quantum mechanically**: Vacuum is unstable. Decay rate is infinite in any Lorentz invariant theory.



Cline, Jeon, Moore '03

$$\Gamma \sim \frac{\Lambda^8}{M_P^4}$$

We take a conservative approach: **forbid ghosts** (negative kinetic energy).

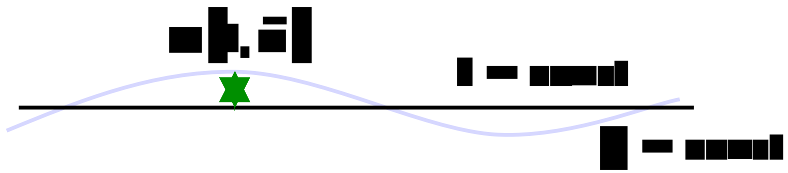
Can we get $w_Q < -1$ with positive kinetic energy?

Beyond a canonical field

K-essence: $S = \int \mathrm{d}^4 x \sqrt{-g}\, P(\phi, X)\ ,\qquad X = -g^{\mu\nu}\partial_\mu\phi\partial_\nu\phi$

Let us expand around: $ds^2 = -dt^2 + a(t)^2 d\vec{x}^2 \qquad \phi = \phi_0(t)$

Convenient parametrization: $\boxed{\phi(t,\vec{x}) = \phi_0(t + \pi(t,\vec{x}))}$



$P_X = \dfrac{\partial P}{\partial X}$ etc,...

$$S = \int \mathrm{d}^4 x\, a^3 \left[P_0 + \dot{P}_0 \pi + \frac{1}{2}\ddot{P}_0\pi^2 + 2 P_X X_0 \dot{\pi} + 2\,(P_X X_0)^{\cdot}\pi\dot{\pi} + P_X X_0 \left(\dot{\pi}^2 - \frac{(\nabla\pi)^2}{a^2}\right) + 2 P_{XX} X_0^2 \dot{\pi}^2 \right]$$

Action for perturbations. Making explicit the **background dependence:**

$$T_{\mu\nu} = -\frac{2}{\sqrt{-g}}\frac{\delta S}{\delta g^{\mu\nu}} \qquad\qquad \rho_Q = 2 X_0 P_X - P_0\ , \qquad p_Q = P_0$$

The action for perturbations

... integrating by parts + using background EOM

Metric perturbations in synchronous gauge: $\mathrm{d}s^2 = -\mathrm{d}t^2 + a^2(t)(\delta_{ij} + h_{ij})\mathrm{d}x^i \mathrm{d}x^j$

$$\boxed{S = \int \mathrm{d}^4x\, a^3 \left[\frac{1}{2}\left(\rho_Q + p_Q + 4M^4\right)\dot{\pi}^2 - \frac{1}{2}(\rho_Q + p_Q)\frac{(\nabla\pi)^2}{a^2} + \frac{3}{2}\dot{H}(\rho_Q + p_Q)\pi^2 - \frac{1}{2}(\rho_Q + p_Q)\dot{h}\pi\right]}$$

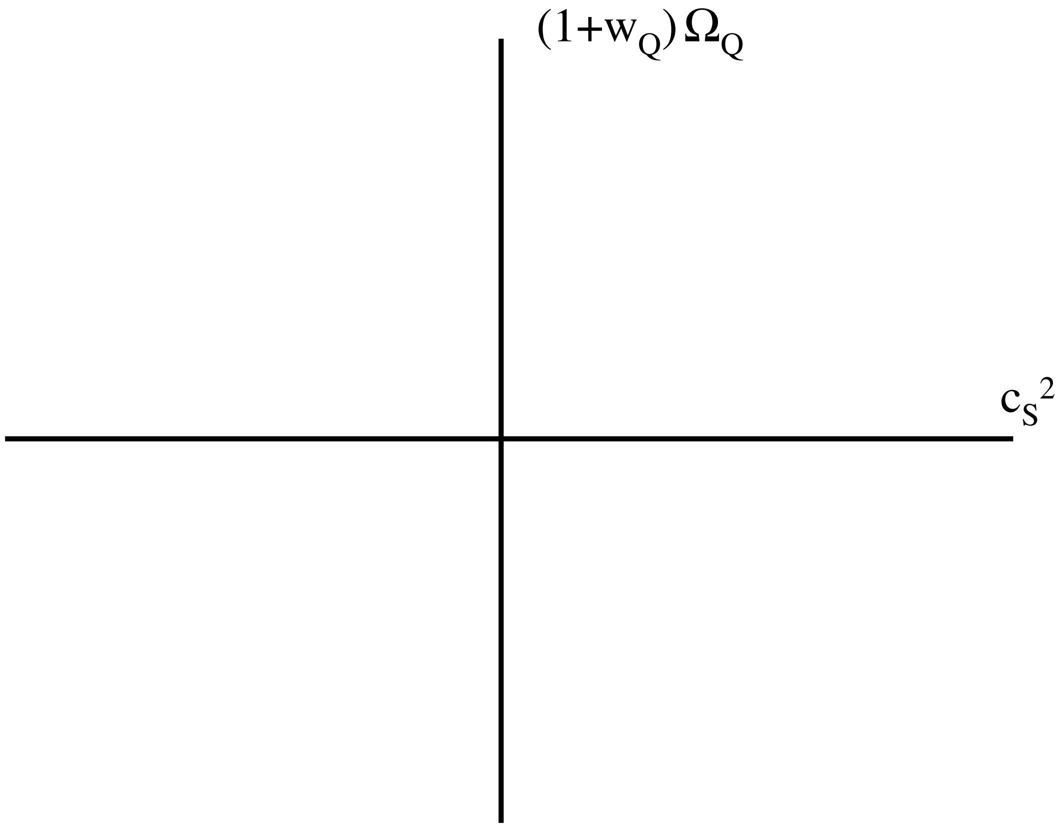
$M^4 \equiv P_{XX} X_0^2$ mass scale

$H = \frac{\dot{a}}{a} = \sqrt{\frac{8\pi G}{3}\rho}$ Hubble parameter

$(\rho_Q + p_Q)(t)$ and $M^4(t)$ are completely unconstrained and related to **observables**:

- **Equation of state:** $(1 + w_Q)\Omega_Q = \frac{\rho_Q + p_Q}{\rho}$
- **Speed of sound:** $c_s^2 = \frac{\rho_Q + p_Q}{\rho_Q + p_Q + 4M^4}$

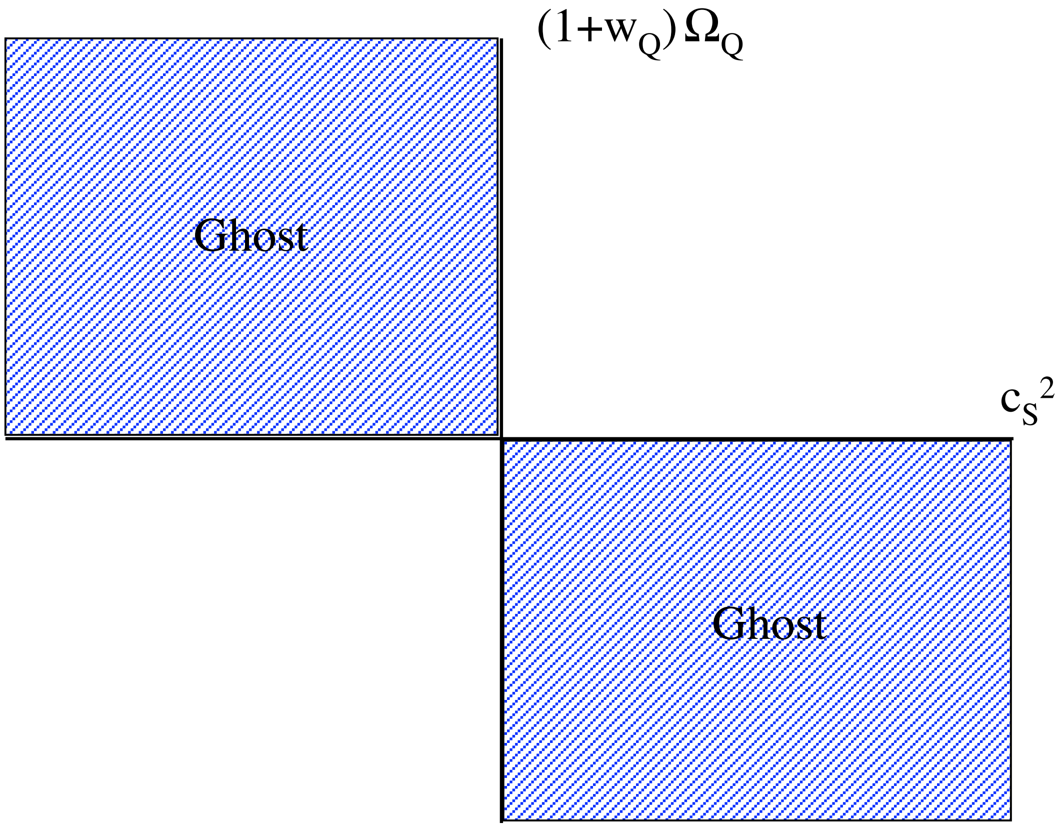
Quintessential plane



Let us study the different theoretical constraints on quintessence:

$$\frac{1}{2}\left(\rho_Q + p_Q + 4M^4\right)\dot{\pi}^2 - \frac{1}{2}(\rho_Q + p_Q)\frac{(\nabla\pi)^2}{a^2} \longrightarrow c_s^2 = \frac{\rho_Q + p_Q}{\rho_Q + p_Q + 4M^4}$$

No ghost and c_S^2

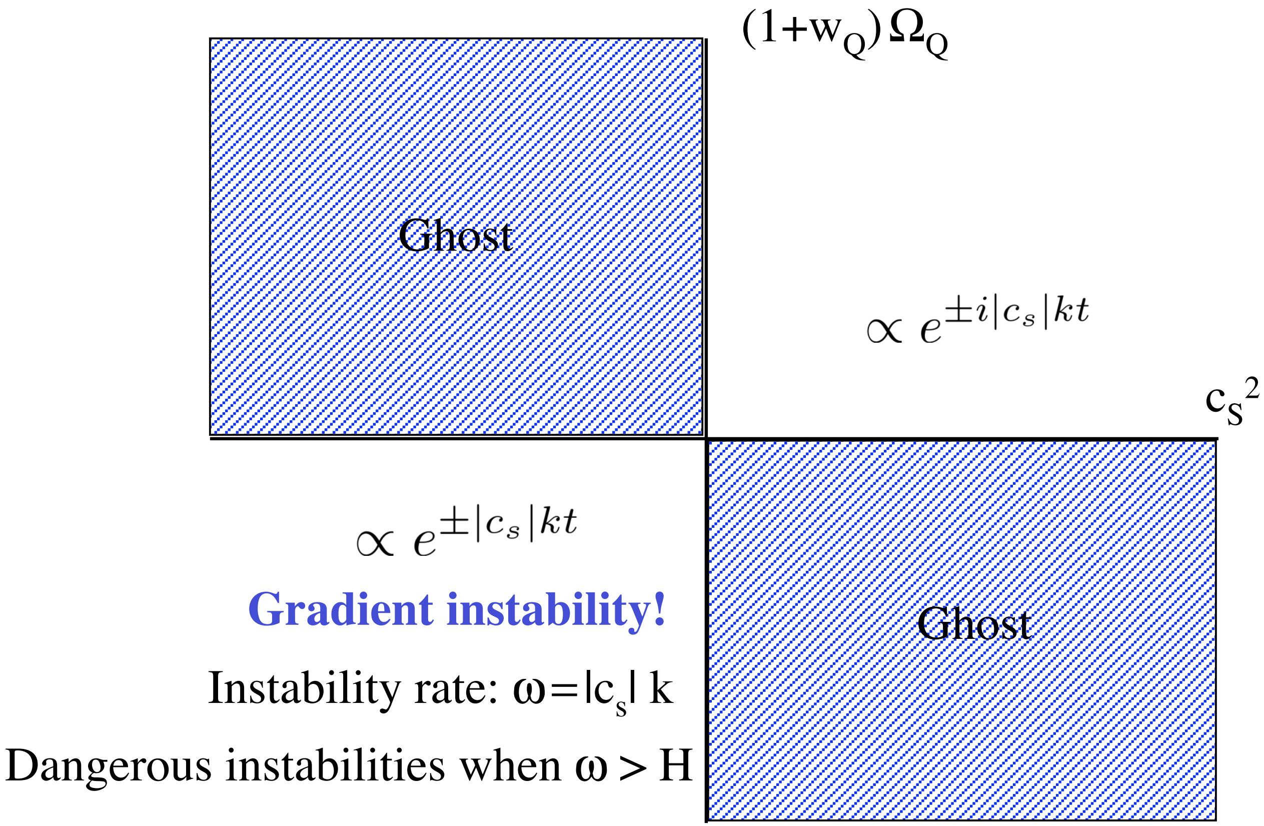


$$\frac{1}{2}\overbrace{\left(\rho_Q + p_Q + 4M^4\right)}^{>0}\dot{\pi}^2 - \frac{1}{2}(\rho_Q + p_Q)\frac{(\nabla\pi)^2}{a^2} \longrightarrow c_s^2 = \frac{\rho_Q + p_Q}{\rho_Q + p_Q + 4M^4}$$

No ghost even for $w_Q < -1$ but **c_S^2 has the same sign of $1+w_Q$!**

w < -1 and gradient instabilities

Wise et al '04
Rattazzi et al '05



$$\frac{1}{2}\left(\rho_Q+p_Q+4M^4\right)\dot{\pi}^2-\frac{1}{2}(\rho_Q+p_Q)\frac{(\nabla\pi)^2}{a^2} \longrightarrow c_s^2=\frac{\rho_Q+p_Q}{\rho_Q+p_Q+4M^4}$$

It is difficult to violate the Null Energy Condition: $T_{\mu\nu}n^\mu n^\nu \geq 0$

Higher derivative

Close to $c_s^2 = 0$ we have to consider also higher derivative operators

For instance: $$-\frac{\bar{M}^2}{2}(\Box\phi)^2$$

$$\rightarrow \quad S = \int d^4x\, a^3 \left[\frac{1}{2}\left(\rho_Q + p_Q + 4M^4\right)\dot{\pi}^2 - \frac{1}{2}(\rho_Q + p_Q)\frac{(\nabla\pi)^2}{a^2} - \bar{M}^2\left(\frac{\nabla^2\pi}{a^2}\right)^2\right]$$

Higher time derivative terms can be neglected for $\omega \ll M \sim \bar{M}$

$$\bar{M}^2\ddot{\pi}^2 \sim \left(\frac{\omega}{M}\right)^2 M^4\omega^2\pi^2 \ll M^4\omega^2\pi^2 \sim M^4\dot{\pi}^2$$

No additional degrees of freedom

Dispersion relation becomes:

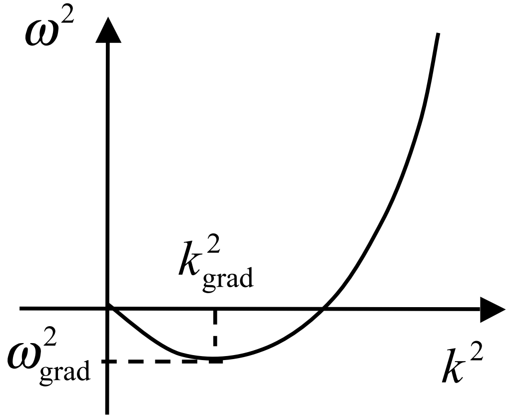
$$\left(\rho_Q + p_Q + 4M^4\right)\omega^2 - (\rho_Q + p_Q)\frac{k^2}{a^2} - \bar{M}^2\frac{k^4}{a^4} = 0$$

Can the higher derivative term k^4 **cure the instabilities**?

Stability analysis

Creminelli, Luty, Nicolis and Senatore '06
Mukohyama '06

Gradient instability: $\left(\rho_Q + p_Q + 4M^4\right)\omega^2 - (\rho_Q + p_Q)\frac{k^2}{a^2} - \bar{M}^2\frac{k^4}{a^4} = 0$



$$\omega^2_{\rm grad} \simeq -\frac{(\rho_Q + p_Q)^2}{\bar{M}^2 M^4}$$

$$|\omega_{\rm grad}| \lesssim H$$

+ Jeans instability: $|\omega_{\rm Jeans}| \lesssim H$

Stability window

$$\boxed{-(1+w_Q)\Omega_Q \lesssim \frac{\bar{M}M^2}{HM_{\rm Pl}^2} \lesssim 1}$$

gradient instab. Jeans instab.

The scales M are the cutoff of my theory; M > (.1mm)$^{-1}$ $\quad -c_s^2 \lesssim \left(\frac{H_0}{M_{\rm Pl}}\right)^{1/2} \sim 10^{-30}$

Extremely small c_s^2

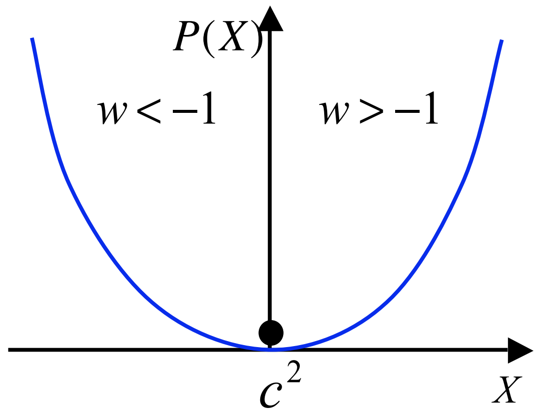
Ghost Condensate

Arkani-Hamed et al '03

Consider the limit $\rho_Q + p_Q = 0$: **Enhanced symmetry**: $\pi \to \pi + \text{const}$

Shift symmetry : $\phi \to \phi + \text{const}$ $\quad \mathcal{L} = \sqrt{-g}\, M^4 P(X)\,, \qquad X \equiv -g^{\mu\nu}\partial_\mu\phi\partial_\nu\phi$

Equation of motion: $\partial_t\left(a^3\dot{\phi}P_{,X}\right) = 0$



In an expanding Universe, we expect $\dot{\phi} \to 0$.

Another possibility:

$\phi = ct, \quad P_{,X}|_{X=c^2} = 0, \quad (\text{with } P_{,XX} > 0)$

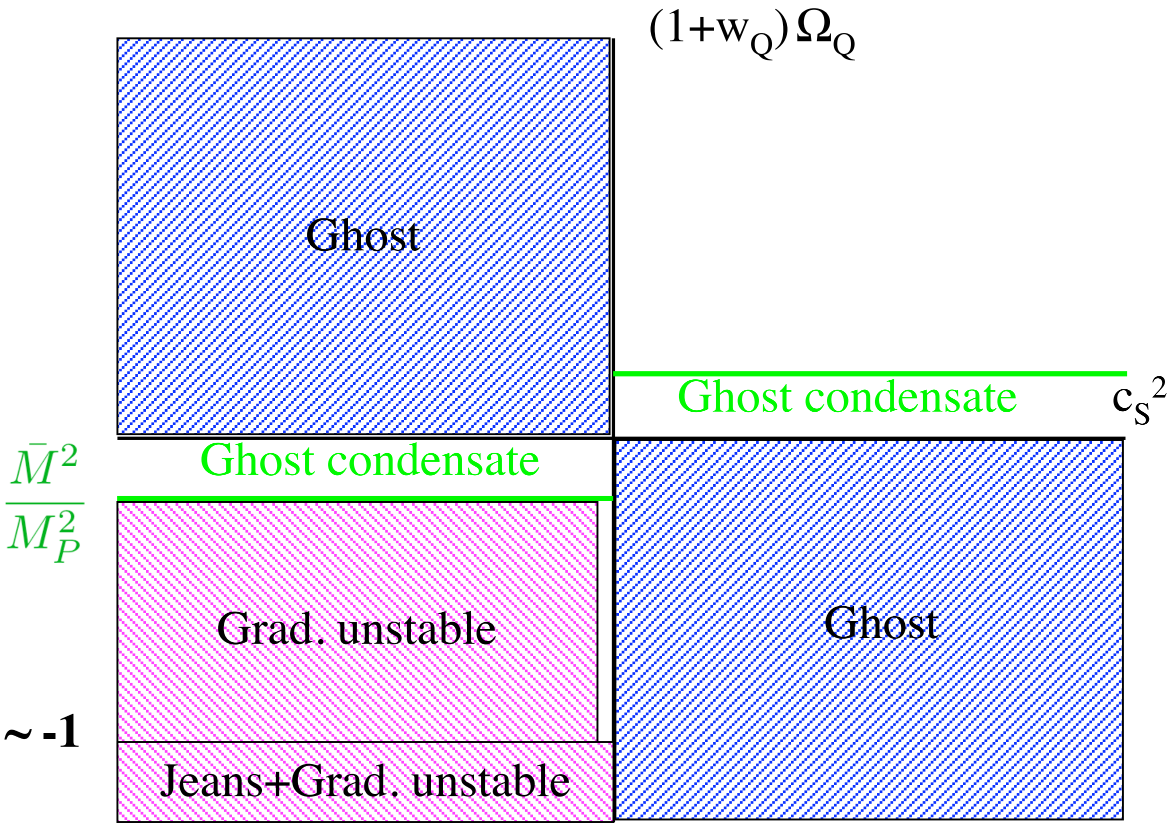
Adding the higher derivative operator the action becomes:

$$S = \int d^4x\, a^3\left[2M^4\dot{\pi}^2 - \frac{\bar{M}^2}{2}\left(\frac{\dot{h}}{2} - \frac{\nabla^2\pi}{a^2}\right)^2\right]$$

Dispersion relation:
$\omega \propto k^2$

A small breaking of the shift symmetry (and thus a small c_s^2) is **technically natural**

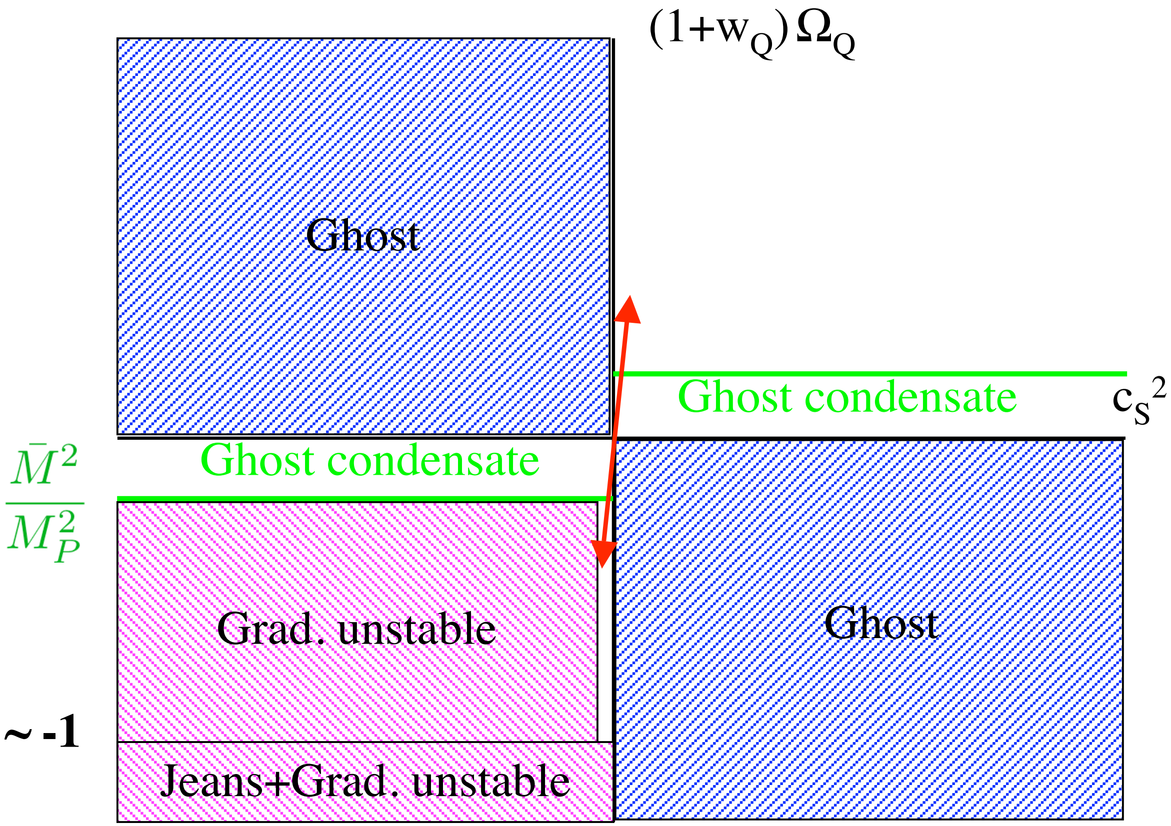
Back to the plane with higher derivatives!



$$S=\int \mathrm{d}^4x\,a^3\left[\frac{1}{2}\left(\rho_Q+p_Q+4M^4\right)\dot{\pi}^2-\frac{1}{2}(\rho_Q+p_Q)\frac{(\nabla\pi)^2}{a^2}-\bar{M}^2\left(\frac{\nabla^2\pi}{a^2}\right)^2\right]$$

There is life at w<-1, but it makes no noise: **extremely small c_s^2!**

Crossing the phantom divide



$$S_Q = \frac{1}{2}\int d^3x dt\ a^3\Big[4M^4\dot{\pi}^2 + \cancel{(\rho_Q + p_Q)}\dot{\pi}^2 - \cancel{(\rho_Q + p_Q)}\frac{(\nabla\pi)^2}{a^2} + 3\dot{H}\cancel{(\rho_Q + p_Q)}\pi^2$$

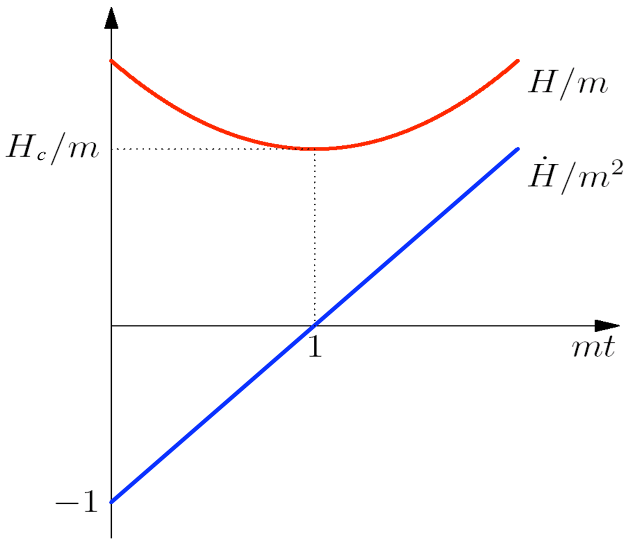
$$-\cancel{(\rho_Q + p_Q)}\dot{h}\pi - \bar{M}^2\big(\frac{\dot{h}}{2} - \frac{\nabla^2\pi}{a^2}\big)^2\Big]$$

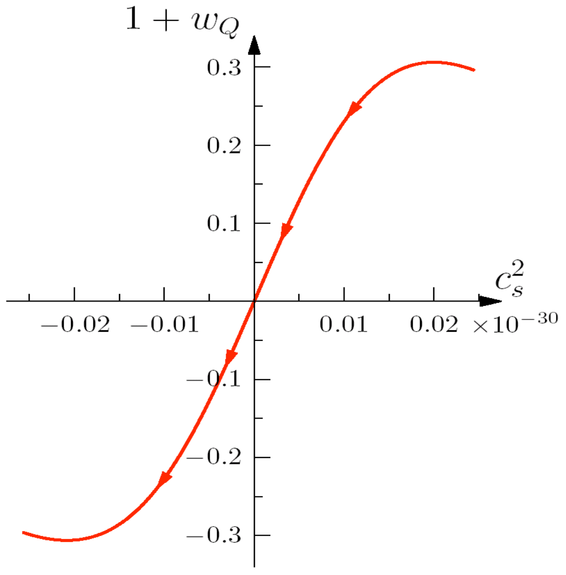
Nothing strange happens when you cross $w_Q = -1$

For example...

$$P(X,\phi) = -3M_{\rm Pl}^2 H^2(\phi) - M_{\rm Pl}^2 \dot{H}(\phi)(X+1) + \frac{1}{2}M^4(\phi)(X-1)^2$$

No other energy components





- The GC strip is very tiny. Higher order operators play a role only on very small scales.
- Effectively **w_Q= -1 is crossed by a k-essence with $c_S^2 = 0$**
- Numerical recipe. When comparing with data $w_Q(t)$ going through w_Q= -1, set $c_S^2 = 0$

Phenomenology with Dark Matter

Quintessence is coupled to DM through the metric: it is dragged in the DM potential wells

Since $c_s^2 = 0$, it clusters on all scales (contrarily to standard quintessence):

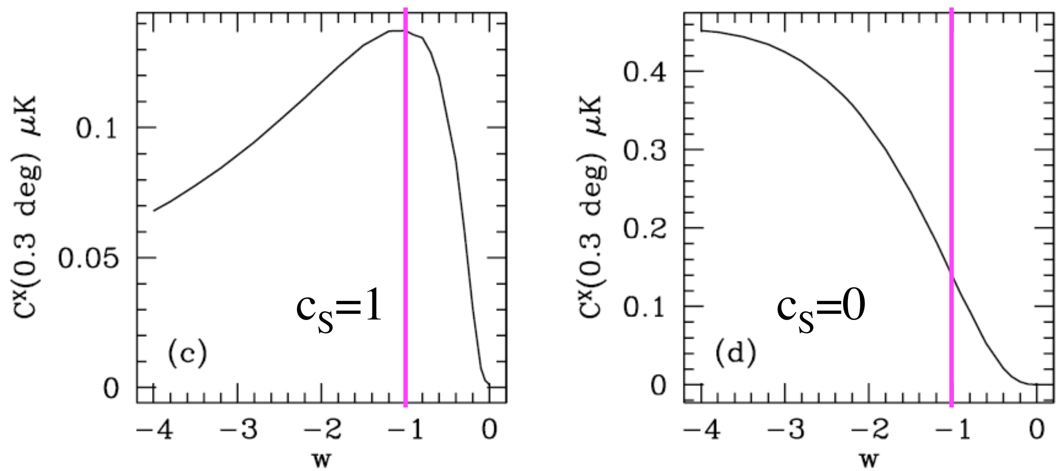
$$1/k_{\mathrm{DE\ s.h.}} = a \int_0^t \frac{c_s}{a} dt \simeq 2 c_s H_0^{-1}$$

Dark energy escapes (!) from DM potential wells: $\delta_Q \simeq \dfrac{1+w}{1-3w} \delta_{\mathrm{DM}}$

Is it possible to experim distinguish $c_s = 0$ from $c_s = 1$? Until which value of $1+w_Q$?

ISW–galaxy correlation: distinction possible for $1+w > 0.05$ (only positive $1+w$ considered)

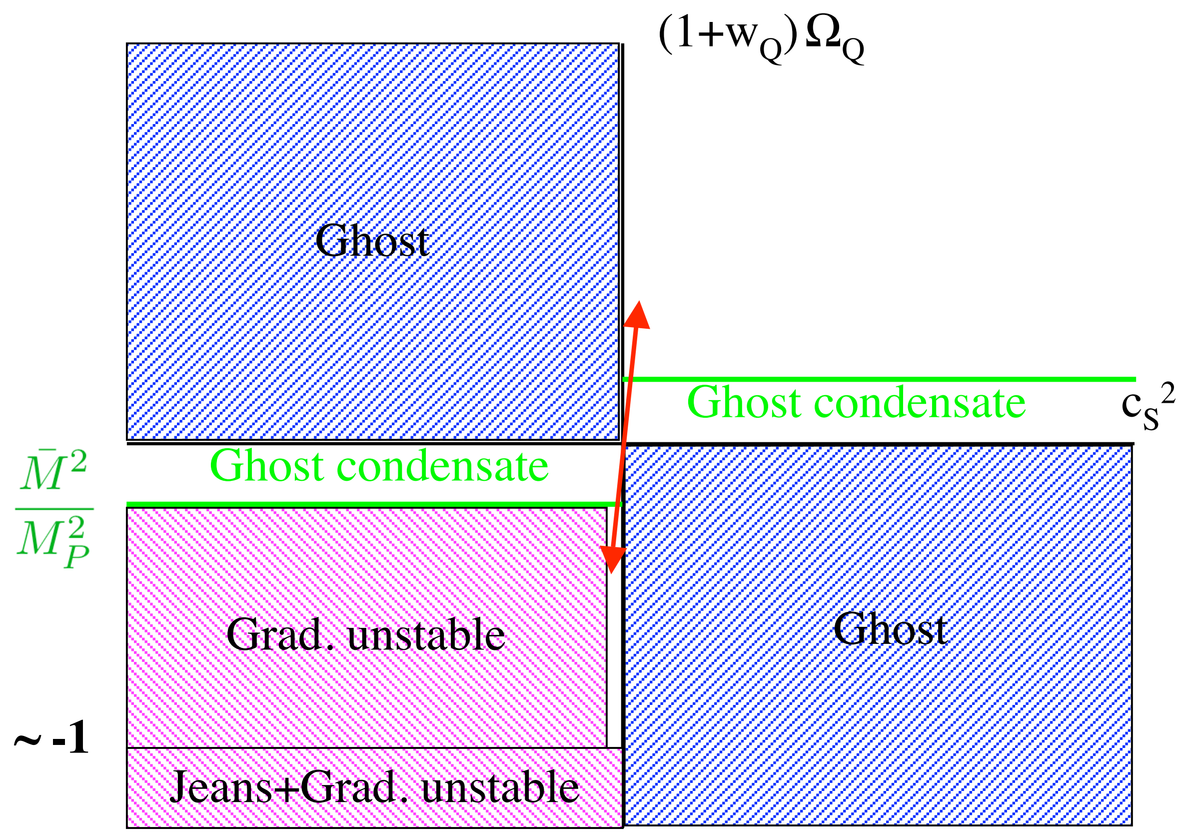
Corasaniti, Giannantonio, Melchiorri '05



Conclusions

- General framework to study single field quintessence models
- Higher derivative terms can stabilize the $w_Q < -1$ region
  Phenomenology is the same as **k-essence models with $c_S^2 = 0$**
- **Quintessential plane**
- The **phantom divide can be smoothly crossed** if one sets $c_S^2 = 0$
- Phenomenology of models with $c_S^2 = 0$ vs $c_S^2 = 1$ must be further explored

Quintessential plane

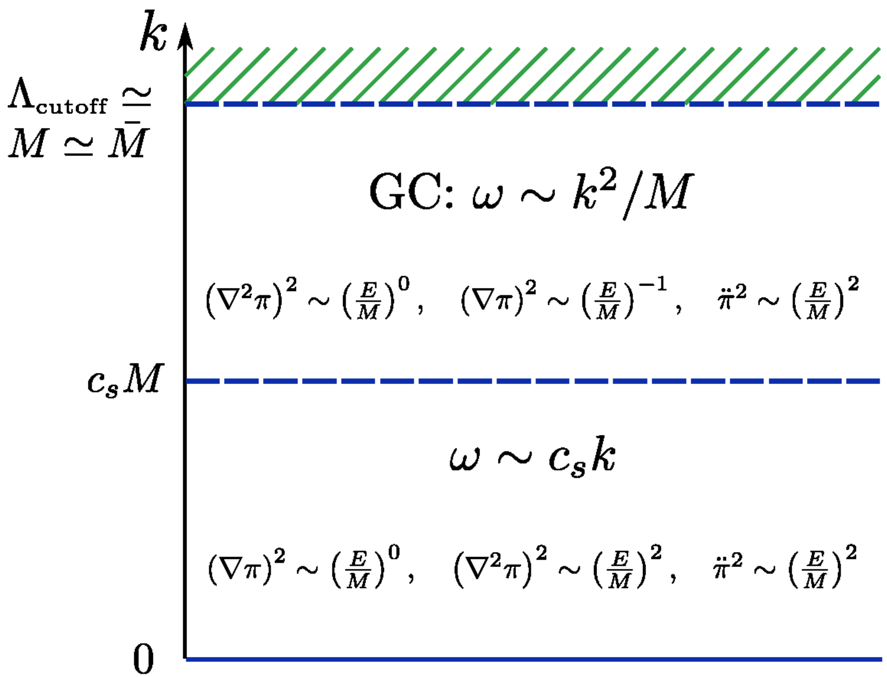


$$S_Q = \frac{1}{2}\int d^3x dt\ a^3 \Big[4M^4\dot{\pi}^2 + (\rho_Q + p_Q)\dot{\pi}^2 - (\rho_Q + p_Q)\frac{(\nabla\pi)^2}{a^2} + 3\dot{H}(\rho_Q + p_Q)\pi^2$$
$$-(\rho_Q + p_Q)\dot{h}\pi - \bar{M}^2\Big(\frac{\dot{h}}{2} - \frac{\nabla^2\pi}{a^2}\Big)^2\Big]$$

Scaling in EFT

Arkani-Hamed et al ‘03, Simon ’91, Weinberg ‘08

$$S = \frac{M^4}{2} \int d^3x dt \left[\dot{\pi}^2 - c_s^2 (\nabla \pi)^2 - \frac{(\nabla^2 \pi)^2}{M^2} + \dots \right.$$



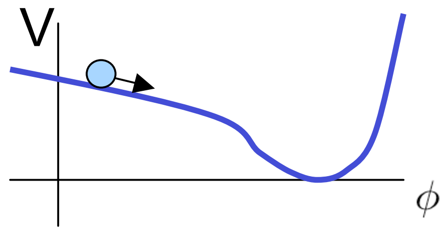
$$\ddot{\pi}^2 \sim c_s^4 (\nabla^2 \pi)^2$$

- scaling transformations:

$$E \to sE \,, \qquad t \to s^{-1} t \,,$$
$$x \to s^{-1/2} x \,, \qquad \pi \to s^{1/4} \pi$$

$$E \to sE \,, \qquad t \to s^{-1} t \,,$$
$$x \to s^{-1} x \,, \qquad \pi \to s^{1} \pi$$

A more general approach



Usual approach to quintessence/inflation:

1. Take a Lagrangian for a scalar $\mathcal{L}(\phi, \partial_\mu \phi, \Box\phi \ldots)$
2. Solve EOM of the scalar + FRW. Find an accelerating solution $\ddot{a} > 0$

$$\phi = \phi_0(t) \qquad ds^2 = -dt^2 + a^2(t) d\vec{x}^2$$

3. Study perturbations around this solution to work out predictions

We want to **focus directly on the theory of perturbations** around the accelerating solution

- Time diffeomorphisms are broken: $t \to t + \xi^0(t, \vec{x}) \qquad \delta\phi \to \delta\phi + \dot{\phi}_0(t)\xi^0$
- In unitary gauge $\phi(t, \vec{x}) = \phi_0(t)$ the scalar mode is eaten by the graviton:

  3 degrees of freedom. Like in a broken gauge theory.
- The most generic action in unitary gauge

$$S = \int \mathrm{d}^4x \sqrt{-g} \Big[\frac{M_{\rm Pl}^2}{2} R + \mathcal{L}_m + p_Q - \frac{1}{2}(\rho_Q + p_Q)(g^{00}+1) + \frac{M^4(t)}{2}(g^{00}+1)^2$$
$$- \frac{\bar{M}^2(t)}{2}\delta K^2 - \frac{\hat{M}(t)^3}{2}\delta K (g^{00}+1) \Big] .$$

The action for perturbations

... integrating by parts + using background EOM

Metric perturbations in synchronous gauge: $\mathrm{d}s^2 = -\mathrm{d}t^2 + a^2(t)(\delta_{ij} + h_{ij})\mathrm{d}x^i \mathrm{d}x^j$

$$\boxed{S = \int \mathrm{d}^4x\, a^3 \left[\frac{1}{2}\left(\rho_Q + p_Q + 4M^4\right)\dot{\pi}^2 - \frac{1}{2}(\rho_Q + p_Q)\frac{(\nabla\pi)^2}{a^2} + \frac{3}{2}\dot{H}(\rho_Q + p_Q)\pi^2 - \frac{1}{2}(\rho_Q + p_Q)\dot{h}\pi\right]}$$

Perturbations cannot be switched off if $\rho_Q + p_Q \neq 0$

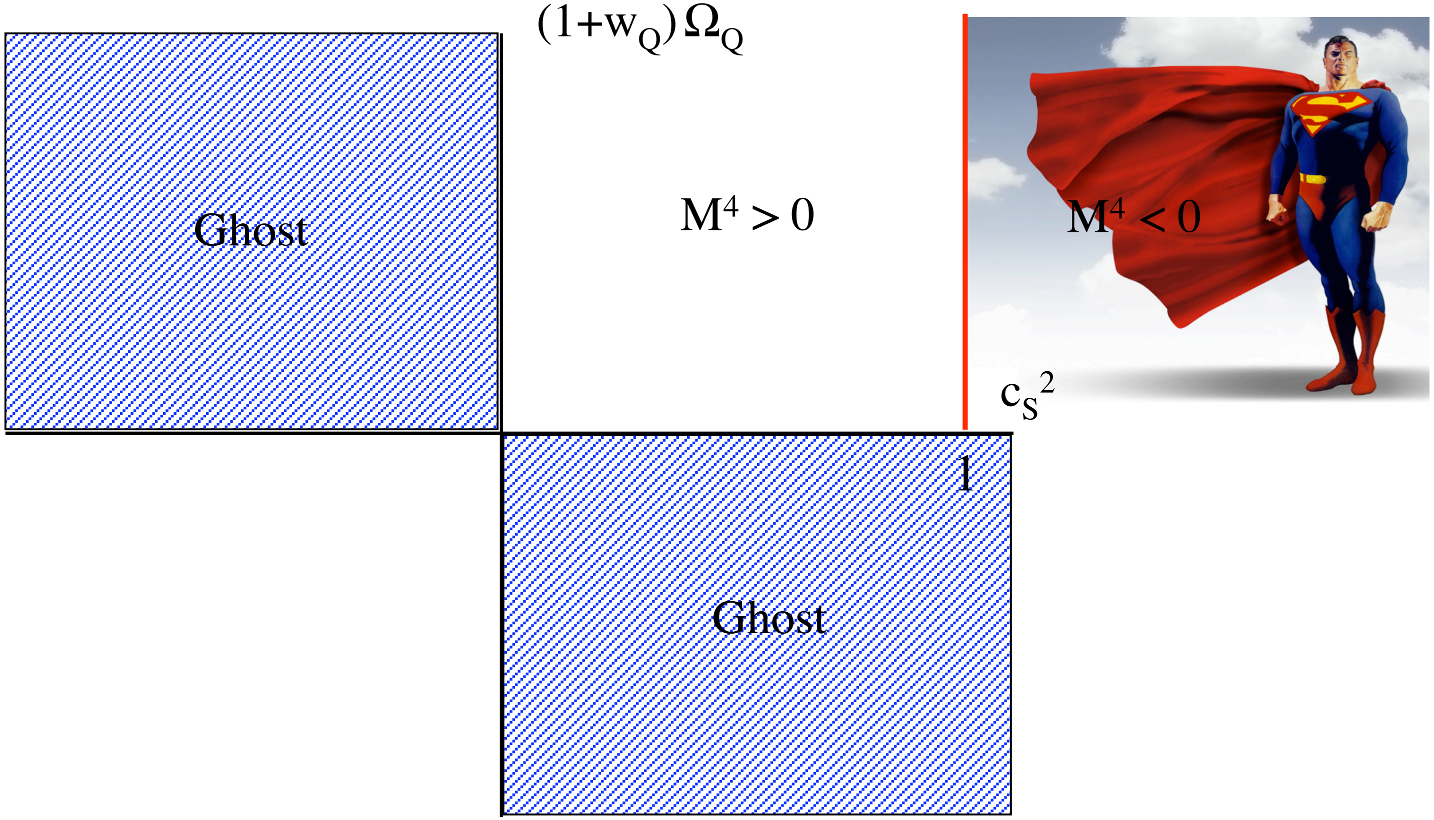
One can always find P(ϕ, X):

$$P(\phi, X) = \frac{1}{2}(p_Q - \rho_Q)(\phi) + \frac{1}{2}(\rho_Q + p_Q)(\phi)X + \frac{1}{2}M^4(\phi)(X-1)^2$$

ϕ=t and the correct $\rho_Q(t)$ and $p_Q(t)$

No field redefinition ambiguities: $\phi \to \tilde{\phi}(\phi)$

Faster than light?



$$\frac{1}{2}\left(\rho_Q + p_Q + 4M^4\right)\dot{\pi}^2 - \frac{1}{2}(\rho_Q + p_Q)\frac{(\nabla\pi)^2}{a^2} \longrightarrow c_s^2 = \frac{\rho_Q + p_Q}{\rho_Q + p_Q + 4M^4}$$

$c_S{}^2 > 1$ ($M^4 < 0$) implies a non-Lorentz invariant UV completion

Arkani-Hamed etal '06
Babichev etal '07

Stability analysis

Creminelli, Luty, Nicolis and Senatore ‘06

Jeans instability: taking into account the mixing with gravity gives rise to a sort of Jeans like instability

$$S = \int \mathrm{d}^4x \left[2M^4 \dot{\pi}^2 - \frac{\bar{M}^2}{2} \left(\frac{\dot{h}}{2} - \nabla^2 \pi \right)^2 \right] \longrightarrow \ddot{\pi} + \frac{\bar{M}^2}{4M^4} \nabla^4 \pi = \frac{\bar{M}^2}{8M^4} \nabla^2 \dot{h}$$

Solving for h: $$\ddot{\pi} + \frac{\bar{M}^2}{4M^4} \nabla^4 \pi = -\frac{\bar{M}^2}{2M_{\mathrm{Pl}}^2} \nabla^2 \pi \longrightarrow \omega_{\mathrm{Jeans}}^2 \simeq -\left(\frac{\bar{M} M^2}{M_{\mathrm{Pl}}^2} \right)^2$$

Phenomenology with Dark Matter

Quintessence is coupled to DM through the metric:

$$S_Q = \frac{1}{2}\int d^3x dt\, a^3 \Big[4M^4\dot\pi^2 + (\rho_Q + p_Q)\dot\pi^2 - (\rho_Q + p_Q)\frac{(\nabla\pi)^2}{a^2} + 3\dot H(\rho_Q + p_Q)\pi^2$$
$$-(\rho_Q + p_Q)\dot h\pi - \bar M^2\Big(\frac{\dot h}{2} - \frac{\nabla^2\pi}{a^2}\Big)^2\Big]$$

Coupling to the metric

Two interesting limits:

1) k-essence with $c_s \sim 0$

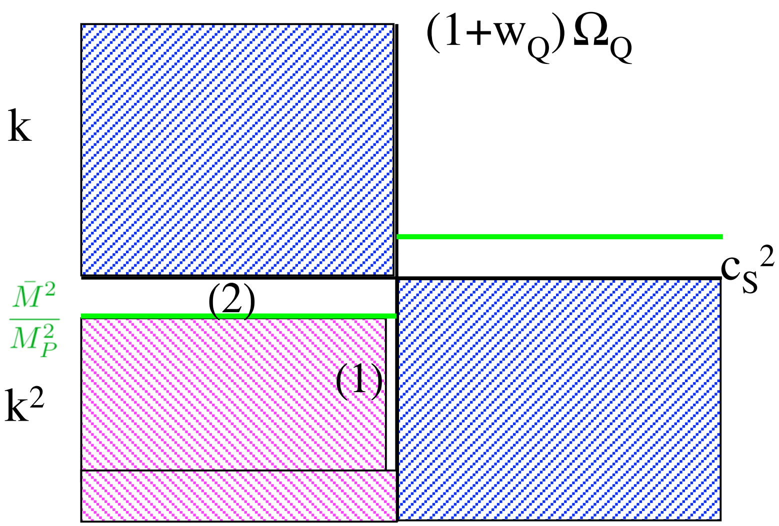
For cosmo scales: $\omega = c_s k$

$$(1+w_Q)\Omega_Q \gg \frac{\bar M^2}{M_P^2}$$

2) Ghost condensate limit

For cosmo scales: $\omega \sim k^2$

$$(1+w_Q)\Omega_Q \ll \frac{\bar M^2}{M_P^2}$$



The phantom divide

- What happens to perturbations when $w_Q = -1$?

Fluid equations:

e.g. Bean and Doré 03

$$\dot\delta = -(1+w)\left\{\left[k^2 + 9\mathcal{H}^2(c_s^2 - c_a^2)\right]\frac{\theta}{k^2} + \frac{\dot h}{2}\right\} - 3\mathcal{H}(c_s^2 - w)\delta$$

$$\frac{\dot\theta}{k^2} = -\mathcal{H}(1-3c_s^2)\frac{\theta}{k^2} + \frac{c_s^2}{1+w}\delta\,.$$

$$\theta \equiv i k^j v_j \qquad c_a^2 \equiv \dot p/\dot\rho = w - \frac{1}{3H}\frac{\dot w}{1+w}$$

$$c_s^2 \equiv \delta\hat p/\delta\hat\rho \qquad T_i^0 = 0$$

The one given by scalar kinetic term

- The phantom psychosis:

- 1st divergence: $c_a^2 \to \infty$ Hu ‘04 — c_a^2 is not observable

- 2nd divergence: in θ equation Caldwell and Doran ‘05 — $c_s^2 \to 0$ at the crossing

- Instability: $c_s^2 \to 0 \quad \Rightarrow \quad c_s^2 < 0$ — Higher derivative terms

Vikman 04, Caldwell and Doran 05, Kunz and Sapone ‘06

Ghost Condensate vs Λ

Ghost condensate limit   For cosmo scales: ω~k^2 $(1+w_Q)\Omega_Q \ll \frac{\bar{M}^2}{M_P^2}$

$$S_Q = \frac{1}{2}\int d^3x dt\, a^3 \left[4M^4\dot{\pi}^2 - \bar{M}^2\left(\frac{\dot{h}}{2} - \frac{\nabla^2\pi}{a^2}\right)^2\right]$$

The scalar degree of freedom does not disappear even for $1+w_Q=0$

$$\ddot{\pi} + 3H\dot{\pi} = -\frac{\bar{M}^2}{12M^4M_P^2}\frac{\nabla^2\delta\rho_{\rm DM}}{Ha^2}$$

The driving of DM is not suppressed by $1+w_Q$ in this limit

$$\delta\rho_Q = 4M^4\dot{\pi} \sim \frac{\bar{M}^2}{M_P^2}\delta\rho_{\rm DM} \lll \delta\rho_{\rm DM}$$

No relevant perturbation!

The ghost condensate is a modification of gravity, but only on very short scales
Irrelevant cosmologically